SGPE Summer School 2016 Mathematics Exam

Question 1:

Let $f(x) = \ln(e^x + e^{-x})$.

(a)[6pt] Compute $f'(x)$ and $f''(x)$.

(b) [2pt] What is the minimum of f?

(c)[2pt] Is f convex, concave, or neither?

Question 2:

Evaluate the following limits:

(a)[2pt] $\lim_{x \to 1} \frac{x^4 - 1}{x^2 - 1}$ (b)[2pt] $\lim_{x \to p} \frac{x - p}{\ln x - \ln p}$ (c)[3pt] $\lim_{x \to \infty} \frac{1}{x} \sin x$ (d)[3pt] $\lim_{x \to -2} \frac{\frac{1}{x} + \frac{1}{2}}{x^3 + 8}$

Question 3:

Determine if each of the following are functions. For those you consider function, determine domain and range; determine if they are continuous.

(a)[5pt] $y = \frac{1}{x}$ (b)[5pt] $y = \sqrt{1 - x^2} + \sqrt{x^2 - 1}$ (c)[5pt] $y^2 = x$

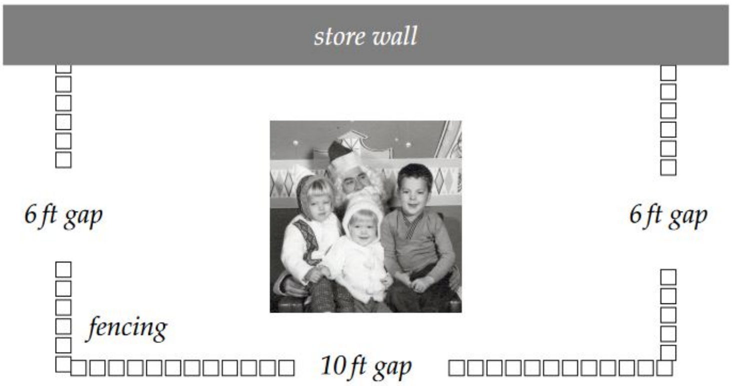
Question 4:

Calculate the following

(a)[5pt] $\int_e^{e^3} (4xlnx)\mathrm{d}x$ (b)[5pt] $\lim_{x \to 0} x \ln x^2$ (c)[5pt]$\int_{-2}^{2} (6x^2 + 10x + 3)\mathrm{d}x$

Question 5:

A department store is fencing off part of the store for children to meet and be photographed with Santa Claus. They have decided to fence off a rectangular region of fixed area 800 ft^2. Fire regulations require that there be three gaps in the fencing: 6 ft openings on the two facing sides and a 10 ft opening on the remaining wall (the fourth side of the rectangle will be against the building wall). Find the dimensions that will minimize the length of fencing used. [10pt]



Question 6:

Consider $f : [1, \infty) \to \mathbb{R}$ defined by $f(x) = 1/x$. Write a general expression for k^{th} derivative $f^{(k)}$. [12pt]

Question 7:

Solve the following linear system of equations[12pt]

$$\begin{aligned} x + y + z &= 3 \\ y - z - 1 &= 0 \\ x + 3z - 2y &= 1 \end{aligned}$$

Question 8:

Optimize the following function by (1) finding the critical value(s) at which the function is optimized and (2) testing the second-order condition to distinguish between a relative maximum or minimum and (3) the values of the relative extrema for the function.[16pt]

$$f(x) = x^2 + 6x - 96$$