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Abstract

We consider theories of interest as they relate to what we will call the Gesell-Keynes (GK) theory which is essentially a real theory of capital accumulation with a monetary constraint that arises because of a liquidity return on holding money. We give brief presentations of the neoclassical theory (Fisher), growth models (Solow and Ramsey) and monetary models of growth (Tobin and Sidrauski). The GK theory is a neoclassical theory of the return on capital with a liquidity return on money hindering the path to the growth theory steady state. The monetary growth models do address the GK liquidity return on holding money and we will highlight a contrast here between how Gesell and Keynes relate to these with different models of the economy and their different solutions to the monetary constraint.

1 Introduction

The main goal of this paper is to consider theories of interest in relation and contrast to what we will call the Gesell-Keynes (GK) theory. This theory contains within it both a real theory and a monetary theory of interest. The real part is that interest arises from the scarcity of capital and that in a system where there are no impediments to capital accumulation, the interest rate will come down to zero once capital has accumulated to the point where the return on it is zero. The impediment to this is money as it competes with capital as a medium of saving at low interest rates with a liquidity return, a *liquidity trap.* We will begin with the neoclassical theory of interest which can be traced back to Fisher and Böhm-Bawerk with a presentation of Fisher (1930) in section 2. Although both Gesell and Keynes explicitly reject the time preference explanation for interest, we will see that the GK theory is in fact consistent with time preference that emerges from scarcity of capital and what is being rejected is pure myopia. We present the GK theory in section 3 and also note crucial differences between how Keynes and Gesell model the economy. Gesell is classical in his real theory but sees a flaw in the monetary system that once corrected makes money the neutral medium of exchange that the classical system assumes and allows decentralized markets to operate freely. Keynes rejects the classical theory for ignoring *liquidity preference* and sees a consequent failure of the market system making both monetary and fiscal management necessary. What Gesell and Keynes have in common is essentially a modern growth theory steady state level of capital combined with a monetary constraint that prevents the steady state from being reached. Section 4 gives a brief presentation of the Solow growth model and the Ramsey growth model and demonstrates how both link directly to the GK theory. In section 5 we present two models that incorporate money into these growth models allowing for an analysis of how money affects the real system and in particular the steady state level of capital. In the Sidrauski model, the economy transitions to the GK steady state and this is not affected by the return on holding money in stark contrast to the GK theory. In the Tobin model the steady state level of capital is inversely related to the return on holding money but this departure from Sidrauski arises because Tobin uses the Solow framework with a fixed savings rate. Section 6 has a discussion of how the GK theory departs from the assumptions of the Sidrauski model and also uses this framework to evaluate the difference between the way Gesell and Keynes model the economy and their very different solutions to the liquidity trap. Keynes's solution is careful monetary and fiscal management to steer the economy to the steady state. Gesell's solution reduces the return on holding money to below the steady state return on capital that allows the unimpeded transition to the steady state. A crucial difference between the two solutions is the ratio of income to money, the velocity of circulation. With Gesell this stays the same in the transition to the steady state while with Keynes this gradually decreases with a corresponding increase in wealth held in the form of money. Our discussion in section 6 will include a focus on the significance of this, including on why Gesell, who considered this alternative solution, argued that it would lead to currency collapse. We end with a brief discussion in section 7.

2 Fisher's Productivity and Thrift

The individual faced with opportunities to invest by forgoing current consumption will choose to allocate resources over their lifetime by accounting for on the one hand the return on capital and on the other hand the gain from the resulting intertemporal trade in consumption. The return on capital will determine the rate at which consumption can be traded over time and preferences over consumption paths will determine the rate at which the individual wishes to trade consumption over time. A key feature of the neoclassical approach is marginal analysis and to illustrate we will consider a simple model of an individual who lives for two periods and consumes one good. Imagine the individual is endowed with 10 units of the good each period and any units not consumed in period 1 will grow by 10% to be consumed in period 2. We will begin by assuming that the individual's preferences over consumption can be represented by a lifetime utility function $U(c_1, c_2) = u(c_1) + u(c_2)$, where c_i is consumption in period i and $u(c)$ is a period utility function that is assumed to be an increasing concave function (diminishing marginal utility). It will help to begin by considering an allocation that is not optimal, the individual plans to simply consume their endowment of 10 units each period. At the current margin, the utility of consuming the 10th unit in period 1 is less than the utility from saving it and consuming it tomorrow as there will be more of it. This logic does not extend to consuming everything tomorrow as that has the individual starving in period 1 and eating to the point where the final unit is giving little additional utility in period 2. Marginal analysis determines the balance between these two forces, the growth on the one hand and diminishing marginal utility on the other. As the marginal utility, $mu(c)$, is decreasing in consumption, $mu(c_1) > mu(c_2)$ when $c_1 < c_2$ and the marginal rate of substitution is greater than 1, $MRS = \frac{mu(c_1)}{mu(c_2)} > 1$. That is, if we decrease c_1 by a unit, we need to increase c_2 by more than a unit to keep to the same total utility. However, so long as the $MRS < 1.1$, it will pay to decrease c_1 and increase c_2 as the growth in c_2 is greater than that required by the MRS to maintain constant utility. By a symmetric argument, it will be optimal to increase c_1 and decrease c_2 when $MRS > 1.1$ and the condition for optimality is $MRS = 1.1$.

Notice that conceptually, this is the same as having two goods in a one period model with prices $p_1 = 1.1$ and $p_2 = 1$ where subscript i now represents good i rather than period i. If we sell one unit of good 1, we can buy 1.1 more units of good 2 and it will be optimal to do this so long as $\frac{mu(c_1)}{mu(c_2)} < 1.1$. In the one period model with 2 goods, we also assume that the rate at which we can substitute one good for another given the production technology is increasing. To produce one more unit of good 1, we need to free up resources from the production of good 2 and therefore sacrifice some units of good 2. This production trade off, the marginal rate of transformation, MRT, is assumed to be increasing so as we shift resources to produce good 1 we have to sacrifice an increasing amount of good 2. The more general condition for optimality is $MRS = MRT$. Mathematically, the diminishing MRS and increasing

MRT are convexities in our model of the economy that ensure the optimal allocation is a balance of the two goods. When we are producing too much of say good 1 then it is both very expensive relative to good 2 to produce and is giving a low marginal utility relative to good 2. In a market economy there will be an excess demand for good 2 with consumers wanting to consume more of good 2 so long as $MRS < MRT$. In equilibrium, $MRS = MRT$ and this is also the market clearing price ratio of good 1 to good 2.

Returning now to our two period model, the production trade off arises because capital gives a return and this gives a MRT greater than 1. Fisher models this at the individual or household level with every household having some return from increasing their production with an increasing MRT. That is, as the household sacrifices more present consumption for capital, the return on the capital decreases (decreasing marginal product). If we consider one household in isolation then we simply have the household's interest rate determined by their return in equilibrium $MRS = MRT$. We can now imagine something like an endowment of 10 units each period with a return of 20% so $MRS = 1$ and $MRT = 1.2$ and then as the household trades consumption in period 1 for period 2, the MRS increases and the MRT decreases until they meet at 1.1 giving an equilibrium interest rate of 0.1.

With many households there is the possibility of borrowing and lending and in equilibrium we will have equality of everyone's $MRSs$ and $MRTs$. What this means is that if a household wants to save in period one but another household has a better return on increasing their production (higher MRT), they will lend money to the other household, to be paid back in period two. This will give rise to a market for loans that will clear at the equilibrium interest rate. Extending this to many periods, Fisher (1930) demonstrates an equilibrium sequence of interest rates. Viewed this way, the interest rate in any period determines the equilibrium price ratio of the commodity between this period and the next with a positive interest rate representing a lower price in the next period. The force that is giving rise to positive interest rates in equilibrium is the return on investing which creates a time preference counter force. Fisher, building on the pioneering work of Böhm-Bawerk, recognized that productivity of capital alone cannot explain a positive rate of interest. There needs to be something that creates a $MRS > 1$ and in our presentation so far, this arises from the concavity of the period utility function. Another way to get this is to have a discounted utility function, $U(c_1, c_2) = u(c_1) + \beta u(c_2)$ where β is the discount rate. With a 10% return, if $\beta = \frac{1}{1.1}$ then we will have an equilibrium with $c_1 = c_2 = 10$ and our balance is achieved just with discounting. This is pure myopia and it is standard to assume that agents have myopic preferences represented by a $\beta < 1$. However, it is not necessary to get the Fisher/Böhm-Bawerk positive interest rate and the notion of time preference is more general, requiring only a way to increase the MRS above 1.

The language used in the presentation above is second nature for an undergraduate economics student. To help make this accessible to a wider audience we will end this section with an attempt to translate terms like the marginal rate of substitution and marginal rate of transformation. Imagine you have £25 and \$60 and there is a fixed £/\$ market exchange rate of 1.2. This wealth can be expressed in a single currency as £75 or \$90. If you exchange some of your pounds for dollars at the fixed exchange rate then this figure for the total won't change. Stretching the example, suppose someone who is desperate for pounds and for some reason does not have access to the market rate offers you the exchange rate 1.6 for your £25. Assuming you have access to the fixed market exchange rate you will accept this favorable offer and your wealth will increase to \$100 or equivalently £83.33. In our presentation above with $U(c_1, c_2) = u(c_1) + u(c_2)$, the MRS is an exchange rate between consuming today and consuming tomorrow that keeps your *utility* rather than your wealth at the same level. At $MRS = 1.1$, if you trade 1.1 units of consumption tomorrow for 1 unit today you will have the same total utility. The reason it is not 1 is that we are at an allocation where you are consuming more tomorrow than today. Unlike our fixed £/\$ exchange rate this exchange rate is diminishing in consumption today as we make the allocation more equal and will come down to 1 when we get to the point $c_1 = c_2$.

From $MRS = 1.1$, if you are offered a more favorable exchange rate, say 1.2, you will take it and trade more of your consumption today for consumption tomorrow as it will increase your level of utility. The MRT is the exchange rate you are being offered. This reflects what is possible given the reality of technology. For example, suppose consumption is in fish and your endowment is labour that allows you to catch 10 fish without a net. Alternatively, you can catch 5 fish in period 1 and also make a net that allows you to catch 16 fish in period 2. This is trading 5 fish today for 6 fish tomorrow or a $MRT = 1.2$. So long as $MRS < MRT$ you are being offered a favorable exchange rate and you will increase your utility by trading.

3 The GK Theory

The real part of the GK theory is essentially the above theory with $\beta = 1$. Gesell explicitly rejects Böhm-Bawerk's time preference theory of interest but recognizes that there will be a positive rate of interest while capital is *scarce*. What is being rejected therefore is $\beta < 1$ or pure myopia. Keynes also sees that scarcity of capital will result in a positive return or marginal efficiency of capital, MEC. This is the discount rate at which the present value of the expected stream of income from an investment is equal to its current price.

Anticipating growth models, the question that both Gesell and Keynes ask is, why doesn't capital accumulate to the point where the return on it is zero? The GK theory answer to this is presented brilliantly in chapter 17 of the General Theory. The following presentation draws heavily on the chapter 17 presentation but also incorporates important aspects from the Natural Economic Order. Holding money gives a non pecuniary return, a liquidity premium. To entice the holder of money to lend it requires an interest rate that compensates the holder for parting with liquidity. Gesell calls this *Basic Interest*. At a return below this threshold, there is a preference to hold money rather than other assets, a *liquidity preference*. We can express the return on any asset in terms of itself as comprising of a yield, q, a carrying cost, c and liquidity, l. Considering wheat, if you are simply storing it then the yield and liquidity are negligible but there is a significant cost, c_w, whereas a house has a significant yield (service), q_h, and money provides liquidity, l_m. To compare these returns we can measure them in monetary terms which requires adding a factor that adjusts for any changes in the monetary value of the asset over the period giving the three returns as $a_w - c_w$, $a_h + q_h$ and l_m. Keynes uses these three examples to represent the components of the return on an asset and in general an asset will have a return $a_g + q_g - c_g + l_g$. By arbitrage, the return on every asset that is held must be the same (interior solution) at some currently ruling return, r. Assets that have a return below this are not held as a medium of saving (corner solution). For wheat to be held, it would need to have a sufficiently high expectation of a price rise in the future so that $a_w - c_w = r$. One thing missing from the chapter 17 presentation is the possibility of a corner solution and Keynes assumes instead that the current price of wheat will fall, increasing a_w, until $a_w - c_w = r$. It is also possible however that wheat will simply be consumed before the price falls so low in which case no wheat is held as a medium of saving. This becomes important when we consider Gesell's demurrage solution which can be thought of as creating a corner solution for money as a medium of saving. With a house, we can imagine the service it is providing is decreasing in the quantity of wealth stored in it and that everyone holds wealth in this form to the point where the marginal service yield is r. Crucially, productive assets will be produced up to the point where the MEC is r. If an entrepreneur has expectations of the stream of income from an asset such that $MEC > r$ then it will pay to borrow at the ruling rate, and buy the newly produced asset. As the asset increases in supply, increased competition will reduce the expectations for the stream of income and also increase the current supply price bringing the MEC down to r. What then is determining this ruling return, r? We can think of this as the equilibrium rate in the Fisher model and without an impediment to capital accumulation, this rate should come down over time as capital becomes less scarce. However, once capital accumulates to the point where the MEC has come down to l_m, capital accumulation is halted and it becomes more attractive to store wealth in money.

This is the GK theory in a nutshell. We no longer have an interest rate that is determined in the real economy but one that is determined by liquidity on money. In chapter 17, Keynes presents this simplified model with a positive lower bound that allows us to see the essence of the problem. The model outside of chapter 17 in the General Theory is of a liquidity preference schedule that together with a fixed money stock determines the interest rate with the schedule becoming elastic at low interest rates. Gesell does this in reverse in the Natural Economic Order with a fixed basic interest that creates the lower bound as the main model but a section that includes a clear description of a downward sloping liquidity schedule that becomes elastic at low interest rates.[1].

There are two ways we can think about this. The first is to decompose the demand for money into a transactions demand that is a function of income and a savings demand that only arises at low interest rates and is an elastic function of the interest rate. The Gesell solution to tax money holdings eradicates the savings demand without affecting the transactions demand. Although Keynes uses this decomposition (with speculative rather than savings demand), he does not see this possibility of a corner solution where the return on holding money can be less than the return on lending money. The second way is that there is a single function for the demand for money that is interest inelastic at high interest rates representing transactions demand and elastic below some threshold representing an additional savings demand. Then for high levels of interest, money is held for transactions purposes and balances do not increase very much as the interest rate comes down. As the interest rate comes down below the threshold, the schedule becomes elastic creating a liquidity trap with more and more money held as savings. Both ways can include as a special case a positive lower bound on liquidity.

There are significant differences between Keynes and Gesell that are discussed

[1] Gesell (1929) Book 3 Chapter 13.

below and in section 6. What they have in common is the liquidity trap, a demand for money that becomes elastic at low interest rates. This is what we are referring to as the GK theory of interest. It is a theory on a monetary determination of the interest rate when interest rates are low. Given the similarities and differences between Keynes and Gesell, we will refer to the two approaches as different models of the GK theory, model K and model G.

A significant difference between the two models is on the determination of the interest rate. Model G has the interest rate determined by real factors until it hits basic interest. From some high starting interest rate, capital accumulation will gradually bring it down and while the demand to hold money is inelastic its return will be equal to this interest rate but will not determine it. We must then either have the money supply adjusting to bring about this equilibrium or the price level adjusting to bring the real value of the money supply to equilibrium. As we are on an inelastic portion of the demand for money schedule, very small adjustments will bring about equilibrium. A natural market process of capital accumulation then brings the interest rate down until it hits *Basic Interest*. When the interest rate is above this rate, Gesell speaks of a different type of interest that we can think of as a real rate determined by the classical investment/savings equilibrium.

Outside of chapter 17 Keynes has the interest rate determined by the stock of money and the demand for money (liquidity preference) schedule. Keynes in chapter 17 is in between models with the liquidity preference theory of interest determination ever present and with this penetrating analysis of the lower bound that constrains the capital accumulation process only when interest is sufficiently low that actually gives a brilliant presentation of model G! For Keynes the interest rate is gradually brought down through monetary policy until it becomes ineffective at low interest rates. Although there is this inconsistency, particularly from chapter 17, our model K will have the interest rate determined by liquidity preference and controlled by monetary management. This interest rate then feeds into determining investment giving the monetary authority a pivotal role in managing the economy.

Keynes and Gesell also have very different solutions to deal with the monetary constraint on capital accumulation. The government is central to the model K solution as an entity that can fill any gap in demand created by a lack of private investment. The combination of monetary and fiscal policy can then continue to bring the interest rate down. This contradicts the positive lower bound of chapter 17 presented above but is consistent with an elastic demand for money that does allow the liquidity return on holding money to come down to zero for sufficiently high balances. One way we can reconcile the contradiction is to observe that Keynes saw the positive lower bound as a theoretical possibility and that there is a conventional aspect to the interest rate. If by the combined monetary and fiscal policy, expectations of future interest rates can be brought down then the lower bound will not arise allowing capital accumulation to continue. Model G addresses the problem directly with a tax on holding money that ensures that there is no demand for money as a vehicle for saving. There is no role for fiscal or monetary policy to bring down the interest rate which is determined by the market. With the root cause of a changing demand for money holdings removed, Say's Law will apply removing the possibility of deficient demand and the need to centrally manage the economy.

4 Growth Theory

The presentation above gives a simple way to think about a world with many vehicles for savings, varying in properties like liquidity, expected yield and risk making up an overall expected return on each one. Arbitrage will ensure that the return on any asset that has been adjusted for these factors will gravitate to the same value that we can call the real interest rate, r. The Fisher analysis gives an answer to what determines this value at a particular point in time. It will reflect an equilibrium intertemporal price ratio that is determined by a combination of return on capital and preferences. A natural question that arises is what determines the return on capital. Although the Fisher analysis includes a diminishing marginal product indicating that with positive net investment it should come down over time, it is not equipped to address the question of a *steady state* quantity of capital, a quantity of capital that once reached will remain constant. The Solow growth model addresses this in a *representative agent* framework with a fixed savings rate. In some ways this is a simplification of the Fisher type of model that has agents with possibly different preferences and production possibilities and the level of savings is determined by intertemporal optimization. However, it allows us to address the question of capital accumulation. Assume that each individual provides a fixed unit of labour and saves a constant fraction of income, s, each period. The income per person is then given by a concave production function (diminishing marginal product), $f(k)$, where k is the current quantity of capital per person. By saving $sf(k)$, capital grows by $sf(k) - \delta k$ where δ is the rate at which capital depreciates. Then so long as $sf(k) > \delta k$, there will be positive net investment and the quantity of capital will grow. The steady state is reached at k^* where $sf(k^*) = \delta k^*$, that is when investment is just covering depreciation. From this point on there will be a constant level of consumption, $c^* = f(k^*) - \delta k^*$ and an unchanging interest rate, r^*, equal to the marginal product of capital at k^*. If we include population growth at rate n then the condition becomes $sf(k^*) = (\delta + n)k^*$ giving a lower steady state quantity of capital and higher interest rate with $c^* = f(k^*) - (\delta + n)k^*$. The intuition is that you now need to add nk^* to the capital stock so that the newly born also have k^* units of capital. To relate this to the GK theory we want to represent zero myopic time preference in the steady state and we can do this by setting the savings rate so that $r^* = (\delta + n)$. This savings rate gives the maximum consumption which corresponds to what would arise in the absence of myopic time preference and is called the golden rule savings rate. The GK model has $n = 0$ and a steady state net return on capital $r^* - \delta = 0$ which fits the Solow model with the golden rule savings rate.

Translating the Solow model, we think of a community that consumes just one good that also serves as capital. We start at the beginning of a period where they have $k = 64$ units of the good, accumulated from previous periods, that will be used for production. Assume that $f(k) = \sqrt{k}$, $s = 0.1$, $\delta = 0.01$. Then they produce an output of $\sqrt{64} = 8$ units of the good, save $0.1 \times 8 = 0.8$ units and lose $0.01 \times 64 = 0.64$ units to depreciation. This process has allowed them to consume 7.2 units for the period and add $0.8 - 0.64 = 0.16$ to the capital stock for next period. The capital stock is growing and will continue to grow every period until we get to $k^* = 100$. Then output is 10, they save $0.1 \times 10 = 1$ unit and lose $0.01 \times 100 = 1$ to depreciation giving zero net additions to the capital stock. From this point on the community is in a steady state with consumption of 9 units per period. The problem with this savings rate is that if the community is not myopic then based on increasing the utility level it is getting that favorable exchange rate for further savings and investment we discussed in translating the Fisher model. The golden rule savings rate is the optimal rate without myopia and maximizes steady state consumption. For this example it is $s = 0.5$ with steady state $k^* = 2500$ and $c^* = 25$. The Solow model is not founded on individual utility maximization based on intertemporal preferences but instead exogenously fixes the savings rate.

The Ramsey growth model also uses the representative agent framework but allows us to combine the capital accumulation of the Solow model with the individual maximization part of the Fisher model. In continuous time, we replace the discount factor, β, with a discount rate, ρ, with $\rho = 0$ corresponding to $\beta = 1$ (no myopic discounting). The steady state is the same, $c^* = f(k^*) - (\delta + n)k^*$ with steady state interest rate $r^* = \delta + \rho + n$ which is now determined by intertemporal optimization. The solution requires $\rho > 0$[2] but if we take $n = 0$ and imagine ρ close to zero then we have a net rate of return $r - \delta$, close to zero again giving the golden rule steady state quantity of capital.[3] In our example above this is $k^* = 2500$, $c^* = 25$ and $r^* = 0.01$.

[2] This is required to ensure the infinite sum of discounted utilities converges.

[3] This assumes utility is being discounted by ρ which doesn't account for the growing population. An alternative is to assume that the household is discounting utility at rate $\rho - n$ to give equal weight to all individuals, future and present. This gives a steady state interest rate $r = \delta + \rho$ and the solution requires $\rho > n$.

The Sidrauski model in the next section uses the first. For our purposes it does not make a

We can now define scarcity of capital in the GK model simply as a level of capital below the steady state level. Although it assumes the special case, $n = 0$ and the golden rule savings rate in the Solow model or $n = 0$ and ρ close to zero in the Ramsey growth model, it also applies to the case where $n > 0$ and more generally to growth models that include other elements that make the rate of interest positive in the steady state such as technological progress. What the GK theory adds is that the steady state level of capital can't be reached if money competes with capital as a medium of saving above the steady state interest rate. Then capital is kept scarce and in place of a process that moves to the steady state, deflationary forces from the increased demand for money create a deflationary cycle.

5 Tobin vs Sidrauski

Starting with Tobin (1965), there is a literature that explores the non neutrality of money in growth models that incorporates a demand to hold money as part of the wealth portfolio. Tobin uses the Solow framework with a fixed savings ratio and shows that the steady state will be at a lower level of capital when savings are split between capital and money. In the steady state, both real money balances and capital grow at the natural rate, n, with the proportion of wealth held as money being determined by the difference between the monetary return on holding money (which we have assumed to be zero) and the return on capital. The model captures the idea that by competing with capital as a store of value, money can have a negative effect on the long term level of capital in the economy and this is called the Tobin effect. Responding to this, Sidrauski (1967) uses the Ramsey framework with money in the utility function to capture the demand to hold money. He shows that the return on holding money does not affect the steady state values derived above and in particular, if we take $n = 0$ and imagine ρ close to zero then we again have a net rate of return

difference as our GK model assumes no myopic discounting which requires $\rho = n$ in the second case giving the golden rule steady state interest rate $r = \delta + n$.

$r-\delta$, close to zero in the steady state. The return on money adjusts to this return so in the steady state, the real money balances are held to the point where the marginal return has also come down to $r-\delta$. This result is referred to as superneutrality of money, that is the real steady state variables are independent of the return on holding money (to distinguish this from the neutrality of money which is independence from the quantity of money).

To evaluate this in the context of the GK theory, assume that $\rho = n = 0$ (ignoring the fact that ρ must be a little above 0) and that the monetary return on holding money is zero giving the arbitrage condition $r - \delta = l_m$. In our example from the last section, we have capital accumulating to the steady state $k^* = 2500$ and $r - \delta = 0$ and the only thing to add with Sidrauski is that in steady state we hold money balances to the point where $l_m = 0$ which requires an assumption that there is not a positive lower bound on the liquidity return. With superneutrality, money is only affecting the level of utility and it is optimal to make the opportunity cost of holding money zero. This follows from the Friedman rule, making the opportunity cost of holding fiat money equal to the zero cost of producing it. Friedman (1969) suggests that a deflation rate equal to the real net return on capital and bonds to achieve this which in our steady state is simply $r - \delta = 0$ and the Friedman rule gives an optimal inflation rate of zero. In the GK model, the demand for money becomes elastic at low interest rates and this optimal inflation tax is accommodating a very large demand to hold money in the steady state.

Tobin has a steady state with return on capital, $r > \delta$, where the proportion of savings held as money is a decreasing function of r with a steady state $(r - \delta) = l_m$. As in the Sidrauski model, this equates to an unbounded decreasing l_m schedule and at the steady state level of capital, the liquidity on holding money, l_m has been brought down to $r - \delta$ by an increased proportion of savings held as money. However, to represent no myopic discounting, the golden rule savings ratio will give $r = \delta$ and $l_m = 0$ which is the same as in the Sidrauski model. The non superneutrality in the Tobin model results from the fixed savings ratio and this is what the Sidrauski model is demonstrating.

6 The Art of Modelling

The GK theory is essentially a growth model with $\rho = n = 0$ and a steady state net interest rate $r - \delta = 0$, combined with a monetary model where the liquidity on money puts a break on the transition to the steady state. Both the Sidrauski and Tobin models incorporate liquidity on money into growth models and demonstrate that the demand for increased real balances can be accommodated in the transition to a steady state and as we have noted above, the reason that the Tobin model does not give a steady state with the golden rule level of capital is that it is assuming an exogenously fixed savings rate. These models are abstractions that give us benchmarks from which to understand the economic phenomena that we observe in our complex reality. The Sidrauski model is a particularly useful benchmark as it gives a set of conditions under which we have both neutrality and superneutrality of money. We can then take the inflation tax on money to be any value including below zero (a deflation subsidy) without affecting the steady state real variables. The larger the tax the smaller the ratio of money balances to income or the higher the velocity of circulation. Within the model the optimal tax is given by the Friedman rule with a zero tax. Turning our head from any model to the real world we will see a multitude of departures from our abstraction. We deal with this with new abstractions or models. Taking for example this result that the real variables are not affected by the inflation rate in the steady state of the Sidrauski model, we see that there are many reasons why money is not in fact superneutral. For example, in models where money facilitate production, the ratio of money to income will affect output. Notice that this gives us two distinct abstractions that suggest reducing the cost of holding money is a good thing, one with superneutrality where it doesn't affect real variables but through the Friedman rule maximizes utility and the other where it is helping to optimize output. In this case the result from using the Friedman rule in the Sidrauski model is supported by a different model where money is not superneutral.

Turning now to the GK theory, there is an important fact of reality that it recognizes and the Sidrauski model does not, prices do not change instantly! This particular abstraction that Sidrauski uses has been really useful for economists. Although it has always been understood as an unrealistic abstraction, classical economists used it to develop the idea of the long run neutrality of money and Sidrauski builds on this with the idea of superneutrality. Resting the assumption, we see a key departure point between Sidrauski and the GK theory is when the demand for money schedule becomes elastic at low interest and enters the liquidity trap where money competes with capital as a vehicle for saving. In the Sidrauski model this does not create a problem as with flexible prices, the demand for money is being constantly equated with an expanding real money supply. Without flexible prices however, excess demand for money creates deficient demand, a deflationary cycle, unemployment etc.

Let's consider first our model K where the liquidity trap is managed by a combination of monetary and fiscal policy that prevents inflationary or deflationary cycles from taking off and is successful in bringing down long term interest rate expectations. Uncertainty is at the heart of Keynes' general theory and both the liquidity schedule that together with the money supply determines the interest rate and the marginal efficiency of capital schedule that determines investment are heavily sensitive to market sentiment. He has an explanation for the depression in a kind of low sentiment equilibrium that monetary policy alone cannot fix or at least not fix it in any reasonable time frame. Much stronger than this, he suggests that the combination will be necessary to steer the economy to the GK steady state.[4] This follows the Sidrauski transition path with the elastic demand for money accommodated on the path to the steady state and if we add a zero inflation target then the Friedman rule is also satisfied in the steady state equilibrium. The key difference is that in

[4] In Keynes (1937), published a year after the General Theory, Keynes draws back on this suggesting that it is the diagnosis of the problem that is general (our GK theory) and that the solution is related to the conditions of the time.

an uncertain world with frictions the economy requires careful management to the steady state.

With model G, the liquidity trap is avoided with the tax on money holdings which shifts the liquidity schedule down by the amount of the tax. Our assumption is that there is some rate of interest at which the schedule begins to become elastic. Above this, it is inelastic and money is held for transactions purposes. The liquidity trap is avoided with a tax that is at least as high as this interest rate, moving the elastic portion of the liquidity schedule to below zero interest. This opens the Sidrauski transition to the GK steady state with the net return coming down to zero. A crucial difference between the model K and model G steady states is that model K requires the accommodation of the increasing demand for liquidity whereas model G maintains liquid wealth at the transaction demand for money. Gesell gives a critique of any solution that accommodates the increased demand for liquidity. He recognizes that this will work in the short run as it prevents a deflationary cycle from starting and allows the economy to continue on its growth path. However, the increased wealth held in liquid form destabilizes the monetary system, priming it for an inevitable inflationary cycle, hyperinflation and currency collapse. The essence of the problem as Gesell sees it is that increasing claims are being created on real goods without a corresponding increase in real goods. He sees this as an abuse of the monetary system that will eventually destroy the trust that it is built on. Monetary models recognize the possibility of *bubble* paths and include stability conditions that relate to expectations. In the Sidrauski model for example, inflationary expectations are formed adaptively and the stability condition requires a sufficient lag between prices changing and expectations being updated to prevent moving onto a bubble path. In monetary management models such as our model K, rational expectations are more commonly used and it is active monetary policy and fiscal discipline that prevent bubble paths. Skeptical about the viability of such management, Gesell is offering an alternative solution where the instability is dealt with by a tax on money that keeps the money supply at the transactions demand. By separating out medium of exchange from medium of saving, Gesell is seeing that it is as a medium of saving that the problem arises. With a tax on money holdings and the money supply at the transactions demand, the velocity of circulation will be high and an inflationary cycle will require some mechanism by which it increases further. However, any movement in that direction will reduce the real value of the money supply below transactions demand checking the increase. On the other hand, a deflationary cycle will require some mechanism by which the velocity falls increasing real balances beyond the transactions demand. Here the tax on money is the check on holding money as a medium of saving. The system therefore has ever present self stabilizing forces not requiring any kind of monetary management. It is the elastic demand as a medium of saving that creates the volatility and the greater the quantity of money that is held as a medium of saving, the greater the potential for an inflationary spiral.

Our model G can be related directly to the Sidrauski model by setting the inflation rate at Gesell's tax rate. We noted above that departing from the Sidrauski model, there are models where money is not superneutral and output increases with a lower inflation rate as higher money balances are held and the additional money balances play some positive role in the economic system. In the Sidrauski model where money is superneutral, the Friedman rule also makes it optimal to have high money balances. Model G is giving the opposite conclusion, a tax on money with low balances and high velocity being optimal and both superneutrality and the Friedman rule failing due to "excess liquidity" externalities related to destabilizing the economic system. There is also a sense in which Gesell's tax is not in conflict with the spirit of these models which is to accommodate a demand for money that is providing transactions services. With money in the utility function we leave open what the motive is for the demand and the assumption is that in a liquidity trap money is being used as a medium of saving rather than a medium of exchange, competing with transactions needs rather than supporting them.

The discussion above has assumed that there is not a positive lower bound on the liquidity return from holding money that has allowed us to use the Sidrauski model as a benchmark to contrast our two models of the GK theory. We will now consider a perfectly elastic demand for money at some positive lower bound, which will allow us to see more clearly the essential difference between model G and model K, by returning to Tobin (1965) who begins with this simple version of his model. Rather than moving to the optimal steady state, the economic system gets stuck at the level of capital that gives the same return on holding capital as the liquidity return on holding money. All savings go into adding to money balances with additional money being created to exactly match this demand. The equilibrium is a kind of knife edge level of capital where both assets give the same return making agents indifferent to the composition of wealth. The equilibrium then involves a suboptimal level of capital and the proportion of wealth held as money increasing forever! This is our model K solution that avoids deficient demand by accommodating the demand for money. Tobin notes, referring to Gesell, that there is another option where by reducing the return on holding money to the steady state return on capital, the constraint on capital accumulation is removed and the economy can transition to the unconstrained steady state. Stating the obvious, we cannot go on creating more claims against real wealth forever, while holding constant the real output of the economy. Gesell's solution on the other hand allows the steady state to be reached without the need to continuously create additional money.

7 Discussion

The modern theory of interest emerged from developments in economic theory with utility foundations for the theory of value and marginal analysis foundations for equilibrium analysis culminating in Fisher's complete general equilibrium theory. Growth theory switched attention to long run analysis of capital accumulation to give a steady state interest rate. These are real models of interest where money is a neutral medium of exchange. The non-neutrality of money in the GK theory arises because a liquidity return on money competes with capital as a store of value and prices cannot adjust sufficiently quickly to keep the money supply up with an elastic demand for money. One way to characterize Keynes's solution is to manage a system where money is not neutral. Gesell who was both classical and neoclassical in his thinking, saw that money can be made the neutral medium of exchange that is being assumed. The tax on money protects the economic system from an elastic demand for money and while capital is scarce, the intertemporal optimization of the Fisher model will determine the interest rate and unimpeded capital accumulation will allow the rate to come down to its steady state value.

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