SGPE Summer School 2022 Mathematics Exam

Question 1 (20 points)

Let $f(x)$ and $h(x)$:

$$f(x) = \sqrt{2x + 5b^2} - ln(x)$$

$$h(x) = 64x^3 \times \left[\frac{y}{x}\sqrt{12b^2} + x^3\right]$$

- Compute $h'(x)$, $f'(x)$, $h''(x)$ and $h''(x)$ [10 pt]

- Find the critical values for f(x) and for h(x) if $y = -8$ and $b = \sqrt{3}$ [10 pt]

Question 2 (20 points)

Calculate the following limits

$$i)\ \lim_{x\to\frac{1}{4}} \sqrt{4x} - 5\ [2\ pt]$$

$$ii)\ \lim_{x\to+\infty} \frac{x^2-1}{x}\ [3\ pt]$$

$$iii)\ \lim_{x\to 1} \frac{x^2-1}{x^3-1}\ [3\ pt]$$

$$iv)\ \lim_{x\to 3^{\pm}} \frac{x^2-8x+12}{x^2-5x+6}\ [5\ pt]$$

$$v)\ \lim_{x\to -1^{+}} \frac{\sqrt{5x+9}-2}{(x+1)^2}\ [7\ pt]$$

Question 3 (20 points)

Calculate the following integrals:

$$i)\ \int (7x+5)^{\frac{3}{2}}\ dx\ [2\ pt]$$

$$ii)\ \int \frac{e^x}{e^x+2}\ dx\ [3\ pt]$$

$$iii)\ \int_0^2 (\frac{2}{3}x^3 - 3x^3 + x^2)\ dx\ [2\ pt]$$

$$iv)\ \int x^2 ln(x)\ dx [5\ pt]$$

$$v)\ \int \frac{x^3}{\sqrt{9-x^2}}\ dx\ [8\ pt]$$

Question 4 (20 points)

(i) Given the function $f(x) = x^2 - 4x - 12$, find 1) the critical values, 2) at which value of $f(.)$ this is optimised, 3) whether this is a relative maximum or minimum using the second-order condition. [5 pt]

(ii) Given the function $f(x) = 9(x-3)^3 + 2$, find 1) the critical values, 2) at which value of $f(.)$ this is optimised, 3) whether this is a relative maximum or minimum using the second-order condition. [5 pt]

(iii) Given the production function $F(K, L) = 3K^{\frac{1}{3}}L^{\frac{2}{3}}$ where input K is capital and input L is labour, find the marginal product of each input. *Hint: you need to take the partial derivative with respect to each variable*. Given the cost function $K + 2L = 600$ find the amount of labour and capital that maximise production. *Hint: you can transform this problem into one of optimisation in only one variable.* [10 pt]

Question 5 (20 points)

Consider the following matrices and perform the required operations where possible.

$$A = \begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & 0 \end{pmatrix}; B = \begin{pmatrix} 4 & 1 \\ 2 & 2 \\ 0 & 3 \end{pmatrix}; C = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}; D = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ -2 & 1 & 2 \end{pmatrix}$$

(i) calculate $A \times B$, $B \times A$, $A \times C$, $C \times B$ and $C \times A$ [2 pt]

(ii) find $A \times B$, $B \times A$, $A \times C$, $B \times C$ and $C \times A$ determinants [5 pt]

(iii) find $A \times B$, $B \times A$, $A \times C$, $B \times C$ and $C \times A$ inverses [7 pt]

(iv) find the inverse of D [6 pt]