



THE UNIVERSITY of EDINBURGH
School of Economics

ECNM11070
Self-Study Mathematics

Exam Date:
Monday 31st July 2023

From and To:
09:30-11:30

Submission deadline:
12:30

Please read full instructions before commencing writing

Exam paper information

- Total number of pages: 6 (including this page).
- There are 5 questions in this exam paper. You must answer ALL questions.

Special instructions

- This is an open-book exam.
- You must complete the exam within the time specified at the top of this page.
- You must start each question on a separate page. Questions must be clearly numbered in the left margin.
- Your answers must be clearly written.
- You are expected to complete this exam within the 2-hour standard exam duration.
- Your exam number (e.g., B123456) must be clearly written at the top of each page. You must not write your UUN or name anywhere.
- Your answers must be clearly written in ink on lined paper.
- All work must be completed individually, without collaboration, as a standard in-person exam would be.

Special items

- Non-programmable calculators are permitted in this exam.

Convenor of Board of Examiners: Mariann Ollar

This examination will be marked anonymously

SGPE Summer School 2023 Mathematics Exam

Question 1 (20 points)

Let $f(x)$ and $h(x)$:

$$f(x) = \sqrt{(2x + 3\sqrt{b})} - 2\ln(x)$$

$$h(x) = bx^2 \left[\frac{y}{x} \sqrt[3]{4b} + x^3 \right]$$

- Compute $f'(x)$, $h'(x)$, $f''(x)$ and $h''(x)$ [10 pt]
- Find the critical values for $f(x)$ and for $h(x)$ if $y = -3$ and $b = 16$ [10 pt]

Question 2 (20 points)

Calculate the following limits

$$i) \lim_{x \rightarrow 1} e^{2x} + 1 \quad [2 \text{ pt}]$$

$$ii) \lim_{x \rightarrow -\infty} \frac{x^4 - 1}{x} \quad [5 \text{ pt}]$$

$$iii) \lim_{x \rightarrow 2} \frac{3x^2 - 7x + 2}{x^2 + x - 6} \quad [6 \text{ pt}]$$

$$iv) \lim_{x \rightarrow 3} \frac{\sqrt{3x} - 3}{\sqrt{2x - 4} - \sqrt{2}} \quad [7 \text{ pt}]$$

Question 3 (20 points)

Calculate the following integrals:

$$i) \int (5x + 3)^{\frac{5}{4}} dx [2 pt]$$

$$ii) \int \frac{3e^x}{5e^x + 1} dx [3 pt]$$

$$iii) \int_0^1 \left(\frac{5}{4}x^4 - 2x^2 + x \right) dx [2 pt]$$

$$iv) \int (x^2 + 1)\ln(x) dx [5 pt]$$

$$v) \int \frac{3x + 2}{x^2 + 5x + 6} dx [8 pt]$$

Question 4 (20 points)

- (i) Given the production function $F(K, L) = K^\alpha L^{1-\alpha}$ where input K is capital and input L is labour, find the marginal product of each input. *Hint: you need to take the partial derivative with respect to K and L .* [10pt]
- (ii) Using the same production function $F(K, L) = K^\alpha L^{1-\alpha}$ and given a production budget B , the firm will spend it on inputs such that $rK + wL = B$, where r and w are prices of inputs. Find the amount of labour and capital that maximise production, as functions of the parameters α , w , r and B . *Hint: you can turn this into a maximisation problem in only one variable. Also, you can treat parameters as if they were given numbers. Your unknowns are K and L .* [10 pt]

Question 5 (20 points)

Consider the following matrices and perform the required operations where possible.

$$A = \begin{pmatrix} 3 & 4 & -1 \\ 2 & -2 & 4 \end{pmatrix}; B = \begin{pmatrix} 3 & -4 \\ 1 & 2 \\ -2 & 4 \end{pmatrix}; C = \begin{pmatrix} -3 & 1 \\ 2 & 4 \end{pmatrix}; D = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{pmatrix}$$

- (i) calculate $A \times B$, $B \times A$, $A \times C$, $C \times B$ and $C \times A$ [2 pt]
- (ii) find $A \times B$, $B \times A$, $A \times C$, $B \times C$ and $C \times A$ determinants [5 pt]
- (iii) find $A \times B$, $B \times A$, $A \times C$, $B \times C$ and $C \times A$ inverses [7 pt]
- (iv) find the inverse of D [6 pt]