Problem Set 3 – Demand

1. The following table gives the current price, quantity, and price elasticities of the linear demand curves for pencils, paper and scissors. The columns E_{rc} under the Price Elasticities heading are calculated as $E_{rc} = \left(\frac{\Delta Q_r}{\Delta P_c}\right)\frac{P_c}{Q_r}$. The terms r and c refer to the row of the table and the column under the price elasticities heading, respectively. For example, if r is one and c is two, the value E_{12} is the responsiveness of pencil demand to changes in the paper price (i.e., a cross- price elasticity). The demand curves for each good are in the form $Q_r = a_r + b_r P_1 + c_r P_2 + d_r P_3$. Using the information in the table, derive the demand curve for each good.

			Price Elasticities		
Demand Item	**Own Price**	**Quantity**	E_{r1}	E_{r2}	E_{r3}
Pencils	0.35	25,000	-1.2	0.25	0
Paper	2.00	90,000	0.01	-0.85	0.45
Scissors	3.15	1,500	0	1.20	-1.75

2. A consumer faces prices for hot dogs and hamburgers of $1 each. Consumption of the two commodities at various weekly income levels are shown below.

Income	Hot Dogs	Hamburgers
$10	3	7
15	6	9
20	10	10

i) Use the information to sketch the income consumption curve on a graph.

ii) Draw the Engel curves for hot dogs and hamburgers.

iii) What is the income elasticity of hot dogs for this consumer as income increases from $10 to $15?

3. Draw a graph with arcade games on the horizontal axis and newspapers on the vertical axis. Joe has \$10 per week to allocate between these commodities. The price of newspapers is \$0.50. At the initial price for arcade games of \$0.25, Joe purchases 10 newspapers and plays 20 games. When the price of games increases to \$0.50, Joe purchases 8 newspapers and plays 12 games. When the price of games increases again to \$0.75, Joe buys 5 papers and plays 10 games.

   i) Use this information to draw the utility maximizing points on a graph. Draw the price-consumption curve.
   ii) Draw the individual demand curve for arcade games.
   iii) Use the information given to calculate Joe's elasticity of demand for arcade games between \$0.25 and \$0.50, and between \$0.50 and \$0.75.

4. Derive demand curve for q2 for Ryan who has a constant elasticity of substitution (CES) utility function

$$U = [q_1^{\rho} + q_2^{\rho}]^{\frac{1}{\rho}}$$

5. The wheat market is perfectly competitive, and the market supply and demand curves are given by the following equations:

$$Q_D = 20{,}000{,}000 - 4{,}000{,}000P$$
$$Q_S = 7{,}000{,}000 + 2{,}500{,}000P$$

   where Q_D and Q_S are quantity demanded and quantity supplied measured in bushels, and
   P = price per bushel.

   a) Determine consumer surplus at the equilibrium price and quantity.
   b) Assume that the government has imposed a price floor at \$2.25 per bushel and agrees to buy any resulting excess supply. How many bushels of wheat will the government be forced to buy? Determine consumer surplus with the price floor.

6. Suzie purchases two goods, food and clothing. She has the utility function U = xy where x is food and y is clothing

   i) Derive the demand curve for clothing
   ii) Is clothing a normal good?

7. Suppose there are exactly two consumers (Nick and Sean) who demand apples. Suppose that Nick's demand for apples is given by

$$q_n(p) = p^{\alpha} f_n(I_n)$$

and Sean's demand is given by

$$q_s(p) = p^{\beta} f_s(I_s)$$

where I_n and I_s are Nick and Sean's incomes, and $f_n(\cdot)$ and $f_s(\cdot)$ are two unknown functions.

i) Find Nick and Sean's (own-price) elasticities of demand, $\xi_{qn,p}$ and $\xi_{qs,p}$.
ii) Suppose that $\alpha > 0 > \beta$. Are apples a Giffen good for Nick? Are apples a Giffen good for Sean?
iii) Are apples an inferior good for Nick? Are appless an inferior good for Sean? Assume that these demands arise from utility maximization given linear budget constraints. [Hint: This question should not require much/any algebra].

8. Suppose your utility function is

$$U(x_1, x_2) = \ln x_1 + x_2$$

i) What is different about this utility function?
ii) Derive the demand curve for x1 and x2
iii) There is a discontinuity demand curve for x2, where does this occur?

9. The demand for hamburgers is estimated from this theoretical model:

$$Q = kP^a I^b A^c e$$

where Q = units per day, P = price per unit, A = advertising budget per month by sellers, I = per capita income of consumers, and e = a random error. In a recent study, one researcher estimated the log-linear form of this equation with regression analysis as:

$$Q = 2.5 - 0.33 \log P + 0.15 \log I + 0.2 \log A$$

Explain what the coefficients of log P, log I, and log A reveal about this product.

10. Referring to the Slutsky equation, what must the relationship be between the substitution elasticity of demand (ε^*) for good x and the income elasticity times the share of budget spent on x ($-\theta\xi$) if x is inferior? If the good is Giffen? What does the demand curve look like if they are equal?

11. If a good is an inferior good, is the compensated demand curve steeper or less steep that the uncompensated demand curve. Why?

12. Suppose an individual is trying to decide how many hours a day to work. This person faces a trade-off between more income (which means more consumption) and leisure. Place consumption on the vertical axis and leisure on the horizontal axis. Use graphs to show the conditions under which the person's labor supply curve will be backward bending (work less hours when the wage is higher).

13. A person's utility function is written below. The price of good 1 rose rises from 5 to 6 and the price of good 2 rises from 4 to 6. If a policy maker wishes to compensate this individual for the change in the true cost of living, how much should she increase the consumer's income by (This is a tough one)?

$$U = (x_1 x_2)^{\frac{1}{2}}$$