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Abstract: I construct a simple matching model that nests transferable utility, non-transferable utility, and imperfectly transferable utility by showing that if the utility possibility frontier of a matched couple satisfies a homogeneity condition it has a CES form, with the elasticity of substitution σ a measure of the degree of transferability. Taking σ as exogenous, I analyse how transferability affects sorting and payoffs. Treating social norms as a source of imperfect transferability, I examine the effect of norms about gender roles within the household.

Keywords: *Matching; marriage market; imperfectly transferable utility; social norms; gender norms.*

JEL Classification Number: C7; D1; D9.

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1 INTRODUCTION

In models of two sided matching with heterogeneous agents, whether utility is transferable or not makes a profound difference to the pattern of sorting and to the value and distribution of agents' payoffs. Most matching models assume either transferable utility (TU) or non-transferable utility (NTU), with a small number looking at the intermediate case of imperfectly transferable utility (ITU). In this paper I analyse a simple model that nests these three possibilities by treating transferability as an exogenous parameter, allowing us to see how transferability affects sorting and payoffs. I illustrate this approach by treating social norms as a source of imperfect transferability, and I analyse the effect of norms about gender roles within the household.

The importance of transferability in matching models is clear when we examine how equilibrium is sustained. In models with some degree of transferability of utility, equilibrium payoffs are such that no pair of agents who are not initially matched to each other can break from their respective partners, form a new couple, and both be better off after a suitable division of their joint output. Thus equilibrium is sustained, and disequilibrium overturned, by transfers of output (or their possibility), opening up a route for the ease with which transfers can be made to affect the equilibrium itself. [1]

With perfectly transferable utility, transfers between a matched couple preserve the sum of their utilities, and this is the basis of the result that equilibrium maximises aggregate utility, and hence aggregate output. With imperfectly transferable utility, transfers are potentially costly: they do not preserve the sum of utilities, and there is no reason to think that aggregate output is maximised in equilibrium, or indeed that equilibrium maximises anything. With non-transferable utility, these considerations apply in extreme form: each possible partnership generates a fixed pair of payoffs, with no role at all for transfers.

The effect of transferability is nowhere more evident than in the equilibrium pattern of sorting. The proposition that when agents' characteristics are complementary there will be positive assortative matching (PAM) regardless of the distribution of types has been described by Legros and Newman (2002) as "probably *the* main idea in the matching literature." But this result assumes transferable utility, and follows from the condition that total output is maximised: complementarity (i.e. supermodularity of output as a function of types) then means that higher types on one side should be matched with higher types on the other. With non-transferable utility, the conditions for PAM are very different. For example, if preferences in a marriage market are based on an always desirable characteristic such as education, then with NTU the most educated man matches with the most educated woman, the second most educated man with the second most educated woman, and so on, generating PAM, regardless of the modularity of output.

In the case of ITU, the supermodularity condition for PAM has been generalised by Legros and Newman (2007); their Generalised Increasing Differences (GID) condition can be interpreted as an extension of the Spence-Mirrlees single-crossing condition. ITU means that it is costly to transfer utility; with GID, higher types find it less costly, and are thus in a better position to secure the advantages of a higher type partner. Similarly, Generalised Decreasing Differences (GDD) generalises the submodularity condition for negative assortative matching (NAM). This approach has been fruitful in a range of applications, for example in the analysis of risk sharing, between couples or between principals and agents; e.g. Legros and Newman (2007), Chiappori

[1] This does not rule out the theoretical possibility that transfers, although available, are not actually used. Then the TU and NTU equilibria coincide. On *No Trade Stability* see Echenique and Galichon (2017)

and Reny (2016), Chade and Eeckout (2017).

As is well known, some ITU problems can be recast in a TU framework, allowing the use of existing results on matching with transferable utility. If there is a representation of agents' preferences, with transformed utility functions, such that for any matched couple the transformed utility possibility frontier (UPF) has a slope of -1, so that the partners' payoffs now sum to a constant, then we have TU. But if the initial problem has imperfectly transferable utility, then this will be reflected not only in the transformations of the original utility functions, but also in the constant to which the payoffs now sum, which in turn could affect both the matching pattern and agents' payoffs, the latter being measured using the original utility functions.

More precisely, let u and v be the utility of men and women respectively and let x and y denote their types. Suppose that if a match produces an output $q = f(x, y)$, the set of possible payoffs is constrained by the relationship

$$g(u, v) = f(x, y) \tag{1}$$

where g is increasing in u and v.[2] We have a TU representation if there exist increasing functions α, β, and γ such that (1) implies

$$\alpha(u) + \beta(v) = \phi(x, y)$$

where $\phi(x, y) = \gamma(f(x, y))$. Then it is the modularity of ϕ not f that determines whether we have PAM or NAM, or some mix of the two, and this in turn depends on the function γ, and thus on the extent and nature of the imperfection in the transferability of utility embodied in the function $g(u, v)$. More precisely, in the case of a TU representation Legros and Newman's GID and GDD conditions reduce to the supermodularity and submodularity respectively of ϕ.[3]

But if the transferability of utility affects sorting and output, it will also affect the distribution of output. Thus an important theme of this paper is the relationship between transferability and payoffs; in particular, does one side of the market or the other lose or gain if transferability changes, and within one side do some types gain or lose more than others? It is here that being able to adapt and use the TU approach is particularly useful. Our approach is to take a situation of imperfectly transferable utility with transferability modelled as an exogenous parameter; we then transform the ITU case to one of perfectly transferable utility; using standard techniques we analyse the pattern of matching and sorting and derive the payoff functions of the transformed set-up; finally we reverse the transformation to get back to the original ITU situation to analyse the effect of transferability on payoffs.

1.1 Illustrative examples

To illustrate how variations in transferability might arise, and before setting out the formal analysis, I consider some simple models of the household. In each, household joint income M depends on the couple's characteristics x and y; i.e. $M = f(x, y)$. Throughout, we take prices of

[2] This formulation embodies an important separability that rules out *marital affinity*, when u or v or both depend directly on x or y or both.

[3] This is essentially Legros and Newman's Corollary 2. Solving $g(u, v) = f(x, y)$ for u and v respectively gives $u = \widehat{u}(x, y, v)$ and $v = \widehat{v}(y, x, u)$. The GID condition is that if $x_2 > x_1$ and $y_2 > y_1$ then for any $u \in [0, \widehat{u}(x_2, y_1, 0)]$, $\widehat{u}(x_2, y_2, \widehat{v}(y_2, x_1, u)) \geq \widehat{u}(x_2, y_1, \widehat{v}(y_1, x_1, u))$. Given the TU representation $\alpha(u) + \beta(v) = \phi(x, y)$, then $\widehat{u}(x, y, v) = \alpha^{-1}(\phi(x, y) - \beta(v))$ and $\widehat{v}(y, x, u) = \beta^{-1}(\phi(x, y) - \alpha(u))$, so the GID condition can be written as $\alpha^{-1}(\phi(x_2, y_2) - \beta(\beta^{-1}(\phi(x_1, y_2) - \alpha(u)))) \geq \alpha^{-1}(\phi(x_2, y_1) - \beta(\beta^{-1}(\phi(x_1, y_1) - \alpha(u))))$. As α^{-1} is increasing this simplifies to the supermodularity condition $\phi(x_2, y_2) + \phi(x_1, y_1) \geq \phi(x_1, y_2) + \phi(x_2, y_1)$.

all goods purchased by the household to be constant, and equal to 1.

(i) one public good and one private good Men get utility $u = a^{\alpha}c^{1-a}$, and women get utility $v = b^{\alpha}c^{1-a}$ where a is the man's consumption of a private good and b is the woman's, c is their joint consumption of a household public good, and $0 \leq \alpha \leq 1$. Their total income is M so $a + b + c = M$. Efficient provision of the public good requires that $c = (1-\alpha)M$ and the allocation of the remaining income to a and b determines u and v. More precisely, $a + b = \alpha M$ implies

$$\left.\begin{array}{lr} \kappa(u^{1/\alpha} + v^{1/\alpha}) = M^{1/\alpha} & 0 < \alpha \leq 1 \\ u = v = M & \text{if } \alpha = 0 \end{array}\right\}$$

where $\kappa = \alpha^{-1}(1-\alpha)^{(\alpha-1)/\alpha}$. Figure 1 shows the UPF in (u, v) space for $\alpha = 1, 0.5$, and 0.

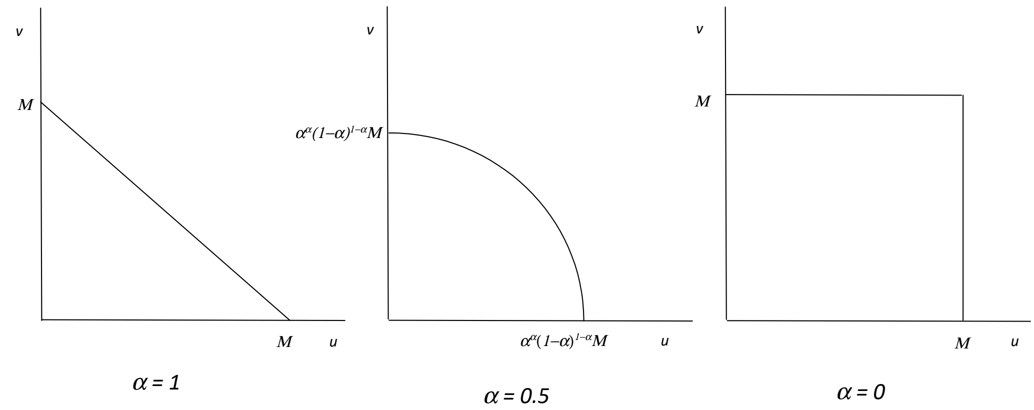

Figure 1: the UPF with one public good and one private good

If $\alpha > 0$, we can put the model in TU form by defining $\widetilde{u} = \kappa u^{1/\alpha}$ and $\widetilde{v} = \kappa v^{1/\alpha}$ so that $\widetilde{u} + \widetilde{v} = M^{1/\alpha}$.[4] There is a clear sense in which the greater is the value of α, the greater is the transferability of utility. This is not an artefact of the way in which we have transformed u and v into $\kappa u^{1/\alpha}$ and $\kappa v^{1/\alpha}$ to get the TU representation, but stems from the underlying economic set-up. If $\alpha = 1$ then $u + v = M$, and since all consumption is private we can transfer utility one-for-one. But in the extreme case where $\alpha = 0$, M is spent entirely on the public good, and there is no mechanism to increase u at the expense of v, or *vice versa*; within the marriage, utility cannot be transferred, and we have NTU.

Note that the transformed utilities sum to $M^{1/\alpha}$, so the pattern of matching will depend not on the modularity of M considered as a function of agents' characteristics but on that of $M^{1/\alpha}$. For example, if $M = (x+y)^r$, then $M^{1/\alpha} = (x+y)^{r/\alpha}$ so for $\alpha > r$ matching will display NAM, but for $\alpha < r$ matching will display PAM. Thus a positive correlation beween spouses' characteristics may be due in part to a low degree of transferability arising from the importance of local public goods in marriage, a point made by many writers, for instance Becker (1973 Ch 4), Legros and Newman (2007, p 1075).

[4] With identical Cobb-Douglas preferences the existence of a TU representation does not depend on the assumption that all prices are 1. See Chiappori and Gugle (2020) Proposition 2.

(ii) Tastes for similarity Suppose a matched couple's output depends negatively on the difference in types. This might reflect a situation where like attracts like, for example a couple are better matched if they are of similar education, or social class, or political outlook. An analysis of the NTU case is given in Clark (2006) and of the TU case in Clark (2020).

Let $M = a - |x - y|^b$; then if $b > 1, M$ is a supermodular function of x and y, and with TU we have PAM. If $b < 1, M$ is a convex function of $|x - y|$ but is in general neither supermodular nor submodular in x and y. As shown in Gangbo and McCann (1996) and McCann (1999) (in the context of concave transport costs) and Clark (2020), in a TU equilibrium with $b < 1$ we have the maximum extent of perfect matching ($x = y$) allowed by the type distributions (this can be seen as an implication of Jensen's inequality applied to convex functions). Then this perfectly matched subpopulation exhibits PAM; the remainder of the population are imperfectly matched, grouped in subpopulations each exhibiting NAM within them and PAM between them.

With tastes for similarity the matching pattern under NTU, analysed in Clark (2006), is very similar to that with TU when $b < 1$; for example if a matched couple can only split their cake equally each agent will seek out a partner of the same type (thus generating PAM). Not everyone can be perfectly matched, and so the remainder will seek partners as close in type as possible, which must generate NAM among any subpopulation whose type distributions do not overlap. Thus going from TU to NTU does not take us straightforwardly from PAM to NAM, or from NAM to PAM, but rather it is as though M as a function of $|x - y|$ has gone from concave or convex to definitely convex. In Section 3.1 I propose a model that can explain this.

(iii) A model of gender norms As a final example, I outline a model of marriage with social norms about how the marital pie should be divided between the man and the woman. The model is more fully developed in Section 4.

A social norm is an unwritten rule about what is acceptable. Couples are expected to conform, and departures from the social norm are costly: they may cause one or both of the couple to feel uncomfortable or guilty or deviant, and they may induce sanctions of some kind. Abnormal behaviour may thus have psychological or material costs, or both. The stronger the norm, the more costly are departures from it.

In the marital context, I assume that in the absence of a norm the household pie M can be cut in any way such that $u + v = M$. The social norm is represented by a particular division, sM for him and $(1 - s)M$ for her; for example if the norm is one of gender equality and neither the man nor the woman should be privileged over the other then $s = 0.5$. If the interests of the man, for example, dominate so he is expected to get more than half the pie, then $s > 0.5$.

Figure 2 shows three utility possibilty frontiers. With UPF1 there is no cost to departing from the norm, so $u + v = M$, as if the norm did not exist; UPF2 goes through the point $(sM, (1 - s)M)$ and departures from the norm are possible but at a cost; with UPF3, the norm is all powerful: any departure that reduces the share of one partner is so costly that the other cannot benefit.

How, then, do norms influence the equlibrium in a marriage market? Suppose that M depends positively on x and y. Then if norms are not important, as with UPF1, we have TU and sorting is determined by the modularity of f. If norms are all powerful, as in the case of UPF3, we have NTU and the highest type on one side will want to match with the highest type on the other, resulting in PAM. With strong norms that still allow for deviations, as with UPF2,

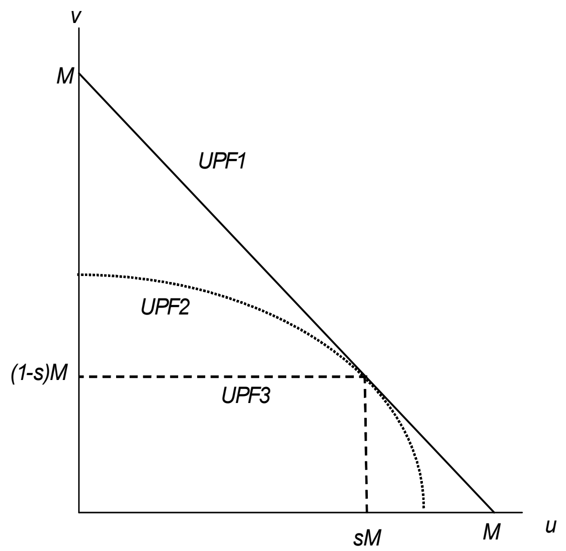

Figure 2: gender norms and the UPF

we have imperfectly transferable utility; intuitively we might expect a mix of the forces driving sorting and equilibrium with UPF1 and UPF 3.

In this approach, a strong norm reduces transferability. As outlined above, this will affect the pattern of sorting; but it will also affect agents' payoffs, and in particular who benefits and who loses from a strong social norms. For example, does a strong norm with $s > 0.5$ necessarily makes all women worse off and all men better off compared to a weak norm or one with $s < 0.5$? To analyse how norms affect both sorting and payoffs, I take advantage of a simple tractable model that allows a parameterisation of the degree of transferability.

1.2 Parameterisation of Transferability

Other writers have recognised that some TU representations allow a parameterisation of transferability e.g. Galichon *et al.* (2019). Their *exponentially transferable utility* model has a single private consumption good and utility functions $u = c + \tau \log a, v = d + \tau \log b$, where $\tau > 0$, with the constraint $a + b = M$. Here, c and d represent marital well-being (non-economic gains to marriage, which may be seen as a local public good available to the married couple)[5]. τ captures the elasticity of substitution between marital well-being and consumption and also parameterises the degree of transferability. If $M = 2$, then as $\tau \to 0$, consumption is not important and we have NTU; as $\tau \to \infty$, only consumption matters and we get the TU model.

In Figures 1 and 2 above, a lower degree of transferability is reflected in utility possibility frontiers with a lower elasticity of substitution, which we denote by σ; as we move from TU to ITU to NTU, σ falls from infinity to zero. In Figure 1, the elasticity of substitution is constant; it equals $\alpha/(1-\alpha)$ if $\alpha < 1$ and is infinite if $\alpha = 1$. This allows us (i) to equate σ with the degree of transferability; and (ii) via the TU representation $\widetilde{u} + \widetilde{v} = M^{1/\alpha}$ to analyse relatively easily how the equilibrium pattern of sorting and payoffs depend on the degree of transferability.

[5] In their models of ITU, Galichon et al (2019) and Galichon and Weber (2024) allow c and d each to depend on x or y or both, thus allowing pair-specific marital affinities.

There is thus a considerable advantage if the UPF has a CES form; not only can we directly equate the elasticity of substitution with the degree of transferability, but the CES formulation is easy to embed in a simple model to show how transferability affects equilibrium. It might be argued that a CES is a very special case, and even though there may exist a TU representation of an ITU set-up, there is no particular reason to suppose that this leads to a CES formulation. I now show that this this is not the case: if a TU representation exists and the UPF is homogenous it has a CES form.

2 Sufficient Conditions for a Utility Possibility Frontier to have a Constant Elasticity of Substitution

Let $g(u, v)$ represent the utility frontier; it denotes the necessary output q for any combination (u, v) of utilities.[6] I assume:

Assumption 1
(i) g is increasing in u and v, differentiable, and quasi-convex.
(ii) There exist increasing transformations functions α, β, and γ such that for any u and v if $g(u, v) = q$ then $\alpha(u) + \beta(v) = \gamma(q)$.
(iii) g is homogeneous of degree 1.
(iv) For some $0 < s < 1$, if $u/v = s/(1-s)$ then $g_u = g_v$ and $g(u, v) = u + v$.

Part (ii) posits the existence of a TU representation. Note that without loss of generality we may take $\alpha(0) = \beta(0) = 0$; e.g. if a TU representation $(\overline{\alpha}, \overline{\beta}, \overline{\gamma})$ has $\overline{\alpha}(0) \neq 0$ and/or $\overline{\beta}(0) \neq 0$ then (α, β, γ) is also a TU representation with $\alpha(0) = 0, \beta(0) = 0$, and $\gamma(0) = \overline{\gamma}(0) - \overline{\alpha}(0) - \overline{\beta}(0)$.

Part (iv) of Assumption 1 says that for any q there is a point on the UPF, with both u and v positive, at which $g_u = g_v$. At this point, $u + v = q$. The couple then share the pie with $u = sq$, and $v = (1-s)q$. Small movements along the UPF from this point incur no loss of overall utility $u + v$ as the UPF has a slope of -1; but with larger transfers $u + v$ may be reduced, g being quasi-convex.[7]

We now have

Proposition 1 *Let Assumption 1 hold. Then for some* $\rho \geq 1, g(u, v) = \left(s^{1-\rho}u^{\rho} + (1-s)^{1-\rho}v^{\rho}\right)^{1/\rho}$.

Proof: see Appendix 1, which also looks at the more general case where g is homothetic.

Proposition 1 says that if Assumption 1 is satisfied, g has a CES form, with parameter ρ such that (i) $\rho \geq 1$ (as g is quasi-convex); (ii) the elasticity of substition σ is $1/(\rho - 1)$; (iii) as $\rho \to 1, \sigma \to \infty$ and $g(u, v) \to u + v$; and (iv) as $\rho \to \infty$ $\sigma \to 0$ and $g(u, v) \to \max\{u/s, v/(1-s)\}$. Then if $g(u, v) = f(x, y)$, and setting $\widetilde{u} = \alpha(u) = s^{1-\rho}u^{\rho}$, $\widetilde{v} = \beta(v) = (1-s)^{1-\rho}v^{\rho}$ and $\phi(x, y) = [f(x, y)]^{\rho}$ we have the TU representation

$$\widetilde{u} + \widetilde{v} = \phi(x, y). \tag{2}$$

[6] Thus g does not depend on x or y, ruling out agent specific preference heterogeneity.

[7] sq and $(1-s)q$ play a similar role to the premuneration values of Mailath et al (2013), in that they can be interpreted as the utilities prior to any transfers away from the *status quo* as represented by the social norm.

3 Matching and Transferability

Given (2) a sufficient condition for PAM (resp. NAM) is the supermodularity (resp.submodularity) of the function $\phi(x,y)$. We now explore the implications of this result.

3.1 ρ and the modularity of ϕ

Wherever f is twice differentiable then so is ϕ, and

$$\phi_{xy} = \rho f^{\rho-2}[(\rho-1)f_x f_y + f f_{xy}]$$

If $\rho = 1, \phi_{xy} = f_{xy}$; this is the TU case. If $\rho > 1$ then the signs of f_x and f_y play a role; we focus on two possibilities.

(a) $\rho > 1$ **with** $f_x f_y > 0$ This is the standard case of vertical differentiation when higher types on either side are always better (or lower types on either side). For $\rho > 1, \phi_{xy} > 0$ if $f_{xy} > 0$, so supermodularity of f is sufficient for that of ϕ. But ϕ_{xy} may be positive even if f_{xy} is negative; as long as $\rho > 1 - \frac{f f_{xy}}{f_x f_y}$ for all x and y, then ϕ is supermodular and we have PAM.

One interpretation of this result is that when $f_{xy} < 0$ and higher types are always better the easier it is to make transfers (a lower value of ρ), the easier it is to sustain NAM. But as ρ increases the necessary transfers are more costly: f_{xy} becomes less important compared to $f_x f_y$, and agents' preference for a more productive partner - and thus a share of a bigger cake - becomes more important. In the limit, as $\rho \to \infty$, the modularity of f plays no role in determining the sign of ϕ_{xy}; only $f_x f_y$ matters and it is always better to match with a higher type.

(b) $\rho > 1$ **with** $f_x f_y < 0$ As discussed in Section 1.1.(ii), one instance where $f_x f_y < 0$ arises is when a couple's output increases the closer are their types; this captures the notion of a taste for similarity, or that "like attracts like".

Suppose that $f(x,y) = h(z)$, where $z = |x-y|$, and h is twice differentiable and decreasing. If $\rho = 1$ and h is concave then ϕ is supermodular: ϕ_x is negative and increasing in y for $y < x$ and positive and increasing in y for $y > x$; thus we have PAM. If $\rho = 1$ and h is convex then typically we have neither super- nor submodularity; ϕ_x is negative and decreasing in y for $y < x$ and positive and decreasing in y for $y > x$.

If $\rho > 1$ then $\phi(x,y) = (h(z))^\rho = \widehat{h}(z)$, so $\widehat{h}$ is convex if h is convex. But if h is concave, we can effectively treat $\widehat{h}$ as convex if ρ is sufficiently high. To see this note that

$$\widehat{h}''(z) = (h(z))^{\rho-2}\left((\rho-1)(h'(z))^2 + h(z)h''(z)\right)$$

If $h'(0) < 0$, then for any $0 \le z \le z_{\max}$ (where $z_{\max}$ is the largest type difference allowed by the distributions of x and y) we can define $\rho^*(z) = 1 + \frac{h(z)h''(z)}{(h'(z))^2}$; then if $\rho > \rho^*(z)$ for all $z \in [0, z_{\max}], \widehat{h}''(z) > 0$ for all $z \in [0, z_{\max}]$ i.e. $\widehat{h}$ is convex. If $h'(0) = 0, \rho^*(0)$ is not defined; but if $\rho > \rho^*(z)$ for all $z \in [\underline{z}, z_{\max}]$, where $0 < \underline{z} < z$, then $\widehat{h}''(z) > 0$ for all $z \in [\underline{z}, z_{\max}]$, so for any $\underline{z}$ positive but arbitrarily close to zero there is a finite ρ such that $\widehat{h}$ is convex over all of the range $[\underline{z}, z_{\max}]$.

Thus in a matching model where like attracts, one justification for assuming TU and that h is convex (perhaps the less natural assumption, reflecting a decreasing marginal loss of output as type differences increase) is that this embodies a degree of non-transferability not explicitly modelled.

3.2 Non-local effects of nontransferability

Complete PAM or complete NAM are at opposite ends of a scale of assortiveness. In many cases the matching pattern may be a mix of PAM and NAM. Furthermore f may not be differentiable. But we can still use the relationship $\widetilde{u}+\widetilde{v}=[f(x,y)]^{\rho}$ to analyse changes in transferability.

Suppose that $f(x,y)$ is increasing in x and y. Let $x_1 < x_2$ and $y_1 < y_2$. If

$$[f(x_1,y_1)]^{\rho_1}+[f(x_2,y_2)]^{\rho_1} \geq [f(x_2,y_1)]^{\rho_1}+[f(x_1,y_2)]^{\rho_1}$$

and $\rho_2 > \rho_1$ then[8]

$$[f(x_1,y_1)]^{\rho_2}+[f(x_2,y_2)]^{\rho_2} > [f(x_2,y_1)]^{\rho_2}+[f(x_1,y_2)]^{\rho_2}$$

Thus higher ρ (or less transferability) increases the tendency to PAM (and indeed could tip the foursome from NAM to PAM) In the language of Anderson and Smith (2024), less transferability increases synergy (the difference in output caused by PAM not NAM).

4 A Household Model with a Social Norm

In this section I set out a simplified model whose purpose is to highlight some of the main issues and possibilities that arise when we treat norms as a source of imperfect transferability. Economists have long recognised that social norms are part of the social context in which economic decisions are made. Adam Smith, in *The Theory of Moral Sentiments* (Smith, 2002) argued that norms arise from our sympathy for others and form part of the social context in which markets operate. That norms as a social force interact and possibly conflict with market forces is a theme addressed by a number of writers.[9] This paper contributes to that literature, using the matching framework set out above. I take social norms to be "unwritten rules shared by members of the same group or society" (Legros and Cislaghi, 2020, p. 62), thereby creating "a shared understanding about actions that are obligatory, permitted, or forbidden" (Ostrom, 2000, pp. 143-144). That is not to say that agents will always abide by such rules; indeed, the model below has a tension between social and market forces whose resolution may well entail a breach of the norm.

[8] Suppose $0 < a < b \leq c < d$ and $a+d > b+c = T$. Now consider the function $x^p + (1-x)^p$, where $0 \leq x \leq 1$ and $p > 1$; this function is convex and has a minimum at $x = 0.5$. Since $\frac{a}{T} < \frac{b}{T} < 0.5$,

$$\left(\frac{a}{T}\right)^p + \left(1-\frac{a}{T}\right)^p > \left(\frac{b}{T}\right)^p + \left(1-\frac{b}{T}\right)^p$$

As $b+c = T < a+d$, $a^p + d^p > b^p + c^p$. The proof is completed by putting $a = [f(x_1,y_1)]^{\rho_1}$, $b = [f(x_1,y_2)]^{\rho_1}$, $c = [f(x_2,y_1)]^{\rho_1}$, $d = [f(x_2,y_2)]^{\rho_1}$ and $p = \rho_2/\rho_1$, where we assume without loss of generality that $f(x_1,y_2) \geq f(x_2,y_1)$.

[9] For example Akerlof (1980), Arrow (1971), Benabou and Tirole (2006), Ostrom (2014), even Becker (1993); for an overview of the literature see Elster (1989), Festre (2010), Burke and Young (2011), Postlewaite (2011).

4.1 Norms and the UPF

I assume that each partner get utility from a single local public good, and disutility from having to undertake household tasks such as cooking, cleaning, and childcare. The allocation of the latter, i.e. who does what around the house, is subject to a social norm, and deviation from the norm reduces utility for both partners.

The public good is bought with household income M , which depends on the couple's characteristics x and y, i.e. $M = f(x, y)$. Time earning money is fixed and the same for both partners; the remaining time for each partner we take to be 1, and is spent either on household tasks or at leisure. The household tasks take a fixed amount of time, 1, which has to be allocated between the man and the woman. We denote by L the amount of leisure time spent by the man, who thus devotes $1 - L$ to household tasks; the woman devotes L to the household tasks and spends $1 - L$ at leisure.

In the absence of any norms, the couple's utilities are given by $u = ML$ and $v = M(1 - L)$, so that $u + v = M = f(x, y)$. Thus utility is costlessly transferable between the couple. It is transferred to the man, for example, by increasing L; he does less within the household and has more leisure; the woman does more housework and has less leisure.

The social norm is represented by $s \in [0, 1]$; it refers to the proportion of household tasks undertaken by the woman, and thus to the amount of leisure taken by the man. If the norm is that housework is split equally $s = 0.5$; if it prescribes that the woman should do all the housework then $s = 1$. A departure from the social norm occurs if $L \neq s$. If norms are important to the couple, this will cause u to be less than ML and v less than $M(1 - L)$. By how much they are reduced depends on the extent of the departure from the norm and on the strength of the norm. Any variation in L transfers utility between agents but also affects the extent to which the norm is adhered to and hence the sum $u + v$; thus utility is transferable but imperfectly so. I model this by constructing a *utility loss factor* $\Lambda \leq 1$, so that

$$u = \Lambda ML \qquad v = \Lambda M(1 - L). \tag{3}$$

where Λ depends on L, s and the strength of the norm ρ.

One way to proceed is to place direct assumptions on Λ. Suppose that:

(i) if $L = s$ or if $\rho = 1$ then $\Lambda = 1$, in which case there is no loss of utility;
(ii) if $L \neq s$ and $\rho > 1$, then $\Lambda < 1$;
(iii) if $L > (resp <)s$ and if $\rho > 1$, then Λ is decreasing (resp. increasing) in L and increasing (resp. decreasing) in s.
(iv) if $L \neq s$, then Λ is decreasing in ρ.

Such a loss factor is given by

$$\Lambda = (L^\rho s^{1-\rho} + (1 - L)^\rho (1 - s)^{1-\rho})^{-1/\rho} \tag{4}$$

Combining (3) and (4) gives the utility possibility frontier:

$$g(u, v) = (s^{1-\rho} u^\rho + (1 - s)^{1-\rho} v^\rho)^{1/\rho} = M = f(x, y) \tag{5}$$

This ITU problem can be given a TU representation by taking $\widetilde{u} = s^{1-\rho}u^\rho$, $\widetilde{v} = (1-s)^{1-\rho}v^\rho$, and

$\phi(x,y) = [f(x,y)]^\rho$, so that (5) implies

$$\widetilde{u} + \widetilde{v} = \phi(x,y). \tag{6}$$

Alternatively, we can arrive at (5) via (3), Assumption 1 and Proposition 1. From (3), u and v are affected in the same proportion by a departure from the norm. This implies that $g(u,v)$ is homogenous of degree 1, since to double both u and v can only be achieved by a doubling of M. In addition, if $L = s$, so that $u/v = s/(1-s)$, then $u + v = M$. Thus parts (iii) and (iv) of Assumption 1 are satisfied. If we further assume (i) and - perhaps most importantly - (ii), then we have Proposition 1 and (5), which in turn implies (4) and the properties (i) - (iv) of Λ listed above.

4.2 Equilibrium

We now embed the UPF given by (5) into an equilibrium model of matching. Agents' types are common knowledge and there are no search frictions. Types are drawn from a type space $T \subseteq R$. Integrable functions ξ and ψ give the density of types amongst men and women respectively, with supports T_ξ and T_ψ. There is an equal mass of men and women. If a type x man matches with a type y woman they produce a positive output $f(x,y)$ and their possible payoffs, u and v, are are constrained by

$$g(u,v) = (s^{1-\rho}u^\rho + (1-s)^{1-\rho}v^\rho)^{1/\rho} \leq f(x,y)$$

I assume that agents' outside options are such that all agents are matched (an assumption which is revisited in Section 4.4.2). Then a *matching* is an element of Θ, the set of measures on T^2 with marginals ξ and ψ, describing which types of men are matched with which types of women. Total output from a matching θ is $\int_{T\times T} f d\theta$.

An *equilibrium* is a matching and payoff functions describing which types get what such that the matching that cannot be blocked; i.e. no unmatched pair can break away from their partners, form a couple, and both be better off. Thus matched pairs satisfy the relevant constraint on their UPF, and pairs outside the support of the matching cannot. More precisely, if the measure θ and the payoffs $u(x)$ and $v(y)$ are an equilibrium then

$$\begin{aligned} g(u(x), v(y)) &\geq f(x,y) \text{ for all } (x,y) \in T_\xi \times T_\psi. \\ g(u(x), v(y)) &= f(x,y) \text{ if and only if } (x,y) \in supp(\theta) \end{aligned}$$

But given the TU representation (6) this is equivalent to

$$\begin{aligned} \widetilde{u}(x) + \widetilde{v}(x) &\geq \phi(x,y) \text{ for all } (x,y) \in T_\xi \times T_\psi. \\ \widetilde{u}(x) + \widetilde{v}(y) &= \phi(x,y) \text{ if and only if } (x,y) \in supp(\theta) \end{aligned}$$

Thus $(x,y) \in supp(\theta)$ if

$$\left.\begin{aligned} \widetilde{u}(x) = \max_y \phi(x,y) - \widetilde{v}(y) \\ \widetilde{v}(y) = \max_x \phi(x,y) - u(x) \end{aligned}\right\} \tag{7}$$

The existence of equilibrium in this type of model is well established; e.g. Keneko (1982), or Galichon (2016), Theorem 7.6. Our strategy now is to analyse the matching pattern and payoffs $\widetilde{u}$ and $\widetilde{v}$ that follow from (7) using standard TU methods. That equilibrium will depend on ρ; we then show how payoffs u and v depend on ρ.

4.3 The model with simple functional forms

To highlight the main issues and possibilities that arise when we treat social norms as a source of imperfect transferability, I take very simple functional forms for ξ, ψ, and f and derive explicit expressions for the matching pattern and the equilibrium payoff functions. I assume

Assumption 2
(i) $f(x,y) = (x+y)^r$, where $r > 0$.
(ii) $T_\xi = [a,b]$ and $\xi(x) = (b-a)^{-1}$; $T_\psi = [c,d]$ and $\psi(y) = (d-c)^{-1}$.

Thus male and female types are both uniformly distributed but on possibly different intervals. An important role is played by $R = \frac{d-c}{b-a}$, the ratio of the type densities.

4.3.1 The matching pattern

From Assumption 2, $\phi(x,y) = (x+y)^{r\rho}$. If $r\rho < 1, \phi_{xy} < 0$ and we have NAM; if $r\rho > 1, \phi_{xy} > 0$ and we have PAM. If $r\rho = 1$, the matching pattern is indeterminate. Denoting by $q_m(x)$ the output produced by a man of type x and his partner, and by $q_w(y)$ the output produced by a woman of type y and her partner, the table below shows, for $r\rho \neq 1$, who matches with whom, and the resulting output.

	$r\rho < 1$	$r\rho > 1$
type x man is matched with a woman of type	$d - R(x-a)$	$c + R(x-a)$
who together produce $q_m(x)$	$[x(1-R) + d + Ra]^r$	$[x(1+R) + c - Ra]^r$
type y woman is matched with a man of type	$a + (d-y)/R$	$a + (y-c)/R$
who together produce $q_w(y)$	$\left[y(1-R^{-1}) + a + d/R\right]^r$	$\left[y(1+R^{-1}) + a - c/R\right]^r$

4.3.2 The payoff functions in $(\widetilde{u}, \widetilde{v})$ space

$r\rho < 1$ This gives us NAM. Then the payoff functions satisfying (7) are

$$\widetilde{u}(x) = \begin{cases} \frac{1}{1-R}\left[(q_m(x))^\rho - (q_m(a))^\rho\right] + \widetilde{u}_a & \text{if } R \neq 1 \\ r\rho(a+d)^{r\rho-1}(x-a) + \widetilde{u}_a & \text{if } R = 1 \end{cases}$$

$$\widetilde{v}(y) = \begin{cases} \frac{1}{1-R^{-1}}\left[(q_w(y))^\rho - (q_w(c))^\rho\right] + \widetilde{v}_c & \text{if } R \neq 1 \\ r\rho(a+d)^{r\rho-1}(y-c) + \widetilde{v}_c & \text{if } R = 1 \end{cases}$$

where $\widetilde{u}_a$ and $\widetilde{v}_c$ satisfy $\widetilde{u}_a + \widetilde{v}(d) = [a+d]^{r\rho}$ and $\widetilde{u}(b) + \widetilde{v}_c = [b+c]^{r\rho}$.[10] Thus

$$\widetilde{u}_a + \widetilde{v}_c = \begin{cases} \frac{1}{1-R}(a+d)^{r\rho} + \frac{1}{1-R^{-1}}(b+c)^{r\rho} & \text{if } R \neq 1 \\ (a+d)^{r\rho-1}(a+d-r\rho(d-c)) & \text{if } R = 1 \end{cases}$$

Note that the sum $\widetilde{u}_a + \widetilde{v}_c$ is continuous in R and converges to $a+c$ as $\rho \to 1/r$.

[10] Note that if $R < 1$, (resp. > 1) then $q_m(x)$ is increasing (resp, decreasimg) in x and $q_w(y)$ is decreasing (resp. increasing) in y.

$r\rho = 1$ Now $\phi(x, y) = x + y$; then although the matching is indeterminate, the payoffs are not, and reflect the absence of any complementarity or substitutability between x and y. More precisely

$$\begin{aligned}\widetilde{u}(x) &= x - a + \widetilde{u}_a \\ \widetilde{v}(x) &= y - c + \widetilde{v}_c\end{aligned}$$

where $\widetilde{u}_a + \widetilde{v}_c = a + c$.

$r\rho > 1$ This gives us PAM. Then the payoff functions satisfying (7) are

$$\begin{aligned}\widetilde{u}(x) &= \frac{1}{1+R}[(q_m(x))^\rho - (q_m(a))^\rho] + \widetilde{u}_a \\ \widetilde{v}(x) &= \frac{R}{1+R}[(q_w(y))^\rho - (q_w(a))^\rho] + \widetilde{v}_c\end{aligned}$$

where $\widetilde{u}_a + \widetilde{v}_c = (a + c)^{r\rho}$; again, as $\rho \to 1/r, \widetilde{u}_a + \widetilde{v}_c \to a + c$.

Agents' outside options We have thus determined payoff functions $\widetilde{u}$ and $\widetilde{v}$, up to a constant. The sum $\widetilde{u}_a + \widetilde{v}_c$ is constrained to ensure that the payoffs of any matched couple add up to their output and that all agents are matched. But there remains a degree of indeterminacy; this can be resolved in a number of ways, each reflecting agents' outside options i.e. their opportunities if they remain single. For the time being, we remain agnostic on the precise determination of $\widetilde{u}_a$ and $\widetilde{v}_c$. However, it should be noted that for a given ρ a higher value of either $\widetilde{u}_a$ or $\widetilde{v}_c$ must be reflected in a lower value of the other, and a higher value of ρ requires either a lower $\widetilde{u}_a$ or a lower $\widetilde{v}_c$ or both. The possibility that for some values of ρ the constraint on $\widetilde{u}_a + \widetilde{v}_c$ cannot be satisfied is addressed in Section 4.4.2.

Continuity of $\widetilde{u}$ and $\widetilde{v}$ Note that the sum $\widetilde{u}_a + \widetilde{v}_c$ is continuous in ρ so that fixing one makes the other continuous in ρ and thus, more generally, makes both $\widetilde{u}$ and $\widetilde{v}$ continuous in ρ. The two sets of payoff functions, when $r\rho < 1$ and $r\rho > 1$, are both continuous in ρ and as $\rho \to 1/r$ they converge to those when $r\rho = 1$. So although the matching pattern switches from NAM to indeterminacy to PAM as ρ increases and passes though $1/r$, the payoff of each agent changes but not discontinuously.

4.3.3 The payoff functions in (u, v) space

We now revert to the original payoff functions by applying the transformations $u = s^{1-1/\rho}\widetilde{u}^{1/\rho}$ and $v = (1-s)^{1-1/\rho}\widetilde{v}^{1/\rho}$. Then

$$u(x) = \begin{cases} \left(\dfrac{s^{\rho-1}}{1-R}\left[(q_m(x))^\rho - (q_m(a))^\rho\right] + u_a^\rho\right)^{1/\rho} & \text{if } r\rho < 1 \text{ and } R \neq 1 \\ (s^{\rho-1} r\rho(a+d)^{r\rho-1}(x-a) + u_a^\rho)^{1/\rho} & \text{if } r\rho < 1 \text{ and } R = 1 \\ (s^{\rho-1}(x-a) + u_a^\rho)^{1/\rho} & \text{if } r\rho = 1 \\ \left(\dfrac{s^{\rho-1}}{1+R}\left[(q_m(x))^\rho - (q_m(a))^\rho\right] + u_a^\rho\right)^{1/\rho} & \text{if } r\rho > 1 \end{cases}$$

$$v(y) = \begin{cases} \left(\dfrac{(1-s)^{\rho-1}}{1-R^{-1}}\left[(q_w(y))^\rho - (q_w(c))^\rho\right] + v_c^\rho\right)^{1/\rho} & \text{if } r\rho < 1 \text{ and } R \neq 1 \\ \left((1-s)^{\rho-1} r\rho(a+d)^{r\rho-1}(y-c) + v_c^\rho\right)^{1/\rho} & \text{if } r\rho < 1 \text{ and } R = 1 \\ \left((1-s)^{\rho-1}(y-c) + v_c^\rho\right)^{1/\rho} & \text{if } r\rho = 1 \\ \left(\dfrac{(1-s)^{\rho-1}R}{1+R}\left[(q_w(y))^\rho - (q_w(c))^\rho\right] + v_c^\rho\right)^{1/\rho} & \text{if } r\rho > 1 \end{cases}$$

Here, $u_a = s^{1-1/\rho}\widetilde{u}_a^{1/\rho}$ and $v_c = (1-s)^{1-1/\rho}\widetilde{v}_c^{1/\rho}$. Then $s^{1-\rho}u_a^\rho + (1-s)^{1-\rho}v_c^\rho = \widetilde{u}_a + \widetilde{v}_c$, so the payoff functions $u(x)$ and $v(y)$ still have a degree of indeterminacy but u_a and v_c inherit the constraints on $\widetilde{u}_a$ and $\widetilde{v}_c$. Then for a given value of ρ a higher value of either u_a or v_c must be reflected in a lower value of the other, and if ρ changes then so must either u_a or v_c or both (in Section 4.4.2 we analyse what happens if they do not).

4.4 Who gains and loses? Market forces versus social forces.

We now identify and isolate some of the key forces at play in the determination of payoffs and illustrate the range of possibilities by analysing a number of special cases.

Market forces If $\rho = 1$, social norms have no effect and the equilibrium is determined by the interaction of technology as given by $f(x, y)$, the distributions of types as given by ξ and ψ, and the specification of u_a and v_c, where $u_a + v_c = (a+c)^r$. We label these variables as market forces. They generate the following payoffs:

$$u(x) = \begin{cases} \dfrac{1}{1-R}\left[q_m(x) - q_m(a)\right] + u_a & \text{if } r < 1 \text{ and } R \neq 1 \\ r(a+d)^{r-1}(x-a) + u_a & \text{if } r < 1 \text{ and } R = 1 \\ x - a + u_a & \text{if } r = 1 \\ \dfrac{1}{1+R}\left[q_m(x) - q_m(a)\right] + u_a & \text{if } r > 1 \end{cases}$$

$$v(y) = \begin{cases} \dfrac{R}{1-R}\left[q_w(y) - q_w(c)\right] + v_c & \text{if } r < 1 \text{ and } R \neq 1 \\ r\rho(a+d)^{r-1}(y-c) + v_c & \text{if } r < 1 \text{ and } R = 1 \\ y - c + v_c & \text{if } r = 1 \\ \dfrac{R}{1+R}\left[q_w(y) - q_w(c)\right] + v_c & \text{if } r > 1 \end{cases}$$

This shows two important features of pure market forces. Firstly, conditions in other markets, insofar as they affect the values of u_a and v_c, feed through to the marriage market very directly.

All men benefit, and all women suffer, from better male, or worse female, outside options that result in higher u_a and/or lower v_c.

Secondly, the relative supply of different types of men and women, as reflected in the ratio of type densities R, affects how much extra payoff a higher type agent gets via two routes; (i) through changes in the output $q_m(x)$ and $q_w(y)$ and (ii) through the share of that changed output (with $r = 1$ or $R = 1$ being regarded as limiting cases). For example, if $r < 1$ we have NAM, and a higher type man gets a share $\frac{1}{1-R}$ of the change to $q_m(x)$ (which for $R > 1$ is decreasing in x); if $r > 1$ we have PAM, $q_m(x)$ is increasing in x, and a higher type man gets a share $\frac{1}{1+R}$ of the increase in output.

Social forces When $\rho > 1$, social norms have an effect via the values of s and ρ. We label these as social forces and for finite values of ρ, market and social forces interact. If $\rho > 1$ then, not surprisingly, almost all men gain and almost all women suffer from a higher value of s (the exceptions being when $x = a$ or $y = c$). But the effect of a higher value of ρ on payoffs is less straightforward.

Revisiting the two features of pure market forces above, we see firstly that men still gain and women lose if u_a is higher and thus (via the constraint connecting them) v_c is lower; but if $\rho > 1$ the effect is less than one-for-one. More precisely, treating $u(x)$ and $v(y)$ as functions of u_a and v_c respectively, then $\frac{du(x)}{du_a} = \left(\frac{u_a}{u(x)}\right)^{\rho-1}$ and $\frac{dv(y)}{dv_c} = \left(\frac{v_c}{v(y)}\right)^{\rho-1}$, both of which, for $\rho > 1$ and $x > a$ or $y > c$, are less than 1 and tend to zero as $\rho \to \infty$.

Secondly, the type densities still plays a role, but the higher is ρ the less important is the second route identified above, the role of R in determining the share of changed output going to higher types. Consider what happens as the social norm becomes increasingly strong: for $\rho > 1/r$, we have PAM and the type densities then determine which male and female types are matched and the resulting outputs $q_m(x)$ and $q_w(y)$. But for $x > a$ or $y > c$, as $\rho \to \infty$

$$\begin{aligned} u(x) &\to sq_m(x) \\ v(y) &\to (1-s)q_w(y) \end{aligned}$$

Thus market forces are overwhelmed and an agent's share of the output they produce is completely given by the social norm s, with no role for u_a or v_c.

However, although an increase in the strength of a given norm reduces the influence of market forces, the effect on payoffs is ambigous. As ρ increases from 1 and approaches ∞ there is a range of possibilities; these are illustrated in the next section.

4.5 The effect on payoffs of a stronger social norm

A stronger norm affects payoffs in two ways: (i) it affects the division of a given output; and (ii) it affects the matching pattern, changing agents' partners and the output to be shared.

4.5.1 The effect of a stronger social norm with a given matching

In order to focus on the effect of stronger norms independently of changes in the matching pattern, we confine ourselves in this section to the case where $r > 1$. We will therefore always have PAM and as ρ changes each agent's partner remains the same, as does the output that each couple produces. What changes is the division of that output and each partner's payoff.

A useful way to summarise the effect of ρ on payoffs is in the form of a locus in (u, v) space that traces out payoffs as x and y increase, where x and y types are matched (so that $q_m(x) = q_w(y)$). With PAM, this locus will be upward sloping, starting at $(u(a), v(c))$ and ending at $(u(b), v(d))$. For finite ρ, it satisfies the equation

$$\frac{(v(y))^\rho - v_c^\rho}{(u(x))^\rho - u_a^\rho} = R\left(\frac{1-s}{s}\right)^{\rho-1}$$

where $s^{1-\rho}u_a^\rho + (1-s)^{1-\rho}v_c^\rho = (a+c)^{r\rho}$. For $\rho = 1$

$$\frac{v(y) - v_c}{u(x) - u_a} = R$$

and as $\rho \to \infty$ the (u, v) locus converges to a line that starts at $(s(a+c)^r, (1-s)(a+c)^r)$, ends at $(s(b+d)^r, (1-s)(b+d)^r)$ and satisfies

$$\frac{v(y)}{u(x)} = \frac{1-s}{s}$$

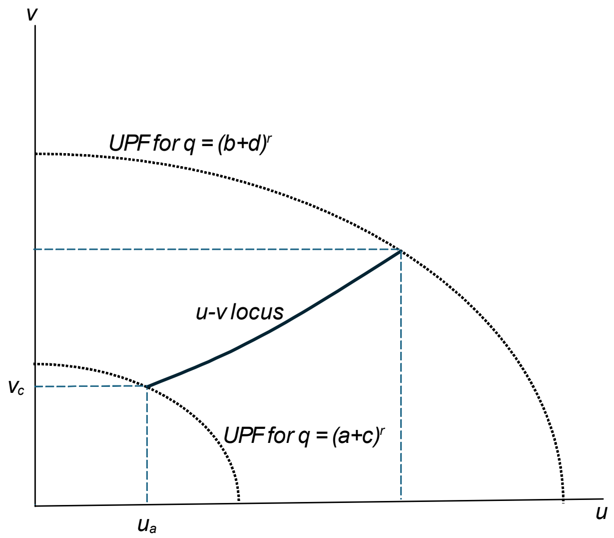

Figure 3: u-v locus when $\rho = 2$

Figure 3 shows the (u, v) locus when $\rho = 2$, with x and y distributed uniformly on $[0, 1]$ and $[2, 4]$ respectively, $s = 0.75, r = 1.5$, and $u_a = v_c = \sqrt{0.75}$.

To see who loses and who benefits as market forces are superceded by social forces. or vice versa, we compare the loci when $\rho = 1$ and as $\rho \to \infty$. In the cases below, u_a and v_c are the values when $\rho = 1$ (i.e. such that $u_a + v_c = (a+c)^r$).

Case 1: The norm has no effect Suppose that $v_c/u_a = (1-s)/s = R$. Then for both $\rho = 1$ and as $\rho \to \infty$

$$\begin{aligned} u(x) &= sq_m(x) = \frac{1}{1+R} q_m(x) \\ v(y) &= (1-s)q_w(y) = \frac{R}{1+R} q_w(y) \end{aligned}$$

Output is always shared according to the norm, or equivalently according to the ratio of densities. Thus market forces and social forces are aligned and work in the same direction.[11] The (u, v) locus when $\rho = 1$ coincides with that when $\rho \to \infty$. . Figure 4 illustrates.

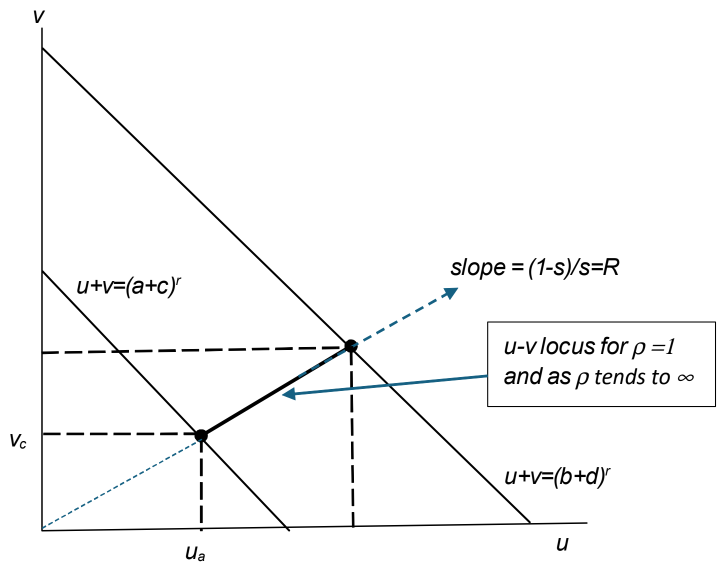

Figure 4: the norm has no effect

This is clearly a very special case and we now examine what happens when the two loci do not coincide.

Case 2a: A stronger norm disadvantages all women Here the (u, v) locus when $\rho = 1$ lies above that when $\rho \to \infty$. This requires that (i) $v_c/u_a > (1-s)/s$ and (ii) $R > \dfrac{(1-s)q_{\max} - v_c}{sq_{\max} - u_a}$, where $q_{\max} = q_m(b) = q_w(d)$. Thus, if market forces operate unimpeded by social forces, good outside options for women resulting in a high ratio of v_c to u_a benefit all women, and disadvantage all men. But if the norm prevails, all women are obliged to settle for a lower share of the marital cake. Figure 5 illustrates.

Case 2b: A strong norm advantages all women This is the reverse of Case 2a but it also throws light on an important effect of strong norms. If women's outside opportunities are poor, reflected in a low value of v_c, then market forces will make all women badly off. A strong norm nullifies this effect, and guarantees all women a fixed share of the household output. Here the norm insulates women from market forces; in Case 2a they cannot take advantage of them.

[11] Treating sq and $(1-s)q$ as premuneration values and using the terminology of Echenique and Galichon (2017), we have a no-trade stable matching.

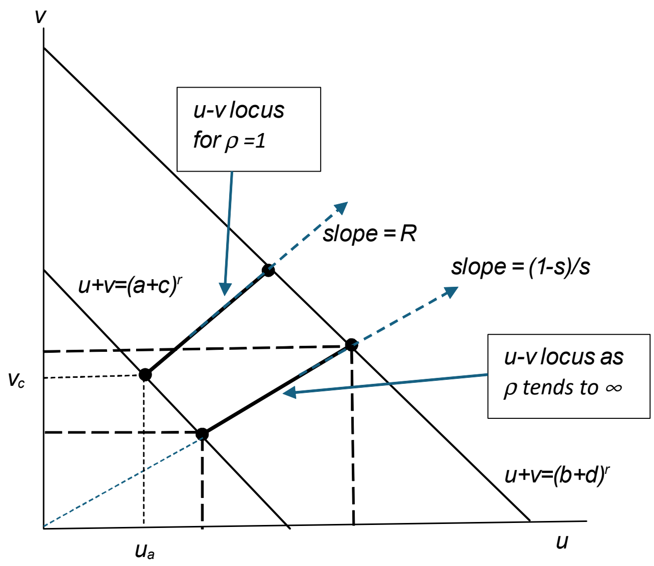

Figure 5: a stronger norm disadvantages all women

Case 3a: Strong norms disadvantage low type women and benefit high type women This is a mix of Cases 2a and 2b in that $v_c/u_a > (1-s)/s$ but now $R < \dfrac{(1-s)q_{\max} - v_c}{sq_{\max} - u_a}$. Diagrammatically, the (u, v) loci intersect; the locus when $\rho = 1$ initially lies above that when $\rho \to \infty$, but eventually falls below it. Thus, the effect of a high density of women at each type, $\frac{1}{d-c} > \frac{1}{b-a}$, is to weaken their market position and reduce their share of the additional output they produce. Low type women are well off with strong market forces, but for sufficiently high types the overall effect is worse than if the social norm prevails. Figure 6 illustrates.

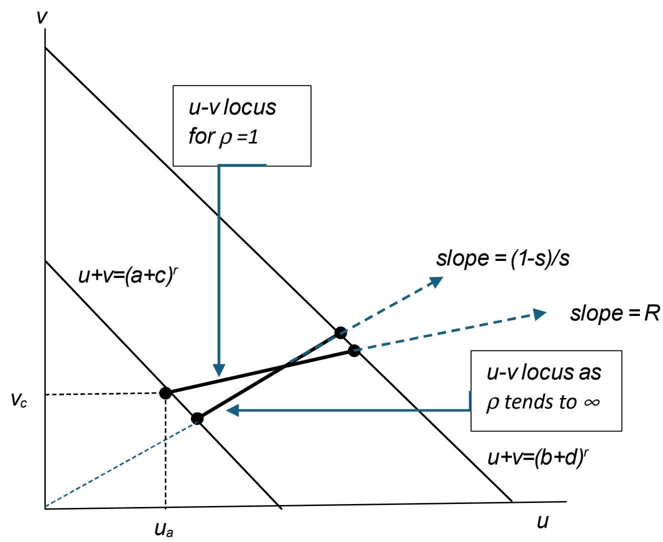

Figure 6: a strong norm disadvantages low type women and benefits high type women

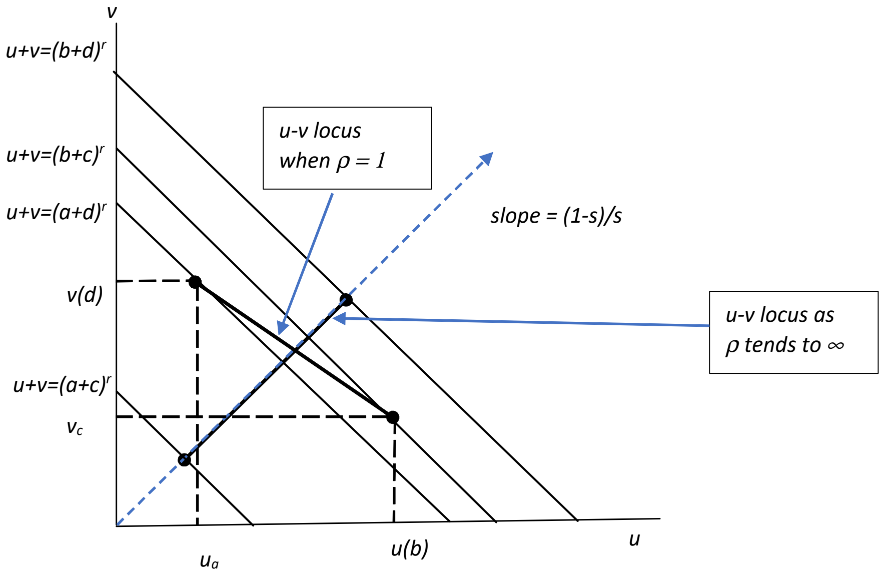

Figure 7: u-v loci when *r < 1* and *R < 1*.

Case 3b: Strong norms advantage low type women and disadvantage high type women This is the reverse of Case 3b: $v_c/u_a < (1-s)/s$ and $R > \frac{(1-s)q_{\max}-v_c}{sq_{\max}-u_a}$. Strong norms insulate lower type women from market forces but higher type women, who would otherwise have a strong market position, are prevented by the norm from taking advantage of this.

4.5.2 The effect of a strong social norm when the matching changes

If $\phi(x,y) = (x+y)^{r\rho}$ and $r < 1$ then we have NAM when the social norm has no force ($\rho = 1$) and PAM when $\rho > 1/r$. As before, we compare payoffs in the two extreme cases, $\rho = 1$ and the limit as $\rho \to \infty$, using the (u,v) locus showing $u(x)$ and $v(y)$ when x and y types are matched, but assuming now that $r < 1$.

With NAM, the (u,v) locus is downward sloping: one end is located at $(u(a), v(d))$, the other at $(u(b), v(c))$. For $\rho = 1$ the locus satisfies

$$\frac{v(y)-v(d)}{u(x)-u_a} = \frac{v(y)-v_c}{u(x)-u(b)} = -R$$

The (u,v) locus as $\rho \to \infty$ is the same as in the previous section.

Figure 7 shows the two loci when $\rho < 1$. Here, $R < 1$, so that $a+d < b+c$. When $\rho = 1$, a type men match with d type women; higher type men match with lower type women and produce a larger q, up to the point where b type men match with c type women; higher types still get higher payoffs. As $\rho \to \infty$, we have PAM, with couples sharing according to the norm.

What additional possibilities arise when the matching pattern changes as a result of a strong norm? Figure 7 illustrates in stark form the most important consequence. The switch from NAM to PAM reduces aggregate output, so the norm must have some effect, in contrast to Case 1 above. In particular, there must be some losers. Figure 7 shows a case where everyone loses. High types get less: $s(b+d)^r < u(b)$ and $(1-s)(b+d)^r < v(d)$; so do low types: $(a+c)^r < u_a$

and $(1-s)(a+c)^r < v_c$. This uniformity of loss occurs because the two sides of the market are very similar in their outside options and their type distributions, and the norm does not strongly favour one side or the other (so $(1-s)/s$ is close to 1). The output loss from PAM thus affects both sides and all types.[12]

As when $r > 1$, there may be some winners from strong norms. We can use Figure 7 to show a range of possibilities by varying s, and thus the slope of the (u, v) locus as $\rho \to \infty$. As s rises, the locus becomes flatter, rotating clockwise; but it always starts from a point such that $u + v = (a + c)^r$ and finishes on a point where $u + v = (b + d)^r$. Then the greater is s, the less is the loss (and the greater is any gain) to men from a strong norm and *vice versa* for women.[13]

4.6 Norms and the size of the marriage market

If social norms make transfers more costly, this reduces the effectiveness and efficiency of the marriage market. The stronger the norm, the more restricted is the ability of a marriage to deliver outcomes better than the alternative. I now explore the possibility that the marriage market will shrink as a result of strong norms.

I take u_a and v_c to be equal to the actual outside options of men and women, their values exogenously given and the same for all types; I therefore relabel them as $\underline{u}$ and $\underline{v}$ respectively, We cannot now assume that $\underline{u}$ and $\underline{v}$ will satisfy the constraints hitherto imposed on u_a and v_c. If agents can do better single than married they will remain unmatched. Any unmatched agents will be those with lower types: an unmatched high type agent could always outbid a matched lower type by offering the lower type's partner a higher share of a larger cake and still get more than their outside option. For simplicity, I assume that if $\rho = 1$ type a men and type c women can just achieve utility $\underline{u}$ and $\underline{v}$ respectively, so in looking at the effect of stronger norms we start from a situation where all agents are married.

Consider first the case where $r \geq 1$. For $\rho = 1$, we have $\underline{u} + \underline{v} = (a + c)^r$. As ρ increases, the matching pattern is always PAM; for type a men and type c women to be matched and get utility no less than $\underline{u}$ and $\underline{v}$ requires that

$$[s^{1-\rho}\underline{u}^\rho + (1-s)^{1-\rho}\underline{v}^\rho]^{1/\rho} \leq (a+c)^r. \quad (8)$$

Unless $\underline{v}/\underline{u} = (1-s)/s$, the LHS of (8) is increasing in ρ;[14] and (8) is satisfied with equality if $\rho = 1$. Then assuming $\underline{v}/\underline{u} \neq (1-s)/s$, (8) cannot be satisfied for $\rho > 1$. The effect of strong norms is thus to remove from the marriage market an equal mass of lower types on each sides of the market; they remain single, exercise their outside options, and get $\underline{u}$ or $\underline{v}$.

Among those who do marry we continue to have positive assortment, with the lowest types remaining in the market being $\underline{x}$ and $\underline{y}$, where $\left(\underline{y} - c\right)/(\underline{x} - a) = R$, and $[s^{1-\rho}\underline{u}^\rho + (1-s)^{1-\rho}\underline{v}^\rho]^{1/\rho} = (\underline{x} + \underline{y})^r$. Thus the output $(\underline{x} + \underline{y})^r$ can be distributed to ensure types $\underline{x}$ and $\underline{y}$ get payoffs equal to

[12] To formally establish this possibility, suppose that $a = c = 0, b = d = 1$, (so $R = 1$), $s = 0.5$, and $u_a = v_c = (1-r)/2$. Then for $\rho = 1$ we have $u(x) = rx + (1-r)/2$ and $v(y) = ry + (1-r)/2$. As $\rho \to \infty, u(x) \to 2^{r-1}x$ and $v(y) \to 2^{r-1}y$. It is readily established that all agents are worse off as $\rho \to \infty$ compared to $\rho = 1$.

[13] In particular, although under PAM a high type man matches with a high type woman to produce a high output, that output cannot be shared to give each more than they get when $\rho = 1$; otherwise NAM would not be a stable outcome when $\rho = 1$. Similarly, under PAM a low type man is matched with a low type woman, and at least one of them is worse off than when $\rho = 1$.

[14] Hardy, Littlewood, and Polya, (1959) Theorem 16. If $\underline{v}/\underline{u} = (1-s)/s$ then norm strength is irrelevant as utilities $\underline{u}$ and $\underline{v}$ can be achieved without departing from the norm.

their outside options $\underline{u}$ and $\underline{v}$ respectively. Higher types get payoffs greater than $\underline{u}$ and $\underline{v}$. Figure 8 illustrates the case where for $\rho = 1$ (8) is satisfied but if $\rho = 2$ utilities $\underline{u}$ and $\underline{v}$ can only be attained with a higher output.

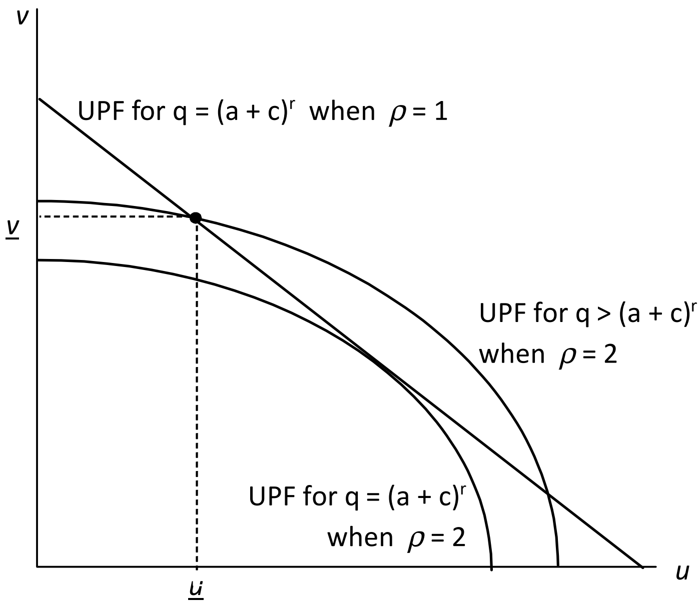

Figure 8: a stronger norm reduces the size of the marriage market

The higher is $\underline{u}$, $\underline{v}$, or ρ, the smaller is the marriage market, which disappears entirely if the very top couple cannot make a go of it:

$$[s^{1-\rho}\underline{u}^{\rho} + (1-s)^{1-\rho}\underline{v}^{\rho}]^{1/\rho} > (b+d)^r \tag{9}$$

As $\rho \to \infty$, $(s^{1-\rho}\underline{u}^{\rho} + (1-s)^{1-\rho}\underline{v}^{\rho}]^{1/\rho} \to \max[\underline{u}/s, \underline{v}/(1-s)]$, in which case for the the marriage market to survive requires that $\underline{u} < s(b+d)^r$ and $\underline{v} < (1-s)(b+d)^r$.

Suppose now that $r < 1$. For $1 \leq \rho < 1/r$, we have NAM. In the case where $R \neq 1$, for type a men and type c women to be matched and get utility no less than $\underline{u}$ and $\underline{v}$ requires that

$$[s^{1-\rho}\underline{u}^{\rho} + (1-s)^{1-\rho}\underline{v}^{\rho}]^{1/\rho} \leq \left[\frac{1}{1-R}(a+d)^{r\rho} + \frac{1}{1-R^{-1}}(b+c)^{r\rho}\right]^{1/\rho} \tag{10}$$

As with (8) the LHS of (10) is increasing in ρ, unless $\underline{v}/\underline{u} = (1-s)/s$: furthermore, the RHS of (10) is decreasing in ρ.[15] Repeating the exercise above, assuming $\underline{v}/\underline{u} \neq (1-s)/s$ and with (10) satisfied with equality for $\rho = 1$, an increase in ρ will induce a shrinking of the marriage market. Given NAM, this will necessitate a rematching of the remaining couples, as it will be the lower

[15]Proof: Let $X = \left[(1-R)^{-1}(a+d)^{r\rho} + \left(1-R^{-1}\right)^{-1}(b+c)^{r\rho}\right]^{1/\rho}$. If $R < 1$, we write $(a+d)^r = [(1-R)X^{\rho} + R(b+c)^{r\rho}]^{1/\rho}$, defining $(a+d)^r$ as a power mean of X and $(b+c)^r$. As $X \neq (b+c)^r$, for a given value of X the value of $[(1-R)X^{\rho} + R(b+c)^{r\rho}]^{1/\rho}$ is increasing in ρ. As $(a+d)^r$ and $(b+c)^r$ are independent of ρ, this implies that X is decreasing in ρ. If $R > 1$, $(b+c)^r = \left[(1-R^{-1})X^{\rho} + R^{-1}(a+d)^{r\rho}\right]^{1/\rho}$, and the same argument applies *mutatis mutandis.*

types who leave. As ρ rises but remains less than $1/r$, we continue to have negative assortment among those who do marry, with the lowest types remaining in the market being $\underline{x}$ and $\underline{y}$, where now $\left(d-\underline{y}\right)/(\underline{x}-a)=R$, and

$$[s^{1-\rho}\underline{u}^{\rho}+(1-s)^{1-\rho}\underline{v}^{\rho}]^{1/\rho}=\left[\frac{1}{1-R}(\underline{x}+d)^{r\rho}+\frac{1}{1-R^{-1}}(b+\underline{y})^{r\rho}\right]^{1/\rho} \tag{11}$$

Eventually $r\rho=1$, at which point the RHS of (11) equals $\left(\underline{x}+\underline{y}\right)^{r}$,the matching switches to PAM, and the marriage market continues to shrink as above, although the market will disappear entirely before the switch to PAM, if $s^{1-\rho}\underline{u}^{\rho}+(1-s)^{1-\rho}\underline{v}^{\rho}>b+d$ for $\rho=1/r$.

If $R=1$, the RHS of (10) is replaced by $\left[(a+d)^{r\rho-1}(a+d-r\rho(d-c))\right]^{1/\rho}$, and the same reasoning applies.[16]

5 DISCUSSION AND CONCLUSION

5.1 What is maximised in equilibrium?

If a TU representation exists, then an equilibrium matching maximises the total not of output as given by f but of transformed output as given by ϕ, where $\phi(x,y)=\gamma(f(x,y)$. Without further knowledge of the transformation γ this is not particularly informative. But if the UPF $g(u,v)$ is homogeneous and quasi-convex then $\phi(x,y)=[f(x,y)]^{\rho}$ for some $\rho\geq 1$, so any matching θ aggregates $[f(x,y)]^{\rho}$ over matched couples, yielding $\int_{T\times T}[f(x,y)]^{\rho}d\theta$. This is the quantity that is maximised in equilibrium. But then equilibrium also maximises $\left(\int_{T\times T}[f(x,y)]^{\rho}d\theta\right)^{1/\rho}$, which we denote by $Q(\rho,\theta)$.

As ρ increases $Q(\rho,\theta)$ attaches increasing weight to the higher values of $f(x,y)$; and as $\rho\to\infty$, $Q(\rho,\theta)\to\max_{(x,y)\in\operatorname{supp}\theta}f(x,y)$.[17] Thus one way to think about the equilibrium matching in the NTU case is that it looks for the couple for which $f(x,y)$ is the greatest, then for the couple from the remaining population with the highest $f(x,y)$, and so on. In the case where a higher type is always better, this means the i^{th} highest type male matches with the i^{th} highest type female. When like attracts like, the couple with the smallest type difference match, then the couple from the remaining population the with smallest type difference, and so on. Note that this leads to a unique equilibrium matching, in contrast to many NTU models in which there are multiple equilibria (see for example Chapter 3 of Roth and Sotomayor, 1990). A norm imposes the same division on any couple's output, s for him and $1-s$ for her, so that an agent always prefers to match with someone with whom they would jointly produce a larger output; agents' preference orderings thus collectively satisfy $\alpha-$*reducibility* (Alcade, 1995), which in turn is sufficient to guarantee uniqueness (Clark, 2006).

$Q(1,\theta)$ is the total output from a matching θ and in equilibrium this is maximised if $\rho=1$; but if $\rho>1$ there is no reason that it should be maximised in equilibrium. Indeed, if f is submodular but f^{ρ} is supermodular then in equilibrium $Q(1,\theta)$ is minimised (conditional on all

[16] To show that $\left[(a+d)^{r\rho-1}(a+d-r\rho(d-c))\right]^{1/\rho}$ is decreasing in ρ if $r\rho<1$, note first that it is homogeneous of degree r in (a,c,d) so w.l.o.g. we set $a+d=1$, implying $d-c<1$. We thus consider how $Y=(1-\rho X)^{1/\rho}$ varies with ρ, where $\rho X<1$. Setting $Z=1-\rho X$, then $dY/d\rho<0$ if $Z<1+Z\ln Z$ which is readily established for $0<Z<1$.

[17] Hardy et al, (1953), Theorem 193.

agents being matched). But for $\rho > 1$ aggregate output and aggregate utility are not the same and from a welfare perspective it is the latter which is more interesting.

Let us denote aggregate utility by $W = \int_T u(x)\xi(x)dx + \int_T v(y)\psi(y)dy$; it is a measure of social welfare that whilst being straightforwardly utilitarian nevertheless reflects individuals' aversion to departures from the norm. Within a household $u + v = \Lambda q$, where Λ is the loss factor defined in (3), so W aggregates Λq over all matched couples; therefore if an omniscient and omnipotent planner wanted to maximise W, the task could be decomposed into (i) maximising aggregate output by a suitable choice of matching, and then (ii) allocating each household's output q so that $\Lambda = 1$. If $\rho = 1$, any division of output will suffice; if $\rho > 1$, (ii) requires output to be divided according to the social norm.

Obviously, if $\rho = 1$ the decentralised matching equilibrium will maximise W. Less obviously, W is also maximised as $\rho \to \infty$ if f is supermodular; this is because f^ρ is then also supermodular (so equilibrium yields PAM and aggregate ouput is maximised); in the limit as $\rho \to \infty$, each marital cake is divided according to the social norm and for all households the achieved level of Λ is 1. However, if f is supermodular and ρ is greater than 1 but finite, then neither market forces nor social forces reign supreme and the achieved level of Λ in each household is typically less than 1. If PAM does not maximise aggregate output (e.g. if f is submodular), aggregate ouput is not maximised as $\rho \to \infty$, although it is allocated equitably within households. Then W is maximised in equilibrium only if $\rho = 1$.

5.2 Norms and inequality

One interpretation of the social norm modelled here is as a form of inequality aversion: a matched couple have a common reference point $(s, 1-s)$, deemed to be an equitable or fair division of how the marital pie is to be divided, and a common degree, ρ, of aversion to any deviation from an equitable division. But the aversion does not extend to inequalities between agents not matched with each other; if a very poor couple and a very rich couple both share their respective outputs in the ratio $s : 1 - s$ then that is not deemed inequitable. Thus a strong gender norm may reduce inter-gender inequality but do little to reduce intra-gender inequality. For example, Case 3a, illustrated in Figure 6, show a strong norm that reduces the payoff of low type women and increases that of high type women.

Case 3a assumes $r > 1$. When $r < 1$, an additional possibility arising from the switch from NAM to PAM as ρ increases is a greater variation in aggregate household incomes, $u + v$. With NAM outputs range from $f(a, d)$ to $f(b, c)$; with PAM they range from $f(a, c)$ to $f(b, d)$. This is accompanied by a lower variation in incomes *within* the household: with $\rho < 1/r$, and thus NAM, well off men are matched with badly off women and *vice versa*, whereas with $\rho > 1/r$ and PAM output is shared according to the norm. So if a strong norm brings about PAM rather than NAM, then even if it is judged to be a socially desirable consequence because it benefits one side of the market (e.g. women) who would otherwise be disadvantaged by market forces, it is still the case that high type women are better off than low types.

This view of the norm is less compelling if s is not close to 0.5, and in particular if the disutility from breaching the norm is due to sanctions or criticism from "society". In Case 2a, illustrated in Figure 5, s is high and advantages men. Then if u_a and v_c are roughly equal, and R is close to 1, the market outcome has greater equality between genders, although, as argued above, whether the norm is strong or weak has little effect on the distribution within each gender.

5.3 Market forces versus social forces

I have drawn a distinction between market forces (technology, the type densities, and the agents' outside options) and social forces (the norm and its strength). But the outside options are themselves the result of market and socal forces elsewhere in the economy. Norms in the marriage market may be very different from those elsewhere; and just as models of general equilibrium allow an analysis of how market forces have effects across the economy, so too would a more general model of norms allow an analysis of how social forces can spread. This is an interesting area for future research and would complement recent work that seeks to link matching patterns in different markets (see for example Calvo et al (2024)). More fundamentally, I do not consider why or how the social norms arise, nor how they might be affected by their interaction with market forces. These are important questions, but beyond the scope of this paper.

5.4 Matching and sorting

One of our main results is that imperfectly transferability of utility relaxes the conditions for PAM, at least when there is vertical differentiation. This stands in contrast to the work of (for example) Eeckhout and Kircher (2010), who analyse how search frictions can increase the tendency to negative sorting. They adopt a framework of vertical heterogeneity with directed search, in which matches emerge, via a search technology, from endogenously formed groups of buyers and sellers. Their key insight relies on the fact that each group need not have the same number of buyers as sellers. If a small number of high type sellers (resp. buyers) form a group with a large number of low type buyers (resp. sellers), then a relatively low proportion of those high types will remain unmatched, reducing the overall loss of output caused by search frictions. The tendency towards PAM induced by supermodularity effect is still present, but is not necessarily strong enough to overcome the benefits of NAM in an environment where not all agents are matched.

Non-transferability, whether partial or complete, can be considered a form of friction: there is an impediment to transfer which is costly, perhaps impossible, to overcome. Then one conclusion of this paper is that whereas search frictions increase the scope for NAM, the effect of transfer frictions is more subtle and depends on both technology and the distribution of types. Thus any empirical finding of PAM (e.g. Eika *et al* (2019) and Chiappori *et al* (2020) on educational assortative mating, Chiappori *et al* (2022) on matching on income) suggests a combination of the following: supermodularity is common; search frictions are rare; transfer frictions are common.

6 APPENDIX

6.1 Proposition 1

Assumption 1
(i) g is increasing in u and v, differentiable, and quasi-convex.
(ii) There exist increasing transformations functions α, β, and γ such that for any u and v if $g(u, v) = q$ then $\alpha(u) + \beta(v) = \gamma(q)$.
(iii) g is homogeneous of degree 1.
(iv) For some $0 < s < 1$,if $u/v = s/(1-s)$ then $g_u = g_v$ and $g(u, v) = u + v$.

Proposition 2 (1) *Let Assumption 1 hold. Then For some* $\rho \geq 1, g(u,v) = \left(s^{1-\rho}u^\rho + (1-s)^{1-\rho}v^\rho\right)^{1/\rho}$.

PROOF: Define
$$\widehat{\alpha}(\omega) = \frac{1}{s}\alpha(s\omega); \qquad \widehat{\beta}(\omega) = \frac{1}{1-s}\beta((1-s)\omega).$$
Then
$$\alpha(u) = s\widehat{\alpha}(\frac{u}{s}); \qquad \beta(v) = (1-s)\widehat{\beta}(\frac{v}{1-s}),$$
and if $g(u,v) = q$ then
$$s\widehat{\alpha}(\frac{u}{s}) + (1-s)\widehat{\beta}(\frac{v}{1-s}) = \gamma(q) \tag{A1}$$
Note that we are taking $\alpha(0) = \beta(0) = 0$ so $\widehat{\alpha}(0) = \widehat{\beta}(0) = 0$ and
$$\begin{aligned}\widehat{\alpha}'(\omega) &= \alpha'(s\omega)\\ \widehat{\beta}'(\omega) &= \beta'((1-s)\omega)\end{aligned}$$

Given the TU representation (α, β, γ), along the UPF for a given level of output q, $\alpha(u) + \beta(v) = \gamma(q)$, so that
$$\frac{du}{dv} = -\frac{\beta'(v)}{\alpha'(u)}$$
If. for any q, $u = sq$ and $v = (1-s)q$, then $\frac{du}{dv} = -1$ and
$$\frac{\beta'(v)}{\alpha'(u)} = \frac{\beta'((1-s)q)}{\alpha'(sq)}$$

so $\alpha'(sq) = \beta'((1-s)q)$ and hence $\widehat{\alpha}'(q) = \widehat{\beta}'(q)$. As $\widehat{\alpha}(0) = \widehat{\beta}(0)$ this implies $\widehat{\alpha} = \widehat{\beta}$.

From(iii) of Assumption 1, if $u/v = s/(1-s)$ then $g(u,v) = u + v$ so $g(sq, (1-s)q) = q$. Thus $\alpha(sq) + \beta((1-s)q) = \gamma(q)$, which in turn implies
$$s\widehat{\alpha}(q) + (1-s)\widehat{\beta}(q) = \gamma(q)$$
As $\widehat{\alpha} = \widehat{\beta}$, we have $\widehat{\alpha} = \widehat{\beta} = \gamma(q)$.Then (A1) can be expressed as
$$s\gamma(\frac{u}{s}) + (1-s)\gamma(\frac{v}{1-s}) = \gamma(q)$$
or
$$\gamma^{-1}(s\gamma(\frac{u}{s}) + (1-s)\gamma(\frac{v}{1-s})) = q \tag{A2}$$
Thus $q = g(u,v)$ is the generalised $\gamma-$ mean of u/s and $v/(1-s)$. As g is homogenous of degree 1 then for any $\lambda > 0$
$$\gamma^{-1}(s\gamma(\frac{\lambda u}{s}) + (1-s)\gamma(\frac{\lambda v}{1-s})) = \lambda\gamma^{-1}(s\gamma(\frac{u}{s}) + (1-s)\gamma(\frac{v}{1-s}))$$
By Theorem 84 of Hardy, Littlewood, and Polya (1959), γ must be a power function, with the form $\gamma(\omega) = \gamma_0\omega^\rho$. Then (A2) can be written as
$$s\gamma_0(\frac{u}{s})^\rho + (1-s)\gamma_0(\frac{v}{1-s})^\rho = \gamma_0 q^\rho$$
Thus
$$s(\frac{u}{s})^\rho + (1-s)(\frac{v}{1-s})^\rho = q^\rho$$

i.e.

$$g(u,v) = \left(s^{1-\rho}u^{\rho} + (1-s)^{1-\rho}v^{\rho}\right)^{-\rho}$$

6.1.1 g is homogeneous of degree k

Assumption 2

(i) g is increasing in u and v, differentiable, and quasi-convex.

(ii) There exist increasing transformations functions α, β,and γ such that for any u and v if $g(u,v) = q$ then $\alpha(u) + \beta(v) = \gamma(q)$

(iii) g is homogeneous of degree k.

(iv) For some $0 < s < 1$,if $u/v = s/(1-s)$ then $g_u = g_v$ and $(g(u,v))^{1/k} = u + v$.

Part (iv) of Assumption 2 replaces part (iv) of Assumption 1, which is clearly incompatible with homogeneity when $k \neq 1$.We now have

Proposition 3 (2) *Let Assumption 2 hold. Then For some* $\rho \geq 1, g(u,v) = \left(s^{1-\rho}u^{\rho} + (1-s)^{1-\rho}v^{\rho}\right)^{k/\rho}$

The proof is the same as that for Proposition 1, with $g(u,v)$ replaced by $(g(u,v))^{1/k}$.

The condition for PAM is now that $(f(x,y))^{\rho/\kappa}$ be supermodular.

6.1.2 g is homothetic

Proposition 2 has a simple generalisation. Suppose $g(u,v)$ is homothetic i.e. it is a strictly increasing transformation τ of a function $\widetilde{g}(u,v)$ which is homogeneous of degree k. Then $(\tau^{-1}(g(u,v)))^{1/\kappa}$ is homogeneous of degree 1 and the constraint $g(u,v) = f(x,y)$ can be expressed as

$$\tau^{-1}(f(x,y)) = \widetilde{g}(u,v)$$

If we generalise part (iv) of Assumption 2 so that for some $0 < s < 1$,if $u/v = s/(1-s)$ then $g_u = g_v$ and $(\tau^{-1}(g(u,v)))^{1/\kappa} = u + v$, we can apply Proposition 2, and the condition for PAM is that $(\tau^{-1}(f(x,y)))^{\rho/\kappa}$ be supermodular.

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