Persuasion and Pricing*

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Abstract

We propose a dynamic model of bilateral trade in which the parties can generate and verifiably disclose (or secretly conceal) 'hard' signals about the good's value. We find that in equilibrium the seller may keep offering a high price, accepted only by a buyer who is concealing a good signal, until eventually settling on a lower price that is accepted for sure. During this 'delay' an uninformed buyer becomes increasingly convinced that the seller is concealing an unfavorable signal. The period of no trade without signal disclosure (interpreted as stubbornness) is shorter if the seller is initially more optimistic (or the buyer more pessimistic) about the good's value, or if the time horizon is longer, or if the parties are more likely to receive a concealable hard signal.

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1 Introduction

We propose a new model of dynamic, bilateral trade (bargaining) where the parties can generate concealable hard signals over time. The hard signals, if obtained, can be used to persuade the other side and secure a more advantageous outcome. In this framework we investigate the price dynamics, the possibility of delay (lack of trade without signal disclosure), and the overall efficiency of the equilibrium outcome. The model can accommodate commonly known but different priors about the amount of surplus to be split.[1] This makes it possible to address the role of excessive optimism or pessimism regarding the gains from trade in this bargaining problem.

The existing and rather extensive literature on bargaining (modeled mostly as dynamic trading games) emphasizes the parties' asymmetries in 'soft' private information, impatience, and risk aversion.[2] We believe that in many real-world bargaining situations *persuasion* (the generation and disclosure of 'hard' signals) plays a similarly important role.[3] In financial disputes such as shareholder lawsuits the courts make their decisions (or settlements are reached) based on objective, hard evidence produced by the parties' experts. Hard evidence is generated by consultants hired by all sides in regulatory consultations as well.[4] Similarly, when pricing an initial public offering of shares (IPO), the seller and potential buyers do not merely engage in cheap talk regarding the firm's assets, or use costly signaling to reveal private information, but often try to generate hard evidence regarding expected future cash flows, market conditions, etc., in order to persuade the other side and make the terms of the deal more attractive for themselves. Our model is intended to capture this feature of bargaining problems in the simplest possible environment.

In our game two players called the buyer (she) and the seller (he) interact over a finite number of periods. The state of nature is binary: either there is a fixed, positive gain from trade (accrued at the buyer conditional on trade), or there is not. The play-

[1] In contrast, Yildiz (2003) considers a model in which non-common priors are held over the probability of making future offers. Thanassoulis (2010) provides another model with non-common priors over the players' bargaining power.

[2] A seminal paper is Rubinstein (1981). Muthoo (1999) provides an overview of the literature.

[3] Optimal rules of persuasion are studied by Glazer and Rubinstein (2004).

[4] For example, in the UK, before the sale of telecommunication licences, Ofcom has sought input from potential market participants in the form of evidence-based analyses concerning auction rules and the effects of various other policy options.

ers' priors about the state are commonly known but may be different.[5] Each period, each player generates a private signal with a fixed, positive probability, drawn independently across time and players conditional on the state. The signal, if generated, perfectly reveals the state and can be verified by the other player. However, the signal is also concealable—it is not possible to prove that a player has generated one unless he or she discloses it. At the end of each period the seller proposes a transfer price that the buyer either accepts or declines (that is, the seller has all the bargaining power). If the offer is accepted then the buyer realizes the gains from trade less the price and the seller receives the transfer price. The players may discount future payoffs. If the game ends without trade then the players get zero payoffs.

Clearly, if the seller learns that the surplus is positive then he will disclose it and offer a price (accepted by the buyer) appropriating all gains, whereas if the buyer learns that there is no surplus then she discloses that and walks away. That is, each side immediately discloses an advantageous signal and the game ends with the first-best outcome. In contrast, if the seller generates a low ('no-surplus') signal then he will conceal it and behave as if he knew nothing. In the pure-strategy perfect Bayesian equilibrium that we construct, the uninformed (or concealing) seller may keep offering high, 'skimming' prices for a while, accepted by the buyer only if she has just learned that the surplus is positive. Depending on the parameters there may be a time when the seller offers a compromise, that is, a price that is accepted by the buyer for sure. The longer the period of 'stubborn skimming', the lower is the settlement price.[6]

Besides deriving this equilibrium under the most general conditions we develop a number of results and applications. We study three classes of the model: the benchmark case of identical priors and no discounting; the leading case where the priors are different but there is no discounting over a finite horizon; and finally the environment where different priors and discounting are both accommodated, but the model is made more tractable (and the results sharp) by assuming symmetrical signal-generating technologies and an infinite horizon.

[5]We allow, but do not require, different priors. Ours is a bilateral trading game—there is no third party who could verify the state ex post and offer state-contingent arbitrage contracts in case the priors are different, therefore different priors do not lead to an exploitation paradox.

[6]The highest price that the uninformed buyer accepts declines over time because she becomes more and more suspicious that the seller is concealing a 'no-surplus' signal.

In the benchmark case (common prior, no discounting) we show that the outcome is no trade without signal disclosure until the end. This 'no-trade' result may disappear if either the common prior assumption is relaxed or discounting is introduced. Our leading case is where the parties' priors are different and discounting is negligible. In this environment the equilibrium can indeed exhibit stubbornness and compromise in the sense mentioned above. The timing of the compromise depends on the parameters. Somewhat counter-intuitively a longer time-horizon, as well as a more-optimistic seller or less-optimistic buyer, can reduce the delay (length of skimming) on the equilibrium path.[7] Finally, we derive sharp predictions in the environment where the horizon is infinite and the parties' signal-generating technologies are identical. The equilibrium involves either skimming in every period, or settling right away—in the latter case, compromise prices are also offered off the equilibrium path for a finite number of periods. Interestingly, a small change in (for example) the seller's prior can lead to a discrete change in the outcome; in particular, if the seller becomes more pessimistic about the likelihood of a positive surplus he may switch from delaying trade to the end to offering a compromise right away.

An extension of the model that we explore at the end involves allowing the parties to trade at the ex ante stage (before either gets a concealable hard signal). In the equal-priors, no-discounting benchmark this leads to immediate trade. An interesting phenomenon, which we illustrate in an example with a common prior and no discounting, but applies more generally, is the following. The seller can increase his payoff by pretending to be overly enthusiastic because this commits him to delay off the equilibrium path, and allows him to charge a greater price at the ex ante stage. More extensions and open research questions are discussed in the last section.

2 Related literature

[to be added]

[7] All else equal, a longer horizon increases the seller's expected profit from skimming in all future periods. The settlement price must compensate for this, which means that the compromise must take place earlier because the uninformed buyer's willingness to pay decreases over time.

3 The model

We study a bilateral trading model with one seller (S, he), one buyer (B, she) and one, indivisible good. The buyer's valuation for the good is $v \in \{\underline{v}, \bar{v}\}$. The seller's prior belief about v is $\sigma_0 = \Pr_S(v = \bar{v})$, the buyer's is $\beta_0 = \Pr_B(v = \bar{v})$; the values of σ_0 and β_0 are commonly known.[8] We normalize $\underline{v} = 0$ and $\bar{v} = 1$ in order to simplify the formulas for expected values. We also assume that the seller's valuation for the good is zero, which is common knowledge. This is not a normalization: It expresses the assumption that trading is ex ante Pareto efficient, and the only uncertainty is whether or not the transfer price should (or could) be positive.[9]

The seller and buyer interact over periods $t = 1, \ldots, T$. The state of nature (buyer's value for the good), v, does not change over time.

At the beginning of every period the seller may receive signal s_t, and the buyer signal b_t. With probability r_S the realization of s_t equals v (i.e., the true value of the good, 0 or 1), otherwise $s_t = \emptyset$ (i.e., no signal). Similarly, $b_t = v$ with probability r_B, and $b_t = \emptyset$ otherwise. All signals are independent conditional on v across players and time; $r_B, r_S \in (0, 1)$. If $s_t = \emptyset$ then we may also say 'S did not observe v at t', likewise for B if $b_t = \emptyset$. We assume that a signal, if observed, can be verifiably disclosed. In other words, s_t and b_t are *hard signals*. A signal can also be concealed, and a player cannot prove that he or she did not receive a signal. The players decide simultaneously, right before the end of period t, whether or not to disclose any signal observed at or before t.

At the end of each period $t = 1, \ldots, T$ the seller proposes a price, p_t, and the buyer either accepts or rejects it. If she accepts then the parties trade; the seller's payoff is p_t and the buyer's is $v - p_t$, and the game ends. If no trade ever occurs then the payoffs are 0. Utilities are transferable, and both players are risk neutral. Both players discount future (expected) payoffs using a discount factor $\delta \in [0, 1]$; an alternative interpretation is that the game exogenously ends at any given $t < T$ with probability $(1 - \delta)$.

[8] If $\sigma_0 = \beta_0$ then the players have common priors. If $\sigma_0 \neq \beta_0$ then they do not.

[9] A different assumption, leading to a different model, would be that the seller's value from not selling the good is c (positive or negative), commonly known. If $c > 0$ then the state of nature determines not only the range of possible transfer prices, but also whether or not trading is efficient.

To summarize, our dynamic trading game is played as follows. At every $t = 1, \ldots, T$ that is reached, verifiable signals $s_t = v$ and $b_t = v$ are generated independently and privately with probabilities r_S and r_B, respectively. The players simultaneously decide whether to disclose any signals that they have observed. Then, still in period t, the seller proposes p_t which the buyer either accepts or rejects. If p_t is accepted then the game ends with payoffs p_t for S and $v - p_t$ for B, discounted at rate δ. Otherwise the game continues with period $t + 1$ until T is reached.

In the next section we derive a pure-strategy perfect Bayesian equilibrium that is the unique one to satisfy certain reasonable robustness criteria. In Section 4 we further discuss this equilibrium and its properties.

4 Equilibrium

The solution concept that we use is perfect Bayesian equilibrium (PBE). In a game like ours PBE requires that (1) the players' strategies be best responses given their beliefs on and off the equilibrium path, and (2) the players' beliefs be consistent with their commonly-known priors and the equilibrium strategies in the usual sense. In our game, the players' priors regarding the probability of $v = 1$ are common knowledge, but may be different. As a consequence we need to keep track of the players' higher-order beliefs over the event $v = 1$.

4.1 General properties and robustness criteria

In any equilibrium, if either player discloses a 1-signal then $v = 1$ becomes commonly known and the only price that S is willing to offer is 1. In the presence of discounting, by strict dominance, $s_t = 1$ must be immediately disclosed by the seller and trade to take place at $p_t = 1$. We will simplify the exposition by saying that $b_t = 0$ is also immediately disclosed and the parties trade at $p_t = 0$. (Whether a worthless good is traded at price 0 or not traded at all is immaterial.)

In the absence of signal disclosure by the seller the buyer can still make an inference from the price that S has offered. Indeed, if in equilibrium the seller were to make a different offer at t depending on whether $s_t = \emptyset$ or $s_t = 0$, then a rational buyer would be able to infer when $s_t = 0$ occurs and reject any positive price in that event.

Consequently, it is a weakly dominant strategy for the seller to conceal $s_t = 0$ and set the exact same price as if he has observed nothing. We impose this dominance criterion throughout the rest of the analysis, that is, assume that S makes the same price offer no matter whether he has observed nothing or a 0-signal.[10]

The buyer could make an inference regarding the seller's signal and the good's value from an out-of-equilibrium offer as well. In order to derive robust results we will assume that the buyer's out-of-equilibrium beliefs satisfy the Intuitive Criterion, putting zero weight on seller types for which the deviation is dominated by his equilibrium payoff. We do not put any other restriction on the buyer's out-of-equilibrium beliefs, in fact, we shall require that the equilibrium be robust to any off-equilibrium beliefs satisfying this dominance criterion.

It turns out that in the equilibrium that we characterize at any given t an uninformed seller either sets $p_t = \bar{p}_t$ that is accepted for sure (i.e., the seller *settles*), or he sets $p_t = p'_t$, which is accepted if and only if the buyer has observed a 1-signal (i.e., the seller *skims*). Clearly, the seller could also set a price so high that no buyer would accept it—however, we will show that this action is dominated by skimming.[11] As a result, an equilibrium price offer is rejected either because the buyer is uninformed (and the price offered was a skimming price), or because the buyer has made a mistake. Consistent with this observation, we will assume that on and off the equilibrium path the seller believes the buyer to be uninformed if she has rejected his offer. (The seller does not infer v from an unexpected rejection of his offer either.) Such out-of-equilibrium beliefs make our results more, not less, robust and attractive.[12] The qualitative features of our equilibrium would survive under alternative specifications of out-of-equilibrium beliefs.

From our constructive derivation it follows that there is a unique PBE with these rather reasonable, desirable properties.

[10]If the buyer's low valuation were greater than the seller's value for the good then the seller could have an incentive to disclose a low signal. We have set up the model to avoid this. Our stylized model captures a situation where the seller wants to disclose good and conceal bad news.

[11]Yet another possibility is that a buyer made indifferent by a price offer accepts it at random. We focus on pure-strategy equilibria even though mixed strategies are allowed.

[12]Clearly, off the equilibrium path a seller cannot believe simultaneously that the buyer has observed a 1-signal and it is still possible for him to receive a 0-signal. Again, the off-equilibrium beliefs we specify are such that they cannot be refuted by any continuation play, and they do not make it easier to induce harsh punishment for a deviation.

We proceed by distinguishing two possible continuation equilibria after an offer is rejected in period t. In Case 1, the seller offers a skimming price in all periods $t+1, \ldots, T$. In Case 2, there is a subsequent period m where the seller settles at price p_m, whereas the seller sets a skimming price in all periods in between t and m. In each case we shall derive the equilibrium price offer p_t and the buyer's equilibrium response. This will complete the derivation of the equilibrium.

Before starting the analysis it is useful to introduce two pieces of notation. Let σ_1 denote an uninformed seller's belief that $v = 1$ conditional on the buyer being either uninformed or concealing a just-received 1-signal having not observed a signal until t:

$$\sigma_1 = \Pr_S(v = 1 | b_t \in \{\emptyset, 1\}, b_\tau = \emptyset, \forall \tau < t) = \frac{\sigma_0}{\sigma_0 + (1 - r_B)(1 - \sigma_0)}. \tag{1}$$

This is the seller's belief regarding $v = 1$ at any t following the buyer's rejection of an offer. Let β_t denote the uninformed buyer's belief at t that $v = 1$ conditional on the seller not having disclosed a 1-signal at or before t:

$$\beta_t = \Pr_B(v = 1 | s_\tau \in \{\emptyset, 0\}, \forall \tau \leq t) = \frac{(1 - r_S)^t \beta_0}{(1 - \beta_0) + (1 - r_S)^t \beta_0}. \tag{2}$$

Note that β_t is strictly decreasing in t. An uninformed buyer becomes increasingly suspicious that the seller who has not disclosed a 1-signal is concealing a 0-signal, making her less and less convinced that $v = 1$.

4.2 Case 1: Skimming expected to the end

Suppose we are at $t = T$. By the construction of the equilibrium strategies and off-path beliefs, the seller believes the buyer is uninformed right before the beginning of the period. If the buyer remains uninformed at T then she believes the good's value is $v = 1$ with probability β_t, which is therefore her maximum willingness to pay for the good. If no signal is revealed at T then an uninformed seller believes that $v = 1$ with probability σ_1, and that the buyer has just generated a 1-signal at T, and hence is willing to pay 1 for the good, with probability $r_B\sigma_1$.

The consequence is that the seller offers either $p_T = 1$ (accepted only if $b_T = 1$, i.e., with probability $r_B\sigma_1$), or $p_T = \beta_T$ (accepted for sure). The seller sets $p_T = 1$ in the final period if and only if $r_B\sigma_1 > \beta_T$.

Let $V_T = r_B \sigma_1$, the uninformed seller's expected profit from skimming at T. If $V_T > \beta_T$, then as we have just said the seller sets $p_T = 1$ and the buyer's continuation value at $T-1$ is zero. Hence at $T-1$, the seller can sell for sure at price $p_{T-1} = \beta_{T-1}$, or alternatively skim with $p_{T-1} = 1$. The seller could also delay (not sell for sure) by setting $p_{T-1} > 1$, but that is inferior to skimming for any $\delta < 1$ because it forgoes a positive profit on a buyer who is concealing a 1-signal.

At any $t < T$, if the uninformed seller is expected to skim with a price of 1 in all future periods, then his payoff at t from skimming using $p_t = 1$ is

$$V_t = \sigma_1 r_B + \sigma_1(1 - r_B)\delta[1 - (1 - r_B)(1 - r_S)] + \dots \\ + \sigma_1(1 - r_B)^{T-t}(1 - r_S)^{T-t-1}\delta^{T-t}[1 - (1 - r_B)(1 - r_S)].$$

The first term of this expression is the probability that the buyer is concealing a 1-signal times the skimming price (which is 1). The second term is the probability that the buyer is not concealing a 1-signal but in the next period either the seller or the buyer will generate a 1-signal, which is again multiplied by the trading price (of 1) and discounted at rate δ. The remaining terms are calculated similarly. This finite geometric series can be summed as

$$V_t = \sigma_1 \left\{ r_B + [1 - (1 - r_B)(1 - r_S)] \times \right. \\ \left. \frac{(1 - r_B)\delta\left[1 - (1 - r_B)^{T-t}(1 - r_S)^{T-t}\delta^{T-t}\right]}{1 - (1 - r_B)(1 - r_S)\delta} \right\}. \quad (3)$$

It is straightforward to verify that V_t is decreasing in t, increasing in T, r_S, r_B, and σ_1. We plot V_t together with β_t and the constant σ_1 in Figure 1.

At $t < T$, if an uninformed seller is expected to skim in all future periods with price 1, then he prefers to skim at t (also with price $p'_t = 1$) whenever $V_t > \beta_t$. In contrast, an uninformed seller prefers to settle with $\bar{p}_t = \beta_t$ when $V_t < \beta_t$. (Again, delay with $p_t > 1$ is dominated for him by skimming at t.) In equilibrium, a seller who has observed a 0-signal behaves as if he were uninformed: skims at t if $V_t > \beta_t$ and settles if $V_t < \beta_t$. This implies that his equilibrium payoff is 0 from skimming, as he knows that the buyer will never observe a 1-signal and take a skimming offer. The reason why a seller who has learnt $v = 0$ does not deviate when he is supposed to skim (e.g., by trying to settle with a price $p_t \leq \beta_t$) is that using the Intuitive Criterion, the buyer infers $v = 0$ from any such out-of-equilibrium offer—the only type of the seller who could potentially gain from such deviation is the one who has observed a 0-signal, as we have just shown that an uninformed seller prefers skimming when $V_t > \beta_t$—therefore $p_t \leq \beta_t$, and in fact all out-of-equilibrium offers, are rejected by an uninformed buyer.

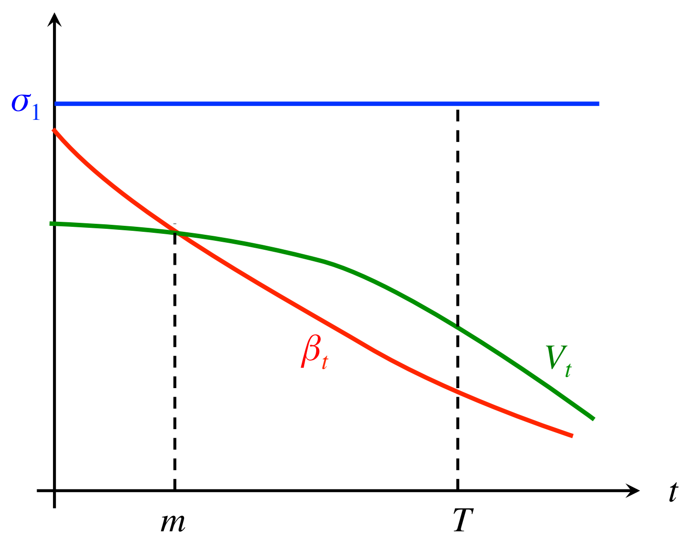

Figure 1: Illustration of V_t, β_t, and σ_1.

Our results thus far are summarized in the following Lemma.

Lemma 1: If $V_t > \beta_t$ for all $t \geq \tau$, then there is an equilibrium in the subgame starting at τ with no prior signal disclosure in which, at all $t \geq \tau$,

(i) $s_t = 1$ or $b_t = 0$ is disclosed, followed by trade at $p_t = v$;

(ii) for all t without such disclosure the seller sets $p_t = 1$, and the buyer buys the good as soon as she observes a 1-signal.

For $\delta = 1$ we have $V_t = \sigma_1 \left[1 - (1 - r_B)^{(T-t)+1}(1 - r_S)^{T-t}\right]$. This is the probability that either the buyer is already concealing a 1-signal or will get such a signal in the future or the seller will get a 1-signal in the future. If this event occurs the seller will sell the good at price 1, otherwise he will not be able to sell (as he is skimming by setting a price of 1 in all periods). Due to the lack of discounting the seller's profit is the expected price at which he (eventually) sells.

4.3 Case 2: Settling expected at a future date

If $V_T \equiv r_B \sigma_1 < \beta_T$ then the seller settles in period T, at price $p_T = \beta_T$. (That is, unless a signal is disclosed at T, the parties trade at price β_T.) In what follows we characterize an uninformed seller's optimal decision at t if he is expected to settle in period $t+k$, at price $p_{t+k} = \bar{p}$, but skim in all periods strictly in between t and $t+k$.

If $k > 1$ then the seller is skimming in period $t+k-1$. A buyer who has seen a 1-signal knows that she can reject the skimming price at $t+k-1$ and buy the good a period later at price $\bar{p}$, unless the seller also discovers a 1-signal by then. Therefore the informed buyer's continuation value at $t+k-1$ is $\delta(1-r_S)(1-\bar{p})$. The highest price that she accepts satisfies $1 - p'_{t+k-1} = \delta(1-r_S)(1-\bar{p})$, which pins down p'_{t+k-1}. By the same argument, for all $i = 0, 1, \dots, k-1$, the skimming price at $t+i$ is

$$p'_{t+i} = 1 - \delta^{k-i}(1-r_S)^{k-i}(1-\bar{p}).$$

If an uninformed buyer rejects the seller's offer at t then her continuation value is

$$\begin{aligned} U_t = \delta\beta_t(1-r_S)r_B(1-p'_{t+1}) + \delta^2\beta_t(1-r_S)^2(1-r_B)r_B(1-p'_{t+2}) \\ \dots + \delta^{k-1}\beta_t(1-r_S)^{k-1}(1-r_B)^{k-2}r_B(1-p'_{t+k-1}) \\ + \delta^k\left[\beta_t(1-r_S)^k(1-r_B)^{k-1}(1-\bar{p}) - (1-\beta_t)(1-r_B)^k\bar{p}\right]. \end{aligned}$$

Here the first term is the discounted payoff the buyer receives when, in the next period, she is skimmed at price p'_{t+1}, which happens only when the buyer observes a 1-signal but the seller does not. All except the last term are analogous. The final term is the discounted expected payoff from the settlement price, $\bar{p}$ in period $t+k$.

After substituting in the formula for p'_{t+i}, for $i = 1, \dots, k-1$, this simplifies to

$$U_t = \delta^k\left[(1-r_S)^k\beta_t(1-\bar{p}) - (1-r_B)^k(1-\beta_t)\bar{p}\right]. \quad (4)$$

The only positive surplus that an uninformed buyer can receive obtains in the event that the seller remains uninformed when $v = 1$; in this case she pays only $\bar{p}$ in period $t+k$ but receives a good of value 1. However, an uninformed agent obtains a negative surplus in the event that $v = 0$ and she remains uninformed about it until $t + k$; in this case she pays $\bar{p}$ at $t + k$ for a worthless good. An uninformed buyer in period t accepts any price $p_t \leq \beta_t - U_t$.

It is easy to show that the maximum price accepted by a buyer concealing a 1-signal, $p'_t = 1 - \delta^k(1 - r_S)^k(1 - \bar{p})$, exceeds that accepted by an uninformed buyer, $\bar{p}_t = \beta_t - U_t$. Therefore, at t an uninformed seller has three choices: (1) Offer p'_t to *skim* the buyer concealing a 1-signal; (2) Offer $\bar{p}_t$, accepted for sure, to *settle*; (3) Offer $p_t > p'_t$ rejected for sure, to *delay*. It can be shown that *skim* dominates *delay* for all $\delta_S < 1$. This is so because delay forgoes profit on the buyer currently concealing a 1-signal.

The seller's payoff from settling at t is $\bar{p}_t \equiv \beta_t - U_t$, which can be written as

$$\bar{p}_t = \beta_t + \delta^k \left[(1 - r_B)^k(1 - \beta_t)\bar{p} - (1 - r_S)^k\beta_t(1 - \bar{p})\right]. \qquad (5)$$

The uninformed seller's payoff from skimming at t, expecting to settle k periods later at price $\bar{p}$, is

$$\begin{aligned} V_t^k(\bar{p}) = \sigma_1 r_B p'_t + \sigma_1(1 - r_B)\delta[r_S + (1 - r_S)r_B p'_{t+1}]+ \\ \ldots + \sigma_1(1 - r_B)^{k-1}(1 - r_S)^{k-2}\delta^{k-1}[r_S + (1 - r_S)r_B p'_{t+k-1}] \\ + \sigma_1(1 - r_B)^k(1 - r_S)^{k-1}\delta^k[r_S + (1 - r_S)\bar{p}] + (1 - \sigma_1)(1 - r_B)^k\delta^k\bar{p}. \end{aligned}$$

The first term of this sum is the expected profit from skimming with price p'_t at t; this price is accepted only when the buyer is concealing a 1-signal. If that offer is rejected then the seller makes a sale in the following period if either he is able to reveal a 1-signal (in which case the price is 1), or he does not observe a 1-signal but the buyer does and accepts his skimming price at $t+1$. The remaining terms are analogous; the final two terms represent the expected profit from selling in period $t + k$, when the seller settles at $\bar{p}$ unless he receives a 1-signal.

This series can also be written in a somewhat more compact form as

$$V_t^k(\bar{p}) = \sigma_1 \left[\lambda + (1-\lambda)(1-r_B)^k(1-r_S)^k\delta^k\right] + \\ \delta^k \left[(1-r_B)^k(1-\sigma_1)\bar{p} - (1-r_S)^k\sigma_1\bar{p}\right], \quad (6)$$

where

$$\lambda = \frac{r_B + (1-r_B)r_S\delta}{1-(1-r_B)(1-r_S)\delta} \in [0,1].$$

Skimming is better than settling for the uninformed seller at t if and only if $V_t^k(\bar{p}) > \bar{p}_t$. In equilibrium, a seller who has observed a 0-signal does the same as if he were uninformed, even though he might get a greater payoff from $p_t \leq \bar{p}_t$, if accepted for sure, than from skimming with p_t'. This is so because any such deviation would make the uninformed buyer infer that the seller has seen a 0-signal (all other beliefs are ruled out by the Intuitive Criterion) and reject the offer.

We summarize our findings in Case 2 in the following Lemma.

Lemma 2: Suppose that in the subgame starting at t, the equilibrium prescribes settling in period $t+k$, at price $\bar{p}$. Then, absent signal disclosure at or before t, if $V_t^k > \bar{p}_t$ then the uninformed seller skims at t with price $p_t' = 1 - \delta^k(1-r_S)^k(1-\bar{p})^k$. Otherwise the seller settles at t, price $\bar{p}_t$ given by (5).

For $\delta = 1$ the formula for V_t^k simplifies to

$$V_t^k = \left[1-(1-r_S)^k\right]\sigma_1 + \left[(1-r_S)^k\sigma_1 + (1-r_B)^k(1-\sigma_1)\right]\bar{p}.$$

Under the same condition the settling price becomes

$$\bar{p}_t = \left[1-(1-r_S)^k\right]\beta_t + \left[(1-r_S)^k\beta_t + (1-r_B)^k(1-\beta_t)\right]\bar{p}.$$

Therefore, if $\delta = 1$, then $V_t^k \geq \bar{p}$ is equivalent to $\sigma_1 \geq \beta_t$. Note that the condition does not depend on the value of the settlement price, $\bar{p}$, nor the period when the settlement is expected to occur. This is particular to the case of no discounting.

4.4 Algorithm for finding the equilibrium

The results of Lemma 1 and Lemma 2 enable us to characterize our equilibrium by the following 'algorithm', or recursive description of the equilibrium strategies in every subgame starting at the end.

1. If $V_t > \beta_t$ for all $t = \tau, \ldots, T$ then, in the subgame starting in period τ without prior signal disclosure, the seller offers $p_t = 1$ for all $t \geq \tau$. Trade occurs at a positive price if and only if a 1-signal is generated by either player at t.

2. Suppose there exists $m = \max\{t \leq T : V_t < \beta_t\}$; that is, m is the last period in which $V_t > \beta_t$ fails. At m the seller settles at price $\bar{p}_m = \beta_m$, that is, trade occurs in period m at price β_m, unless either the seller generates (and discloses) a 1-signal or the buyer sees and discloses a 0-signal.

3. If m exists then, going backward, in each period $t < m$ we can determine whether the seller skims (offers a price only accepted by a buyer concealing a 1-signal) or settles (offers a price accepted for sure) by comparing V_t^k of equation (6) and $\bar{p}_t$ of equation (5), where k is such that the next time of settling after t is $t + k$.[13]

In the next section we further characterize this unique 'skimming-settling' equilibrium in three general classes of the model.

5 Results

In this section we develop sharper predictions for the skimming-settling equilibrium in three classes of our general model. First, we study the benchmark case of a common prior and no discounting. We prove a 'no-trade' theorem: The seller offers a skimming price in all periods except perhaps the last one, therefore there is no trade without hard signal disclosure until the end.

The no-trade theorem fails to hold in our model if either there is (sufficient) discounting, or the parties have different priors. In the second (and leading) case of our general model we consider the latter departure from the benchmark. We find that in

[13] At $t = m - 1$, the value of k is clearly 1. In earlier periods the value of k depends on t, as well as on whether settling occurs even before m.

equilibrium, depending on the parameters, the seller either skims to the end, or skims until a specific period and settles then. Settling (compromise) comes earlier if the time horizon is longer, or the seller's prior about the good state is greater (or the buyer's prior is lower), or if the parties are more likely to generate a hard signal.

Finally, we consider the case where the parties may have different priors and a discount factor that is less than 1. In order to make the model more manageable we make the signal-generating technologies equally precise for both players, and let the time horizon tend to infinity. Due to these specific assumptions the model's prediction becomes very sharp: The seller makes a settlement offer in the first period (which is accepted and the game ends), and off the equilibrium path the seller keeps making settlement offers until a specific date, at which he reverts to skimming for all future periods. The final date of settling (the last opportunity for compromise) and the buyer's belief about the surplus being positive at that date determine the price offered and accepted in the first period.

5.1 Benchmark: Common prior and no discounting

The very special case of no discounting ($\delta = 1$) and a common prior ($\beta_0 = \sigma_0$) yields a stark prediction which is not entirely surprising based on our intuition and the received literature (e.g., Milgrom and Stokey, 1982; Morris, 1994). If the parties start persuading each other and the seller making trade offers only *after* they first have a chance to observe a fully-revealing, hard signal, then the resulting adverse selection makes trade without signal disclosure impossible until the very end.

Theorem 1. If $\beta_0 = \sigma_0$ and $\delta = 1$, then in the skimming-settling equilibrium there is no trade without signal disclosure at any $t < T$.

Trade without disclosure is possible at $t = T$, indeed if $V_T < \beta_T$ then $\bar{p}_T = \beta_T$. In the proof of Theorem 1 we establish that if $V_T > \beta_T$ then $V_t > \beta_t$ for all $t < T$; moreover, if $V_T < \beta_T$ then trade occurs at any $t < T$ if and only if $\sigma_1 < \beta_t$, which is false under the assumption of common prior. The proof is presented in Appendix A.

The benchmark case and the no-trade theorem that obtains are indeed what they are: benchmarks. Either discounting or different priors are needed for anything interesting to happen in our dynamic trading model. The extensive literature on bargaining establishes the role that discounting (also interpreted as a positive probability of the game suddenly ending) plays in reaching an agreement. In the next subsection we focus on how different priors lead to compromise with δ sufficiently close to 1. In our model, in contrast to many models of bargaining, agreement may not take place right away, and in particular the comparative statics results on the date of the (final) compromise turn out to be interesting and insightful.

5.2 Different priors and negligible discounting

In this paper our goal is to study bargaining (intertemporal price discrimination in bilateral trade) when the parties' motivation to cut a deal is not their impatience (or anxiety about the probability of the game ending) but rather their desire to persuade their counterpart who may have a different prior about the state of nature and therefore may agree to disagree. Consequently the case where δ is close to 1, but the priors are not equal, is central to our analysis.

A key, qualitative feature of the equilibrium in this case is that, absent signal disclosure, the seller may offer prices (not necessarily 1) that are only acceptable to a buyer concealing a 1-signal for some period of time. After this period of time the seller settles, that is, he offers a price that is accepted for sure. If that is rejected (off the equilibrium path) then the seller reverts to skimming in all subsequent periods.

We first state the Theorem and then discuss its content and implications. The proof of the theorem is relegated to Appendix A.

Theorem 2. For δ close to 1 the skimming-settling equilibrium is as follows:

1) If $V_t > \beta_t$ for all $t = \tau, \dots, T$ then the seller offers $p_t = 1$ for all $t \geq \tau$. Trade occurs at a positive price if and only if a 1-signal is generated at t.

2) Suppose there exists $m = \max\{t \leq T : V_t < \beta_t\}$.

Assume $\sigma_1 > \beta_1$. There is no trade before m unless either the seller observes $s_t = 1$ (disclosed, trade at $p_t = 1$), or the buyer observes $b_t = 0$ (disclosed, trade at $p_t = 0$), or the buyer observes $b_t = 1$ (concealed) in which case she accepts $p'_t = 1 - \delta^{m-t}(1-r_S)^{m-t}(1-\beta_m)$. In all other cases trade takes place at m, price $\bar{p}_m = \beta_m$.

If $\sigma_1 < \beta_1$ then, in the absence of signal disclosure at $t = 1$, trade occurs at $\bar{p}_1 = \beta_1 - \delta^{m-1}\left[(1-r_S)^{m-1}\beta_1(1-\beta_m) - (1-r_B)^{m-1}(1-\beta_1)\beta_m\right]$.

As we discussed in the previous section, if either S generates $s_t = 1$ or B generates $b_t = 0$, then the signal and hence the value of the good is revealed and trade occurs immediately at $p_t = v$. The main question that Theorem 2 answers is whether in a given period trade also occurs *without* signal disclosure (when either the players observed no signals, or one of them observed a disadvantageous signal and concealed it). The main result of the theorem is that for discount factors close to 1, there are essentially three possibilities, depending on the parameter values:

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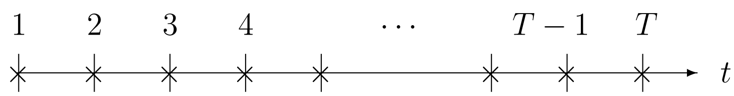
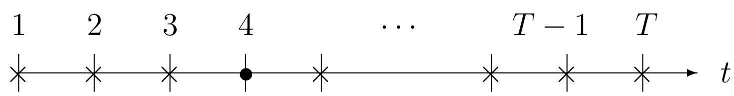
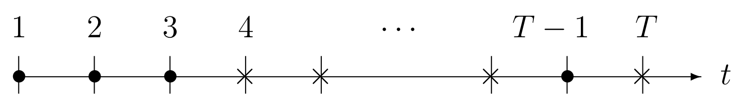
1) If $V_t > \beta_t$ for all t, then skim with $p_t = 1$ to the end:



2A) Otherwise, if $\sigma_1 > \beta_1$ then skim for all $t < m = \max\{t | V_t < \beta_t\}$ and settle at $\bar{p}_m = \beta_m$:



2B) If $V_t < \beta_t$ for some t and $\sigma_1 < \beta_1$, then settle at $t = 1$ and at all $t < m$ such that $\sigma_1 < \beta_t$, then settle again at m, skim otherwise:



Case 2A is illustrated in Figure 1, seen above. In this example $\sigma_1 > \beta_1$, meaning that the seller is more enthusiastic about the good's value than the buyer at the time they meet. Then, provided $\beta_1 > V_1 \approx [1 - (1 - r_B)^T (1 - r_S)^{T-1}]\sigma_1$, trade in the absence of signal realization takes place exactly at $m = \max\{t : V_t < \beta_t\}$. Assuming $r_B \sigma_1 > \beta_T$ such $m < T$ exists.

If m exists (i.e., in Case 2A or 2B), then it is the last period in which trade occurs with certainty (i.e., even in the absence of signal disclosure). The following Proposition describes how this "last period of compromise" changes as a function of the parameters of the model.

Proposition 1: (i) If the seller is a priori more optimistic, or if the buyer is less optimistic, then m is lower (compromise is reached earlier).

(ii) A larger T (longer horizon) shifts V_t right inducing a lower m.

(iii) Larger r_S or r_B shifts V_t up, β_t down (constant in r_B), decreasing m.

Notice that this Proposition has particular impact when Case 2A obtains (i.e., the equilibrium involves $m-1$ periods of skimming followed by settling at $\bar{p}_m = \beta_m$). Of course, it also affects the equilibrium price in Case 2B where the parties settle immediately.

An interesting consequence of Theorem 2 is that the equilibrium mapping is discontinuous in the players' priors. (However, equilibrium payoffs are not discontinuous.) To see this, start from a situation where $\sigma_1 > \beta_1$ (e.g., common prior), and $\beta_T > V_T$ (e.g., r_B not too large). Then, for δ close to 1, the equilibrium outcome is for the seller to skim for all $t < T$ and settle at $p_T = \beta_T$.

Suppose the seller becomes somewhat skeptical relative to the buyer, i.e., σ_0 is decreased while β_0 remains the same. All else equal, σ_1 as well as V_t for all t decrease. If, as a result of this change in the prior, σ_1 falls just below β_1, then the outcome changes drastically. The parties immediately settle at price

$$p_1 = \beta_1 - \delta^{T-1}\left[(1-r_S)^{T-1}\beta_1(1-\beta_T) - (1-r_B)^{T-1}(1-\beta_1)\beta_T\right].$$

5.3 Discounting and different priors, with infinite horizon

In this subsection we consider the case of (substantial) discounting and possibly different priors. This is clearly a very rich environment, and all types of equilibria seen above can occur. However, if the horizon approaches infinity (technically, we assume finite T as before, but take the limit $T \to \infty$ in the relevant formulas), the comparison of different continuation values becomes more straightforward and the predictions rather sharp. If δ is greater than a given threshold $\bar{\delta}$ then the seller skims in all periods. However, if δ is less than $\bar{\delta}$, then the seller offers a settling price at $t = 1$ and settles off the equilibrium path in each period until $t = m$. If no deal is reached at or before m then the seller offers a skimming price in all subsequent periods.

The main technical simplification is that as T tends to infinity, the seller's continuation value from skimming in all future periods becomes constant over time, $V_t \approx V(\delta)$ for all t. It is easy to check that $V(0) = r_B\sigma_1$, $V(1) = \sigma_1$, and $V(\delta)$ is strictly increasing in δ. When the additional assumption of $r_B = r_S = r \in (0,1)$ is made, we obtain

$$V(\delta) = \frac{r + (1-r)\delta - (1-r)^2\delta}{1-(1-r)^2\delta}\sigma_1. \tag{7}$$

The last period in which a compromise price (settlement) can be agreed on is m such that $\beta_{m+1} < V(\delta) \leq \beta_m$, and the seller offers $\bar{p}_m = \beta_m$. Such period does not exist if $V(\delta) > \beta_1$, or equivalently, if $\delta > \bar{\delta}$ where $V(\bar{\delta}) = \beta_1$.

If $T \to \infty$ and $r_B = r_S = r$ then the seller's profit from skimming at t given that he will offer a settlement price $\bar{p}_{t+1}$ exactly one period later also becomes a constant for all t,

$$V^1(\delta) = [r + (1-r)\delta]\,\sigma_1 - (1-r)\delta(\sigma_1 - \bar{p}_{t+1}).$$

The seller's payoff from settling at t, given that in the continuation he settles at $t+1$ at price $\bar{p}_{t+1}$ is

$$\bar{p}_t \equiv \beta_t - U_t = \beta_t - (1-r)\delta(\beta_t - \bar{p}_{t+1}).$$

The seller prefers settling at t, given that he expects to be settling at price $\bar{p}_{t+1}$ a period later, if $\bar{p}_t > V^1(\delta)$ holds. This condition is equivalent to

$$\frac{r}{1-(1-r)\delta}\sigma_1 < \beta_t.$$

The following theorem characterizes the skimming-settling equilibrium for the case of arbitrary δ and priors, $T \to \infty$, $r_B = r_S = r$.

Theorem 3. Assume $T \to \infty$ and $r_B = r_S = r \in (0,1)$. Define $\bar{\delta}$ to satisfy $V(\bar{\delta}) = \beta_1$.

(i) If $\delta > \bar{\delta}$ then in the skimming-settling equilibrium $p'_t = 1$ for all t: No trade without signal disclosure in every period.

(ii) If $\delta < \bar{\delta}$ then the seller settles at $t = 1$ and (off the equilibrium path) all $t \leq m$, where m is such that $\beta_{m+1} < V(\delta) \leq \beta_m$. At m, the settling price is $\bar{p}_m = \beta_m$; for all $t < m$ it is determined recursively by $\bar{p}_t = \beta_t - (1-r)\delta(\beta_t - \bar{p}_{t+1})$.

The result is illustrated in Figure 2. Compromise is reached at $t = 1$, and off the equilibrium path at all $t \leq m$ as well. In the proof (relegated to Appendix A) we establish that if $V(\delta) < \beta_m$ for some m then $V^1(\delta) < \bar{p}_t$ for all $t < m$; as a result the seller prefers to settle in every period before m.

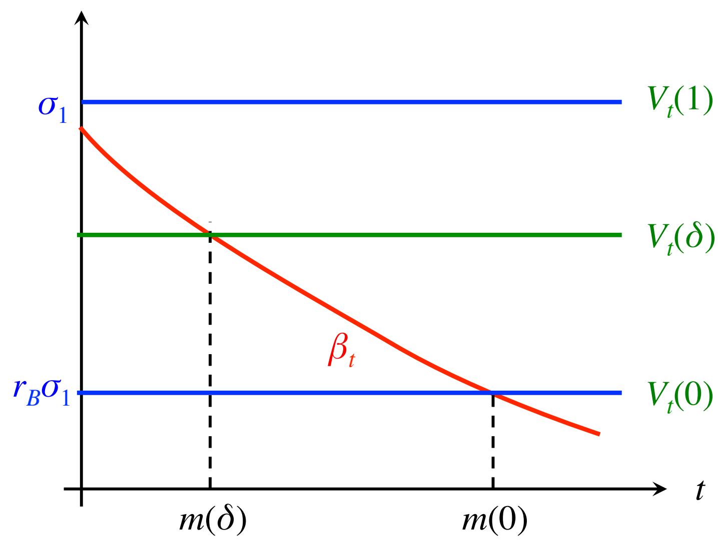


Figure 2: Long horizon and discounting.

6 Extensions

The main extension of the model that we explore in this section is one where the seller can make an offer at the ex ante stage, before either player receives a concealable signal. In the case of a common prior, with no discounting, it immediately follows that a settlement is reached at $t = 0$. However, the price at which trade occurs depends on what happens off the equilibrium path. This insight opens up the possibility that the seller could gain by committing to (an agent with) a prior either more optimistic or more pessimistic than the buyer's. It turns out that the seller may benefit from *excessive optimism* when an ex ante trade is available.

Consider an example with a common prior, $T = 1$, and assume the seller can make an ex ante offer p_0. Assume there is no discounting and that $\beta_1 > V_1$. Since

$\sigma_0 = \beta_0$ we have immediate agreement at time 0. Off the equilibrium path the parties settle at $\bar{p}_1 = \beta_1$. Hence the ex ante trading price (which equals the seller's profit) is $\bar{p}_0 = \beta_0 - [\beta_0(1 - r_S)(1 - \beta_1) - (1 - \beta_0)(1 - r_B)\beta_1] = \beta_0 r_S + \beta_0(r_B - r_S)\beta_1 < \beta_0$.

Now suppose that the seller hires an agent with $\sigma_0' > \sigma_0$ such that $V_1' = r_B \sigma_0' > \beta_1$. At $t = 1$ the agent would skim with $p_1' = 1$. Anticipating this, the settlement price at 0 is $\beta_0 = \sigma_0 > p_0$. In this example the seller can gain from pretending to be overly enthusiastic because this commits him to delay off the equilibrium path.

7 Conclusions

We have developed a dynamic model of bilateral trading with persuasion (probabilistic generation of concealable hard signals) and possibly different, commonly-known priors. The equilibrium that we identify may exhibit stubbornness: the seller offering a high price rejected by an uninformed buyer who grows more and more suspicious over time. However, compromise may be reached when the seller becomes sufficiently worried that no sale will take place at all.

We examined the effects of discounting, the probability of generating hard signals, and the length of the time horizon on the date of compromise. We also looked at the way in which the buyer's and seller's priors (skepticism or optimism) affect the equilibrium outcome. In particular, we have found that excessive seller optimism may be profitable when ex ante trades are feasible.

There are multiple interesting avenues for further research. We plan to consider a more general offer structure to represent bargaining power (e.g., alternating or random offers). A similar structure with concealable hard information could also be incorporated in other, related models of trade and bargaining, for example, ones with the possibility of a negative surplus, auctions, or models of debate.

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Appendix A: Omitted proofs

Proof of Theorem 1 (no trade under common prior and no discounting).

For $\delta = 1$, we have $V_t = \sigma_1 \left[1 - (1-r_B)^{T-t+1}(1-r_S)^{T-t}\right]$. Therefore

$$\begin{aligned} V_t - V_{t+1} &= \sigma_1 (1-r_B)^{T-t}(1-r_S)^{T-t}\left[1-(1-r_B)(1-r_S)\right] \\ &= (\sigma_1 - V_{t+1})\left[1-(1-r_B)(1-r_S)\right], \end{aligned}$$

hence V_t is strictly decreasing and concave.

We show that $V_T > \beta_T$ implies $V_t > \beta_t$. For $\beta_0 = \sigma_0$, $V_T > \beta_T$ can be written as

$$\frac{r_B \sigma_0}{1 - r_B + r_B\sigma_0} > \frac{(1-r_S)^T \sigma_0}{1-\sigma_0 + (1-r_S)^T\sigma_0}.$$

Simplifying by σ_0, then cross-multiplying with the denominators and then cancelling terms yields

$$r_B(1-\sigma_0) > (1-r_B)(1-r_S)^T. \tag{8}$$

Equation (8) can also be written as $\sigma_0 < 1 - (1-r_B)(1-r_S)^T/r_B$.

Suppose towards contradiction that $V_t \leq \beta_t$. By definition, and using $\beta_0 = \sigma_0$, this is equivalent to

$$\left[1 - (1-r_B)^{T-t+1}(1-r_S)^{T-t}\right]\frac{\sigma_0}{1-r_B+r_B\sigma_0} \leq \frac{(1-r_S)^t\sigma_0}{1-\sigma_0+(1-r_S)^t\sigma_0}.$$

Simplifying by σ_0 and cross-multiplying by the denominators, we get

$$\left[1-(1-r_B)^{T-t+1}(1-r_S)^{T-t}\right]\left\{1-[1-(1-r_S)^t]\sigma_0\right\} \leq (1-r_S)^t\left[1-r_B(1-\sigma_0)\right].$$

By (8), we have

$$\begin{aligned} 1-[1-(1-r_S)^t]\sigma_0 &> (1-r_S)^t + \frac{1}{r_B}(1-r_B)(1-r_S)^T[1-(1-r_S)^t], \\ 1-r_B(1-\sigma_0) &< 1-(1-r_B)(1-r_S)^T. \end{aligned}$$

Therefore $V_t \le \beta_t$ implies

$$\left[1-(1-r_B)^T(1-r_S)^{T-1}\right]\left[(1-r_S)^t + \frac{1}{r_B}(1-r_B)(1-r_S)^T[1-(1-r_S)^t]\right] < (1-r_S)^t\left[1-(1-r_B)(1-r_S)^T\right].$$

Subtract $(1-r_S)^t$ from both sides and divide by $(1-r_B)(1-r_S)^T/r_B$,

$$-r_B(1-r_B)^{T-t} + \left[1-(1-r_B)^{T-t+1}(1-r_S)^{T-t}\right][1-(1-r_S)^t] < -r_B(1-r_S)^t.$$

After rearranging we get

$$r_B(1-r_S)^t + [1-(1-r_S)^t] < (1-r_B)^{T-t}\left\{r_B + (1-r_B)(1-r_S)^{T-t}[1-(1-r_S)^t]\right\}.$$

The left-hand side can be further rewritten as $r_B + (1-r_B)[1-(1-r_S)^t]$, which is greater than the expression in curly brackets on the right-hand side, which in turn is greater than the entire right-hand side. Therefore the inequality is false. We have derived a contradiction from $V_t \le \beta_t$, therefore $V_t > \beta_t$ for all t.

In the main text we noted that if $V_T < \beta_T$ then (because of $\delta = 1$), in all periods $t < T$ the seller settles if and only if $\sigma_1 < \beta_t$, which is false because of the common prior assumption. This completes the proof.

Proof of Theorem 2.

If $V_t > \beta_t$ for all t then the claim follows from Lemma 1. Otherwise define $m = \max\{t \le T : V_t < \beta_t\}$, the last period in which settling can occur on or off the equilibrium path. By Lemma 2, the uninformed seller skims at $t < m$ if and only if $V_t^{m-t} > \bar{p}_t$. For $\delta = 1$ this condition is equivalent to $\sigma_1 > \beta_t$ (note that the time and price of settlement do not matter). Since β_t is decreasing, $\sigma_1 > \beta_1$ implies $\sigma_1 > \beta_t$ for all t, therefore the seller skims at all $t = 1, \ldots, m$. If $\sigma_1 < \beta_1$ then the seller skims at $t = 1$ and in every subsequent period (off the equilibrium path) such that $\sigma_1 < \beta_t$. This completes the proof.

Proof of Theorem 3.

As stated in the main text, we want to show that if $V(\delta) \le \beta_m$ then for all $t < m$

we have $V^1(\delta) < \bar{p}_t$. The former conditions is equivalent to

$$\frac{r + (1-r)\delta - (1-r)^2\delta}{1-(1-r)^2\delta}\sigma_1 \leq \beta_m$$

whereas the latter is

$$\frac{r}{1-(1-r)\delta}\sigma_1 < \beta_t, \text{ for all}\, t < m.$$

Since $\beta_m < \beta_t$ for all $t < m$ it is sufficient for us to prove that

$$\frac{r}{1-(1-r)\delta} < \frac{r + (1-r)\delta - (1-r)^2\delta}{1-(1-r)^2\delta}.$$

Cross-multiplying with the denominators we get, equivalently,

$$r\left[1-(1-r)^2\delta\right] < \left[r + (1-r)\delta - (1-r)^2\delta\right]\left[1-(1-r)\delta\right].$$

Expanding terms and simplifying yields

$$-(1-r)r < 1-(1-r) - r - (1-r)\delta + (1-r)^2\delta.$$

By rearranging we get $(1-r)(\delta - r) < (1-r)^2\delta$, equivalently $\delta - r < \delta - r\delta$, which is true. This completes the proof.

Appendix B: Miscellaneous calculations

The seller's continuation value from skimming

In Case 1,

$$\begin{aligned} V_t &= \sigma_1 r_B + \sigma_1(1-r_B)\delta[1-(1-r_B)(1-r_S)] + \\ &\quad \ldots + \sigma_1(1-r_B)^{T-t}(1-r_S)^{T-t-1}\delta^{T-t}[1-(1-r_B)(1-r_S)] \\ &= \sigma_1 r_B + \sigma_1\left[1-(1-r_B)(1-r_S)\right](1-r_B)\delta \times \\ &\quad \left[1 + (1-r_B)(1-r_S) + \ldots + (1-r_B)^{T-t-1}(1-r_S)^{T-t-1}\delta^{T-t-1}\right] \\ &= \sigma_1 r_B + \sigma_1\left[1-(1-r_B)(1-r_S)\right](1-r_B)\delta\frac{1-(1-r_B)^{T-t}(1-r_S)^{T-t}\delta^{T-t}}{1-(1-r_B)(1-r_S)\delta}, \end{aligned}$$

as claimed in equation (3).

In Case 2, substitute p'_{t+i} for $i = 1, \ldots, k-1$ into the formula for V_t^k to get

$$\begin{aligned}
V_t^k = {} & \sigma_1 r_B \left[1 - \delta^k (1-r_S)^k (1-\bar{p})\right] \\
& + \sigma_1 (1-r_B)\delta \left\{r_S + (1-r_S) r_B \left[1 - \delta^k (1-r_S)^k (1-\bar{p})\right]\right\} + \\
& \ldots + \sigma_1 (1-r_B)^{k-1} (1-r_S)^{k-2} \delta^{k-1} \left\{r_S + (1-r_S) r_B \left[1 - \delta(1-r_S)(1-\bar{p})\right]\right\} \\
& + \sigma_1 (1-r_B)^k (1-r_S)^{k-1} \delta^k [r_S + (1-r_S)\bar{p}] + (1-\sigma_1)(1-r_B)^k \delta^k \bar{p}
\end{aligned}$$

Rearrange terms to get

$$\begin{aligned}
V_t^k = {} & \sigma_1 r_B \left[1 + (1-r_B)(1-r_S)\delta + \ldots + (1-r_B)^{k-1}(1-r_S)^{k-1}\delta^{k-1}\right] \\
& + \sigma_1 (1-r_B) r_S \delta \left[1 + (1-r_B)(1-r_S)\delta + \ldots + (1-r_B)^{k-1}(1-r_S)^{k-1}\delta^{k-1}\right] \\
& - \sigma_1 r_B (1-r_S)^k \delta^k \left[1 + (1-r_B) + \ldots + (1-r_B)^{k-1}\right](1-\bar{p}) \\
& + \sigma_1 (1-r_B)^k (1-r_S)^k \delta^k \bar{p} + (1-\sigma_1)(1-r_B)^k \delta^k \bar{p}.
\end{aligned}$$

Therefore

$$\begin{aligned}
V_t^k & = \sigma_1 \left[r_B + (1-r_B) r_S \delta\right] \frac{1 - (1-r_B)^k (1-r_S)^k \delta^k}{1 - (1-r_B)(1-r_S)\delta} \\
& \quad - \sigma_1 (1-r_S)^k \delta^k \left[1 - (1-r_B)^k\right](1-\bar{p}) \\
& \quad + \sigma_1 (1-r_B)^k (1-r_S)^k \delta^k \bar{p} + (1-\sigma_1)(1-r_B)^k \delta^k \bar{p} \\
& = \sigma_1 \lambda - \sigma_1 \lambda (1-r_B)^k (1-r_S)^k \delta^k - \sigma_1 (1-r_S)^k \delta^k (1-\bar{p}) \\
& \quad + \sigma_1 (1-r_B)^k (1-r_S)^k \delta^k - \sigma_1 (1-r_B)^k (1-r_S)^k \delta^k \bar{p} \\
& \quad + \sigma_1 (1-r_B)^k (1-r_S)^k \delta^k \bar{p} + (1-\sigma_1)(1-r_B)^k \delta^k \bar{p},
\end{aligned}$$

where $\lambda = \left[r_B + (1-r_B) r_S \delta\right] / \left[1 - (1-r_B)(1-r_S)\delta\right]$. Canceling the last term of the penultimate line and the first term of the last line yields

$$V_t^k = \sigma_1 \lambda - \sigma_1 (1-\lambda)(1-r_B)^k (1-r_S)^k \delta^k - \delta^k \left[(1-r_S)^k \sigma_1 (1-\bar{p}) - (1-r_B)^k (1-\sigma_1)\bar{p}\right],$$

which is indeed equivalent to (6).

The uninformed buyer's continuation value in Case 2

Substituting in p'_{t+i} for $i = 1, \ldots, k-1$ into the formula for U_t we get

$$\begin{aligned} U_t = \delta\beta_t(1-r_S)r_B\delta^{k-1}(1-r_S)^{k-1}(1-\bar{p}) + \delta^2\beta_t(1-r_S)^2(1-r_B)r_B\delta^{k-2}(1-r_S)^{k-2}(1-\bar{p}) \\ \ldots + \delta^{k-1}\beta_t(1-r_S)^{k-1}(1-r_B)^{k-2}r_B\delta(1-r_S)(1-\bar{p}) \\ + \delta^k\left[\beta_t(1-r_S)^k(1-r_B)^{k-1}(1-\bar{p}) - (1-\beta_t)(1-r_B)^k\bar{p}\right]. \end{aligned}$$

The first two lines equal $\beta_t(1-r_S)^k\delta^k r_B(1-\bar{p})\left[1 + (1-r_B) + \ldots + (1-r_B)^{k-2}\right]$, so

$$\begin{aligned} U_t = \beta_t(1-r_S)^k\delta^k(1-\bar{p})\left[1-(1-r_B)^{k-1}\right] \\ + \delta^k\beta_t(1-r_S)^k(1-r_B)^{k-1}(1-\bar{p}) - \delta^k(1-\beta_t)(1-r_B)^k\bar{p}, \end{aligned}$$

which indeed becomes (4).

Skimming price exceeds settling price

This is obvious in Case 1 (skimming with 1). In Case 2, in period t with settling expected at price $\bar{p}$ in period $t+k$, the skimming price is $p'_t = 1 - \delta^k(1-r_S)^k(1-\bar{p})$ and the settling price is $\bar{p}_t = \beta_t - \delta^k\left[(1-r_S)^k\beta_t(1-\bar{p}) - (1-r_B)^k(1-\beta_t)\bar{p}\right]$. The difference can be written as

$$\begin{aligned} p'_t - \bar{p}_t &= 1 - \beta_t - \delta^k(1-r_B)^k(1-\beta_t)\bar{p} \\ &= (1-\beta_t)\left[1 - \delta^k(1-r_B)^k\bar{p}\right] > 0. \end{aligned}$$