THE UNIVERSITY of EDINBURGH

SCHOOL OF ECONOMICS

SELF-STUDY MATHEMATICS
ECNM11070

Exam Date:	**Exam End Time:**	**Submission Deadline:**	Exam Diet:
09:30 (BST), 02 August 2021	11:30 (BST), 02 August 2021	**12:30 (BST),** **02 August 2021**	**April/May 2021**

Please read full instructions before commencing writing

Exam paper information

- Total number of pages: **4 (including this cover page)**
- Number of questions: **7**
- **Students should answer ALL questions.**
- **Answers that do not show procedure will not be considered.**

Special instructions

- This is an open-book exam.
- You should start EACH question on a separate page. Questions must be clearly numbered in the left margin.
- You are expected to complete this exam within the 2-hour standard exam duration.
- You have one additional hour immediately following the exam to photograph, and upload your answers to Learn.
- You must number each answer page.
- Your exam number (e.g. B123456) must be clearly written at the top of each page.
- Your answers must be clearly written in ink on lined paper.
- Answers will be subject to checks through TurnItIn.
- All work must be completed individually, without collaboration, as a standard exam would be.

Special items

- Non-programmable calculators may be used.

Examiner(s) or Chairperson or Convenor of Board:
Marianne Ollar

This examination will be marked anonymously

Instructions: Answer ALL questions.
Answers that do not show procedure will not be considered.

Question 1

i) For each function $f(x)$ find $f'(x)$, $f''(x)$ as well as their critical values [4pt]:

$$f_1(x) = x^3 + 2x^2 - 7x$$
$$f_2(x) = (x^2 - 1)^2$$

ii) for the following utility function:

$$u(W) = \frac{W^{1-\gamma}}{1-\gamma}$$

with $\gamma \neq 1$, find $-\frac{u''(W)W}{u'(W)}$ [5pt]

Question 2

Calculate the following limits [10pt]

$$i) \lim_{x\to\infty} = \frac{x^4 + x^3 + 1}{\sqrt{x} + x^2 + x^3}$$
$$ii) \lim_{x\to 16} = \frac{\sqrt{x} - 4}{x - 16}$$
$$iii) \lim_{x\to 1} = \frac{x^2 - 1}{x^3 - 1}$$

Question 3

Calculate the following integrals [15pt]

$$i) \int (7x+5)^{\frac{3}{2}}\, dx$$

$$ii) \int \frac{x^2}{\sqrt[3]{(x^3+7)^2}}\, dx$$

$$iii) \int \sqrt{x} ln(x)\, dx$$

Question 4

i) A local football club is playing the play-off final hoping to fill all the 2,000 seats of the stadium and make a revenue of £14,000. The public is split into over- and under-18-year old. The over-18-year old ticket's price is £10, whereas the one for the under-18-year old is £5. Assuming the stadium is full, how many people from each group will attend? [7pt]

ii) A recent report on the housing market in your city declared that, on average, an additional bedroom is associated with an increase in house prices by £1,100 and that a three-bedroom flat is expected to be sold for approximately £173,300. Write down the equation describing this relationship and draw a graph. [4pt]

iii) 20 years ago you bought a car for an amount £x. Assuming exponential depreciation at 30%, and that by selling the car today for £3000 you're at least breaking-even, how much did you buy the car for, back 20 years ago? [4pt]

Question 5

You have recently been hired by a company producing high-quality barbells. In particular, you estimate the following production function $Q = f(x) = 6\sqrt{x}$, where Q is the quantity of barbells produced and x is the amount of input to produce Q barbells. The unit-cost of the input is £50 whereas each barbell is sold at a price of £200 per unit. According to the production function you estimated, *i)* What is the amount of input that maximises the profit, how many barbells are produced and what is the (maximised) profit? (Hint: write the profit as a function of the amount of input) [8pt] *ii)* the owner of the company states that from previous experience, at that price and cost of input, by using more input they would produce more barbells, and make higher profits. Formally verify whether this statement is correct. [8pt]

Question 6

For the following matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} B = \begin{pmatrix} 3 & 0 \\ -1 & 4 \end{pmatrix}$$

Find
i) their determinants $det|A|$ and $det|B|$ [4pt];
ii) their inverse A^{-1} and B^{-1} [5pt];
iii) verify $AA^{-1} = I$ and $BB^{-1} = I$ [6pt].

Question 7

The headteacher of a certain primary school needs to set up classrooms for 1st- and 2nd-graders, ensuring that students' performance is maximised. The headteacher hires you, a famous education economist who has recently conducted outstanding research on the educational benefits of attending a multi-grade class, i.e. a class combining students from different year groups. You advise the headteacher that, after considering various plausible factors of school performance, the classroom-level performance can be modelled with the following function:

$$A(x, y) = x^{\frac{1}{3}} y^{\frac{2}{3}}$$

Where x is the number of 1st-graders and y is the number of 2nd-graders. Find the number of 1st- and 2nd-graders that maximises the classroom-level performance, keeping in mind that a classroom has to host 30 pupils. [20pt]

[End of Examination]