

**THE UNIVERSITY OF EDINBURGH**

**SCHOOL OF ECONOMICS**

**MATHEMATICS**

**SGPE SUMMER SCHOOL 2018**

Exam Date: **30 July 2018**

From and To: **09:30-11:30**

**Please read full instructions before commencing writing**

**Exam paper information**

- Total number of pages: **8 (not including this cover page)**
- Number of questions: **8**

**Special instructions**

- Students should answer ALL questions.
- Answers that do not show procedure will not be considered.

**Special items**

- None.

**Examiner(s):** **Dr Tatiana Kornienko** (Chair)  
**Professor Martin Sefton** (External)

This examination will be marked anonymously



UoE - SGPE Economics Summer School (2018)  
Mathematics Examination

July 2018

**Question 1**

Let  $h(x)$ :

$$h(x) = 64x^3 \times \left[ \frac{y}{x} \sqrt{12b^2 + x^3} \right]$$

- Compute  $h'(x)$  and  $h''(x)$  [6 pt]
- Find the critical values for  $h(x)$  if  $y = -8$  and  $b$  is a constant  $= \sqrt{3}$  [4pt]

**Question 2**

(a) Let  $g(x)$ :

$$y = (\sqrt{x} - 1)^2 - (x^2 + 1)^4$$

- Find  $g'(x)$  [5pt]

(b) Consider the following matrix  $A$ :

$$A = \begin{bmatrix} 9 & 7 \\ 7 & 8 \end{bmatrix}$$

- Find:  $A^{-1}$  [2pt];  $\det(A^{-1})$  [2pt];  $A^{-1} * I_2$  [1pt], where  $I_2$  is the identity matrix of size  $[2 \times 2]$ .

**Question 3**

Evaluate the following limits:

(a)  $\lim_{x \rightarrow \frac{1}{2}} \frac{x^2 - 20x + 91}{x - 7}$  [3pt]

(b)  $\lim_{t \rightarrow -8} \frac{\sin(\pi t)}{t^2 - 64}$  [4pt]

(c)  $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 8}{x - 64}$  [3pt]

**Question 4**

(a) Determine which of the followings are functions. Specify the domain and range. Also mention if the function is continuous. [2.5pt, each]

- $y^2 = x + 2$

- $y = \ln(x)$

(b) Calculate the following integrals. [2.5pt]

$$\int \frac{x^3}{(x^2 + 5)^2} dx$$

(c) Calculate the following integrals. [1pt]

$$\int_2^4 x^2 dx$$

(d) Graph the integral (area under the function) in part (c) on the X,Y plane. [1.5pt]

### Question 5

- There are 50 employees in the new offices of a Scottish economics consultancy startup. Each employee brings approximately £8,000 in revenue per month. For each new employee in the office, the revenue per employee drops by £100. How many employees should be hired for the new Edinburgh office in order to maximize the total monthly revenue? [5pt]
- The CEO decides to hire the required new employees, but the startup is short on cash and needs a loan of £10,000. The CFO asks for a loan of £10,000 pounds today. The interest rate for the loan is  $r$ . Find  $r$  if the total amount owed after ( $n=10$ ) years is equal to £200,000 (Note: The sum was continuously compounded for  $n$  years.). [5pt]

**Question 6**

Solve the following linear system of equations [10pt]:

$$2x - 4y + 5z = -33$$

$$4x - y = -5$$

$$-2x + 2y - 3z = 19$$



**Question 7**

Consider  $f : \mathbb{R} \setminus \{q\} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{1}{x-q}$ .

- Find the general expression for the  $k^{\text{th}}$  derivative  $f^{(k)}(x)$  [7pt].
- Also, find the  $6^{\text{th}}$  derivative  $f^{(6)}(x)$  [3pt].

(**Hint:** Write the general expression first and use it to compute the  $6^{\text{th}}$  derivative.)

### Question 8

Consider the system of linear equations:

$$\begin{aligned}2x - y &= 0 \\ -x + 2y &= 3\end{aligned}$$

- Write down the coefficient matrix and the matrix form for this system of equations [3pt].
- Graph the row picture for these system of equations in the X,Y plane [3pt].
- Write the system of equations in column form and graph the column picture. find the solution for the system of equations [4pt].