

Problem Set 8 - Game Theory

1. Two firms at the St. Louis airport have franchises to carry passengers to and from hotels in downtown St. Louis. These two firms, Metro Limo and Urban Limo, operate nine passenger vans. These duopolists cannot compete with price, but they can compete through advertising. Their payoff matrix is below:

		United Limo	
		Increase Advertising	Don't Increase Advertising
Metro Limo	Increase Advertising	25, 15	30, 0
	Don't Increase Advertising	15, 20	40, 5

- a) Does each firm have a dominant strategy? If so, explain and what that strategy is.
- b) What is the Nash equilibrium? Explain where the Nash equilibrium occurs in the payoff matrix.
2. Suppose we have a football player taking a penalty kick. The kicker can kick right or left, and the goalie can defend right or left (assume kicker's right). Write the game using general parameters. Find all the equilibria, pure and mixed. Draw the best response curves.
3. Two firms are considering entering a new market. Entrance requires construction of a highly specialized plant. Demand is sufficient for either one to be profitable, but not both. A newspaper writer, observing the posturing of the two firms, each stating that they are planning to go ahead with plans for the new facility, noted, "sunk costs make for credible threats." What does she mean by this statement?
4. A frog and a scorpion were standing on the bank of the river trying to figure out a way to cross. "I know," said the scorpion "I will climb on your back and you can swim across the river." The frog said "But what if you sting me with your stinger?" The scorpion said, "Why would I do that? Then we would both die." The frog found this convincing, so the scorpion climbed on his back and they started across the river. Halfway across, at the deepest point, the scorpion stung the frog. Writhing in pain, the frog cried out, "Why did you do that? Now we are both doomed!" "Alas," said the scorpion, as he sank into the river, "it is my nature." Draw this game in extensive form, what could the

frog have done to prevent this?

5. The widget market is controlled by two firms, Acme Widget Company and Widgetway Manufacturing. The structure of the market makes secret pricecutting impossible. Each firm announces a price at the beginning of the time period and sells widgets at the price for the duration of the period. There is very little brand loyalty among widget buyers so that each firm's demand is highly elastic. Each firm's prices are thus very sensitive to inter-firm price differentials. The two firms must choose between a high and low price strategy for the coming period. Profits (measured in thousands of dollars) for the two firms under each price strategy are given in the payoff matrix below. Widgetway's profit is before the comma, Acme's is after the comma.

		Acme	
		Low Price	High Price
Widgetway	Low Price	60, 60	250, -20
	High Price	-20, 250	130, 130

- a) Does either firm have a dominant strategy? What strategy should each firm follow?
- b) Assume that the game is to be played an infinite number of times. (Or, equivalently, imagine that neither firm knows for certain when rounds of the game will end, so there is always a positive chance that another round is to be played after the present one.) Would the tit-for-tat strategy would be a reasonable choice? Explain this strategy.
- c) Assume that the game is to be played a very large (but finite) number of times. What is the appropriate strategy if both firms are always rational?
6. Boeing and Airbus compete over the quantity of airplanes they produce. The inverse demand curve for airplanes is

$$P = 100 - Q$$

Where Q is the total amount of airplanes produced. Suppose the marginal cost of producing an airplane (in millions) is 10. What is the Nash equilibrium level of production for both firms? (This is a bit tough as we haven't covered Cournot competition, but we have all the tools to answer it)

7. Instead, what happens if Boeing and Airbus compete over the price of airplanes they produce (assume they are perfectly substitutable and they have the same marginal cost)? What is the Nash equilibrium?
8. In class we played a game where if all people play heads, everybody gets a prize. If one person plays tails and the others play heads, the one person who plays tails gets a big prize and all the other people in class keep their small prize. If two or more people play tails, everybody gets nothing. What are all the Nash equilibria of this game?
9. Let's play the dictator game. I give you \$10 and you can offer any amount to your opponent. If your opponent accepts the amount you both get the payoff, if he rejects, you both get nothing. What is the equilibrium offer? Experiments show that people generally reject offers less than 1/3 of the amount, what is game theory missing?
10. Mitchell Electronics produces a home video game that has become very popular with children. Mitchell's managers have reason to believe that Wright Televideo Company is considering entering the market with a competing product. Mitchell must decide whether to set a high price to accommodate entry or a low, entry-detering price. The payoff matrix below shows the profit outcome for each company under the alternative price and entry strategies. Mitchell's profit is entered before the comma, and Wright's is after the comma.

		Wright Televideo	
		Enter	Don't Enter
Mitchell Electronics	High Price	60, 25	85, 0
	Low Price	30, -20	60, 0

- a) Does Mitchell have a dominant strategy? Explain.
- b) Does Wright have a dominant strategy? Explain.
- c) Mitchell's managers have vaguely suggested a willingness to lower price in order to deter entry. Is this threat credible in light of the payoff matrix above?
- d) If the threat is not credible, what changes in the payoff matrix would be necessary to make the threat credible? What business strategies could Mitchell use to alter the payoff matrix so that the threat is credible?

11. Chevron and Arco are two petrol stations in direct competition with each other. Each firm can set a low price or a high price. If they each set a high price, they will split the market. If they each set a low price, they will split the market again, but at a lower profit. If one firm sets a low price and the other firm sets a high price, the low-price firm will capture all of the customers.

		Chevron	
		A	B
Arco	A	10, 10	0, 15
	B	15, 0	5, 5

- i) What is the Nash equilibrium if this game is played once?
 - ii) If this game is repeated indefinitely what will happen?
 - iii) If there is a prediction that the world will end in December, 2015. Both the manager of Chevron and Arco firmly believe this. Does your result in b change?
12. In race to the bottom games (where everybody tries to undercut each other) people will end up playing the bottom value. If I suspect my opponents have bounded rationality (they aren't that smart) should I play the Nash equilibrium?
13. The k^{th} order statistic of n draws of a distribution is the expected value of the k^{th} highest draw. Suppose everybody's valuation is drawn from the same distribution. In some auctions, people bid their valuations and in others they shade their bids. For revenue equivalence, the expected revenue from any auction is the same. What must the relationship be between the first and second order statistic?
14. Suppose you are bidding over rights to drill a certain oilfield. People aren't entirely certain how valuable this oilfield is, but they will come up with estimated valuations. How would your bidding strategy be different than in the private value auctions we have seen?
15. (Bonus): Show that each of the following statements is true:

- i)
$$\sum_{t=0}^{\infty} \delta^t = \frac{1}{1-\delta}$$

$$\text{ii) } \sum_{t=0}^T \delta^t = \frac{1 - \delta^{T+1}}{1 - \delta}$$

$$\text{iii) } \sum_{t=T}^{\infty} \delta^t = \frac{\delta^T}{1 - \delta}$$

$$\text{iv) } \sum_{t=T}^S \delta^t = \frac{\delta^T - \delta^{S+1}}{1 - \delta}$$