

A Critique of Shareholder Value Maximization

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ABSTRACT

The majority of academic economists share the view that a corporation should serve the exclusive interests of its shareholders (shareholder value maximization). This view is firmly grounded on the extension, by Arrow (1953) and Debreu (1959) of the two welfare theorems to production economies with uncertainty and complete markets. This paper considers a variant of the Arrow-Debreu model where uncertainty is endogenous: probabilities of productive outcomes depend on decisions made by firms. In that case, a competitive equilibrium with shareholder value maximizing firms (capitalist equilibrium) is never Pareto optimal. This is because endogenous uncertainty implies that firms exert externalities on their consumers and their employees. When firms are stakeholder oriented, in that their managers are instructed to maximize a weighted sum of their shareholder value and of their contributions to consumer and employee welfares, the new competitive equilibrium (stakeholder equilibrium) improves upon the capitalist equilibrium.

1 Introduction

Everyone knows that corporations are not just cash machines for their shareholders, but that they also provide goods and services for their consumers, as well as jobs and incomes for their employees. Everyone, that is, except most economists. Indeed in the debate on the social responsibility of corporations, the majority of academic economists share the view expressed in unambiguous terms by Friedman (1970) “there is one and only one social responsibility of business—to use its resources and engage in activities designed to increase its profits”. By contrast, proponents of the ‘stakeholder’ view of corporations assert that managers should pay attention not only to the profits of the shareholders but also to the welfare of their employees and consumers. The orthodox view held by most economists is a tradition inherited from the Anglo-American view of corporations, while the so-called non-orthodox stakeholder view is that held in countries such as Japan and most continental European countries, in particular Germany and France.

The way in which a society views the role of a corporation can be traced to its legal system and to the social norms which shape the way individuals think about the role of institutions. Common Law countries such as the UK and the US view a corporation as a piece of private property and through their legal structure place exclusive emphasis on the shareholders as the owners of the firm. Civil Law countries such as Germany and France view corporations as ‘mini-societies’ and place emphasis on the responsibility of the firm to its employees as well as its shareholders. Social norms have pushed this view of the corporation to its extreme form in Japan where the responsibility to the interest of employees and other stakeholders such as

suppliers outweighs that to the shareholders (see Yoshimori (1995)).

When taken in historical perspective the stakeholder view of the corporation has been gaining momentum in all advanced economies over the last hundred years: the changing legal structures and the evolution of social norms have come to make most large corporations aware that they need to expand the focus of their responsibilities to a larger group than their shareholders, to include employees and consumers as well as other groups such as suppliers and subcontractors involved in their long-term productive relationship. While the view that corporations, like all institutions in a modern society, would need to democratize and serve the interests of the wider base of all stakeholders with whom they deal on a regular basis was clearly articulated in the prescient final chapter of Berle and Means (1932) on the “new concept of the corporation”, the idea did not catch on with economists. Indeed what is remarkable is the hegemony of the Anglo-American view of the corporation among economists: to this day the idea that the corporation should serve the exclusive interest of its shareholders remains the dominant paradigm for corporate governance (Schleifer-Vishny (1997)). Although recently there are some signs of a willingness to change (Tirole (2001, 2006), Allen (2005), Allen-Carletti-Marquez (2009)) mainstream economics has not kept abreast of the evolution of society’s view of the role and responsibilities of a corporation, and continues to advocate shareholder value maximization as the primary responsibility of the management of a corporation.

How do we account for these apparently orthogonal views of the objective of corporate governance? Presumably the justification that underlies the orthodox view can be found in the standard inventory of theorems asserting the efficacy of the price system when firms maximize profits: these however are all directly or indirectly based on the assumption that firms are infinitesimal. The decisive insight of Berle and Means (1932) is that since the end of the 19th century a significant share of US output is produced by enterprises (corporations) which have become very large and that their size makes them very different from the small enterprises which populate standard economic models.

One aspect of the influence of size has long been recognized by monopoly and oligopoly theory which has shown that profit maximization by large firms with market power leads to lower production, higher prices and less innovation than would be optimal. This however has not shaken the faith of economists in the virtue of profit maximization. It has rather led to the adoption of laws restricting the behavior of large corporations—e.g. Antitrust Law in the US, Competition Law in Europe—and to the creation of agencies charged with their

enforcement, like the Federal Trade Commission and the Directorate General for Competition of the European Commission.

The message of this paper is that even if these laws and the associated agencies created to implement them are successful in enforcing competition on the product and labor markets, and even if, at the cost of being unrealistic we assume that corporations do not create externalities of the pollution type on the physical environment, there would still be a systematic way in which the actions of large corporations have a significant effect on agents with whom they interact, in particular on the consumers they serve and on the workers they employ. For it is a fundamental fact of business that all firms operate in an environment of uncertainty and that exposure to risk induces a new way in which a large corporate enterprise differs from a small firm. The success or failure of a large corporation may have a significant impact on its consumers and/or its workers: if a firm can be closed (fail) and its consumers can buy elsewhere at the same price, and its workers can find employment elsewhere at the same wage, then we say that the firm is ‘small’; otherwise, and it is clearly a matter of degree, we say that the firm is ‘large’.

To formalize this idea we present a simple model in which firms can invest resources to influence the probability of success, where success/failure is identified with a more or less productive technology. We assume perfect competition on the product and labor markets and no standard externalities created by any of the firms’ production processes. In much of the paper, to focus on the kernel of the argument, we study a “benchmark” model in which there is a single firm exposed to risk and a second firm which is a stand-in for the rest of the production sector. In this setting profit maximization always leads to under-investment: the profit maximizing level of investment is less than the social optimum.

From a ‘modeling’ perspective this result is surprising. After all the model is close to an Arrow-Debreu model of a production economy with uncertainty and complete contingent markets for which we know that equilibria are always Pareto optimal regardless of the number and size of the firms (provided firms act as price takers). The main difference is that in our model a firm’s investment affects the probability of its outcome while in an Arrow-Debreu model states of nature with fixed probabilities combine with investment to determine the firm’s outcome. Thus an apparently small difference in modeling dramatically changes the normative properties of the equilibrium: the two Welfare Theorems which are the basis for the faith in markets and profit maximization no longer hold. To understand exactly where the difference lies, we embed our benchmark economy in an Arrow-Debreu economy with the

same characteristics. We obtain the striking result that an Arrow-Debreu equilibrium **never** exists in our economy. This comes from non-convexities that are inherent to the simple and natural modeling of endogenous uncertainty by a two state random variable. This is in line with the following quotation of Arrow (1971): “ we have seen that it is possible to set up formal mechanisms which under certain conditions will achieve an optimal allocation of risk by competitive methods. However, the empirical validity of the conditions for the optimal character of competitive allocations is considerably less likely to be fulfilled in the case of uncertainty than in the case of certainty and, furthermore, many of the economic institutions which would be needed to carry out the competitive allocation in the case of uncertainty are in fact lacking.”

When we use the market structure which is natural for our benchmark model—competitive spot markets and financial markets—there is an external effect created by the firm’s investment decision, which is more subtle and less noticeable than the standard externalities on the physical environment which accompany many production processes. For by shifting probability towards the outcome where it is more productive, and thus reducing the price of its output and increasing the wages of its employees, the firm’s investment influences the expected utilities of its consumers and employees: since this external effect is not internalized by the markets, profit maximization ceases to be the correct “social criterion” for the firm.

It thus seems natural to explore ways in which the firm can be led to internalize the externality by including the interests of consumers and workers in the criterion it uses for the choice of investment. In Section 4 we study the criterion that an "ideal" stakeholder firm should adopt if investment is to be socially optimal. We show that if the firm can be considered independently of other firms—for example if it is a “natural monopoly” with no competition—then the criterion consists of the sum of the surpluses of its stakeholders, namely its shareholders, consumers and workers. The main problem is then to obtain information on the surpluses of consumers and workers. We suggest that implementing the Coasian approach of creating marketable rights, “consumer rights” or right to buy from the firm, and “worker rights” or right to work for the firm, can serve to elicit these surpluses and provide a measurable way of evaluating the performance of the stakeholder firm.

When there are several firms competing on the same product and labor markets, if a stakeholder firm were to maximize the sum of the surpluses of its own stakeholders, it would exaggerate the difference in benefit between the good and the bad outcome, and would be led to over-invest to make the successful outcome more likely. For such a calculation would

exclude the consumers, workers, and shareholders of competing firms who are also affected by the firm's outcome, and their interests are at odds with those of the stakeholders of the firm under consideration.

Thus there is under-investment when only profit is taken into account, and over-investment if the total surplus of its own stakeholders is used as the firm's criterion. This leads to the striking result that there is an ideal weight to be placed on its consumer and worker surpluses which, when added to the profit of the shareholders, gives a criterion which induces the socially optimal investment. Furthermore if the firm applies any positive weight to its consumer and worker surpluses below this level, then the resulting stakeholder equilibrium outcome improves on the capitalist equilibrium.

A complementary approach to this paper is provided by the recent contribution of Allen-Carletti-Marquez (2009). ACM are motivated the cost incurred by workers who are laid off when a firm goes bankrupt. They consider a setting with imperfect competition where firms must commit to a price for their output before demand or cost shocks are realized, and firms can go bankrupt when hit by an unfavorable shock. ACM study how the pricing strategy of a "stakeholder firm", which takes into account in its objective function the cost of layoffs for its workers, differs from that of a "shareholder firm" maximizing profit.

The paper is organized as follows. Section 2 introduces the benchmark model and the concept of a capitalist equilibrium in which firms use the criterion of profit maximization. We show that there is always under investment in a capitalist equilibrium. Section 3 embeds the economy in an Arrow-Debreu framework and compares the capitalist equilibrium with the Arrow-Debreu equilibrium of the same economy. Section 4 studies if a stakeholder approach can resolve the problem of under-investment in a capitalist equilibrium and Section 5 concludes.

2 Inefficiency of Capitalist Equilibrium

Consider a two-period stochastic production economy with J firms. There are three goods: a produced good, a composite good called "money" (used as the numeraire) and labor. At date 0 the only available resource is money, a part of which can be used to finance investment expenditures by the firms. Each firm faces production risk and operates in an environment where its projects can be more or less successful. While a firm cannot completely control its environment it can invest resources to increase the probability of better outcomes. To keep the analysis simple we assume that each firm $j = 1, \dots, J$ has two possible outcomes, a good technology f_g^j or a less productive technology f_b^j , and that incurring expenditures γ^j can

augment the probability π^j of the good outcome f_g^j . Our objective is to study whether the standard criterion of profit maximization by firms leads to socially optimal choices $(\gamma^1, \dots, \gamma^J)$ or if some other criterion is required.

2.1 Benchmark Model

The analysis of the problem is decomposed into two steps: in the first only firm 1 is subject to risk and the outcomes of the other firms are fixed so that the productive sector, consisting of all firms other than firm 1, can be summarized by a surrogate second firm with deterministic technology \hat{f} : we take this as the benchmark model. In a second step we show how the results of the benchmark model can be extended to the symmetric version of the model outlined above.

At date 1 the first firm's technology will be one of the two production functions $y_s = f_s(l)$ where s is either g or b . Each production function $f_s : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is differentiable, increasing, concave and satisfies $f_s(0) = 0$, $s = g, b$. The marginal product of f_g is uniformly higher than that of f_b : $f'_g(l) > f'_b(l), \forall l > 0$, which implies that $f_g(l) > f_b(l), \forall l > 0$, thus justifying the terminology that "g" is the good and "b" the bad outcome. When firm 1 incurs the investment expenditure γ at date 0 the probability of having the good outcome at date 1 is $\pi(\gamma)$, where $\pi : \mathbb{R}_+ \rightarrow [0, 1)$ is differentiable, increasing, concave and satisfies $\pi(0) = 0$ and $\pi'(0) = \infty$. Choosing the investment γ , which leads to the probability $\pi(\gamma)$ of the good outcome, is equivalent to directly choosing the probability $\pi \in [\pi(0), 1)$ by incurring the cost $\gamma(\pi)$ at date 0: using this approach is convenient for writing first-order conditions and is used in most of the analysis that follows. It follows from the assumptions on $\pi(\gamma)$ that the cost function $\gamma : [0, 1) \rightarrow \mathbb{R}_+$ is differentiable, increasing, convex, and satisfies $\gamma'(0) = 0$, $\gamma'(\pi) \rightarrow \infty$ as $\pi \rightarrow 1$. To retain the symmetry of notation, we let π_s denote the probability of outcome s , $s = g, b$, with $\pi_g = \pi$ and $\pi_b = 1 - \pi$.

Firm 2 does not face risk and makes no investment at date 0. It operates a technology $\hat{f}(\hat{l})$ at date 1 where \hat{f} is differentiable, increasing, concave and satisfies $\hat{f}(0) = 0$. To avoid boundary solutions we assume that all production functions satisfy the Inada condition $f'_g(0) = f'_b(0) = \hat{f}'(0) = \infty$.

There are three "classes" of agents: workers/employees, consumers and capitalists. Each consists of a continuum of identical agents of mass 1. Each worker is endowed with 1 unit of labor at date 1, consumes only money and has the utility function

$$U^w(m, \ell) = m_0 + \delta \sum_{s=g,b} \pi_s (m_s - v(\ell_s)),$$

where $m = (m_0, m_g, m_b)$ is a worker's consumption of money and ℓ_s is the quantity of labor sold to the firm in outcome s , $s = g, b$. The discount factor satisfies $0 < \delta \leq 1$ and the disutility of labor, $v(\ell) : \mathbb{R}_+ \rightarrow \mathbb{R}$, is differentiable, convex and increasing, with $v(0) = 0$, $v'(0) = 0$ and $v'(\ell) \rightarrow \infty$ if $\ell \rightarrow 1$. Throughout we will use the symbol “ ℓ ” for the labor supplied by the representative worker and “ l ” for the demand for labor by the firms.

Each consumer, who consumes both money and the produced good, has the utility function

$$U^c(m, c) = m_0 + \delta \sum_{s=g,b} \pi_s (m_s + u(c_s)),$$

where $c = (c_g, c_b)$ is the consumption of the produced good in the two outcomes, and u is differentiable, strictly concave and increasing, with $u(0) = 0$ and $u'(c) \rightarrow \infty$ if $c \rightarrow 0$.

Finally there are capitalists, who own the firms, consume only money and have the same (linear) utility function

$$U^k(m) = m_0 + \delta \sum_{s=g,b} \pi_s m_s.$$

For reasons that we explain later, we assume that the two firms are owned by distinct subsets of shareholders. The money endowments $e^i = (e_0^i, e_1^i)$, $i = w, c, k$ are assumed to be sufficiently large so that non-negativity constraints on consumption never bind. We let $e_0 = e_0^w + e_0^c + e_0^k$, $e_1 = e_1^w + e_1^c + e_1^k$ denote the aggregate endowment of money at date 0 and 1, and denote by $\mathcal{E} = (U, e, f, \gamma, \hat{f})$ the economy with preferences and endowments $(U^i, e^i)_{i=w,c,k}$ and technologies (f, γ, \hat{f}) for the firms.

2.2 Socially Optimal Investment

Given the quasi-linearity of the agents' preferences, a Pareto optimum is an allocation¹ $(\pi^*, m^*, c^*, \ell^*, l^*, \hat{l}^*)$ that maximizes the sum of the agents' utilities

$$\max_{(\pi, m, c, \ell, l, \hat{l}) \geq 0} \sum_{i=w,c,k} \left(m_0^i + \delta \sum_{s=g,b} \pi_s m_s^i \right) + \delta \sum_{s=g,b} \pi_s [u(c_s) - v(\ell_s)]$$

subject to the resource constraints for money, consumption and labor

$$\begin{aligned} \sum_{i=w,c,k} m_0^i + \gamma(\pi) &= e_0, & \sum_{i=w,c,k} m_s^i &= e_s, \\ c_s &= f_s(l_s) + \hat{f}(\hat{l}_s), & \ell_s &= l_s + \hat{l}_s, \quad s = g, b. \end{aligned} \tag{1}$$

¹We use the following notational convention: a letter without superscript or subscript summarizes the vector of indexed values of the corresponding variable. For example, $m = (m_0^i, m_s^i)$, $i = w, c, k$, $s = g, b$ and $\ell = (\ell_s)_{s=g,b}$

This is equivalent to finding $(c^*, \ell^*, \pi^*, l^*, \hat{l}^*)$ that solves

$$\max_{(c, \ell, \pi, l, \hat{l}) \geq 0} e_0 - \gamma(\pi) + \delta \sum_{s=g, b} \pi_s [e_1 + u(c_s) - v(\ell_s)],$$

or more simply solves

$$\max_{(c, \ell, \pi, l, \hat{l}) \geq 0} \delta \sum_{s=g, b} \pi_s [u(c_s) - v(\ell_s)] - \gamma(\pi), \quad (2)$$

subject to the resource constraints (1). The maximum problem (2) decomposes into the choice of consumption-labor $(c_s^*, \ell_s^*, l_s^*, \hat{l}_s^*)$ within each outcome state $s = g, b$ that maximizes the social welfare

$$W_s = u(c_s) - v(\ell_s) \quad (3)$$

subject to the resources constraints (1), and firm 1's choice of investment, or more directly the choice of the probability of success π^* that maximizes

$$\delta(\pi W_g^* + (1 - \pi)W_b^*) - \gamma(\pi) \quad (4)$$

where W_g^*, W_b^* are the optimized values of (3). The first-order conditions for the choice of consumption-labor at date 1 are

$$\begin{aligned} u_s'^* f_s'(l_s^*) &= v'(\ell_s^*), & u_s'^* \hat{f}'(\hat{l}_s^*) &= v'(\ell_s^*), & s &= g, b \\ c_s^* &= f_s(l_s^*) + \hat{f}(\hat{l}_s^*), & \ell_s^* &= l_s^* + \hat{l}_s^*, & s &= g, b \end{aligned} \quad (5)$$

Since the social welfare W_s in each outcome s is a strictly concave function, there is a unique solution to the FOCs (5), which are necessary and sufficient for characterizing the optimal allocation. Since $f_g(l) > f_b(l)$ for all $l > 0$, $W_g(l, \hat{l}) = u(f_g(l) + \hat{f}(\hat{l})) - v(l + \hat{l}) > u(f_b(l) + \hat{f}(\hat{l})) - v(l + \hat{l}) = W_b(l, \hat{l})$ so that

$$W_g^* = \max_{(l, \hat{l})} W_g(l, \hat{l}) > W_b^* = \max_{(l, \hat{l})} W_b(l, \hat{l}).$$

Again, this justifies our notation that “g” is indeed the good social outcome.

The FOC for the optimal choice of investment by firm 1 at date 0 is given by

$$\delta (W_g^* - W_b^*) = \gamma'(\pi^*), \quad (6)$$

and this has a unique solution π^* since γ' increases from 0 to ∞ . (6) requires that the marginal cost of increasing the probability of success equals the discounted social benefit of realizing the good rather than the bad outcome of firm 1.

2.3 Capitalist Equilibrium

We now analyze a market equilibrium of the above economy in which both firms choose their labor and firm 1 chooses its investment in the best interests of their shareholders. As we will see, Consumers buy the firms' output and workers sell their labor services on spot markets. Agents can also trade on asset markets to redistribute their income. We will show that the real side of such a market equilibrium can be summarized by a vector $(\bar{\pi}, \bar{l}, \bar{\hat{l}})$ consisting of the probability of the good outcome, and the labor choices in each productive outcome. This vector can be compared with the Pareto optimal choice (π^*, l^*, \hat{l}^*) derived above.

At each date the price of the composite commodity (money) is normalized to 1. At date 0 agents trade a riskless bond promising one unit of money in each outcome $s = g, b$ at date 1 with price $\frac{1}{1+r}$ where r is the interest rate. There is also an equity market at date 0 on which the capitalists trade the shares of the firms, the price of equity being q for firm 1 and \hat{q} for firm 2. At date 1 for each outcome $s = g, b$ there are spot markets for labor and the produced good with prices $(w_s, p_s), s = g, b$.

Firm 1 makes two choices: at date 0 it selects the probability π and at date 1 it chooses the amounts of labor $l = (l_g, l_b)$ to hire in each outcome. The labor is chosen in each outcome to maximize its profit

$$R_s(l_s; w_s, p_s) = p_s f(l_s) - w_s l_s, \quad s = g, b$$

taking the spot prices (w_s, p_s) as given.² Assuming that the firm correctly anticipates the spot prices and its future labor decision, it chooses the probability π at date 0 to maximize the (net) present value of profit, which in this case is just the discounted expected profit net of the investment cost since agents are risk neutral. Firm 1's combined choice problem amounts to choosing (π, l) to maximize its value for the shareholders, which we denote by SV :

$$SV(\pi, l; w, p) = \sum_{s=g,b} \frac{\pi_s}{1+r} R_s(l_s; w_s, p_s) - \gamma(\pi). \quad (7)$$

In the same way firm 2, which has no date 0 investment decision, maximizes its value for the shareholders

$$\widehat{SV}(\hat{l}; w, p) = \sum_{s=g,b} \frac{\pi_s}{1+r} \hat{R}(\hat{l}_s; w_s, p_s), \quad (8)$$

²By assuming that the firms behave competitively on the labor and output markets we abstract from potential distortions created by monopolistic or oligopolistic behavior on the spot markets at date 1, focusing instead on the investment decision of firm 1. As we mentioned in the introduction there are institutions that have been created to prevent firms from exercising their market power: thus we assume that a "Competition Authority" or an "Antitrust Agency" knows enough about the production possibilities of the firm to penalize any excess profit due to restrictive practices.

by choosing \hat{l}_s at date 1 which maximizes its profit

$$\hat{R}(\hat{l}_s; w_s, p_s) = p_s f(\hat{l}_s) - w_s \hat{l}_s.$$

The three groups of agents trade on the spot and financial markets and have sequential budget equations at date 0 and in each outcome at date 1 of the form

$$\begin{aligned} m_0^i &= e_0^i - \frac{1}{1+r} z^i - q\theta^i - \hat{q}\hat{\theta}^i + \xi^i \\ m_s^i &= e_s^i + z^i + R_s\theta^i + \hat{R}_s\hat{\theta}^i + w_s\ell_s^i - p_s c_s^i, \quad s = g, b \end{aligned} \quad (9)$$

where z^i is the bond holding and $\theta^i, \hat{\theta}^i$ are the ownership shares of the firms purchased by agent i . Finally

$$\begin{aligned} \bullet \quad \xi^i &= 0, & \text{if } i = w, c, & \quad \xi^i = [q - \gamma(\pi)]\theta_0^i + \hat{q}\hat{\theta}_0^i & \text{if } i = k \\ \bullet \quad c_s^i &= 0, & \text{if } i = w, k, & \quad c_s^i = c_s & \text{if } i = c \\ \bullet \quad \ell_s^i &= 0, & \text{if } i = c, k, & \quad \ell_s^i = \ell_s & \text{if } i = w, \end{aligned} \quad (10)$$

where θ_0^i and $\hat{\theta}_0^i$ denote the initial endowments of shares by capitalist i (these endowments sum to one across all capitalists and cannot be simultaneously positive for any of them, given our assumption that the firms are owned by distinct subsets of individuals). The owners of firms 1 participate in the financing of the cost $\gamma(\pi)$ proportionally to their endowments of shares of firm 1. All capitalists get income from the sale of their ownership shares ($\xi^i = [q - \gamma(\pi)]\theta_0^i + \hat{q}\hat{\theta}_0^i$), while only the consumers purchase the produced good ($c_s^c = c_s$) and only workers sell their labor services ($\ell_s^w = \ell_s$). While shareholders are assumed to finance the investment of firm 1, any mode of financing whether by debt or by issuing new shares would lead to the same equilibrium in view of the Modigliani-Miller theorem. All agents are assumed to know firm 1's choice of π at date 0 and to correctly anticipate future spot prices and the firms' profits R_s and \hat{R}_s in each outcome s at date 1.

Given the linearity of the agents' preferences in the numeraire composite commodity, the first-order conditions for the optimal choice of bond and equity holdings imply

$$\frac{1}{1+r} = \delta, \quad q = \delta \sum_{s=g,b} \pi_s R_s = \sum_{s=g,b} \frac{\pi_s}{1+r} R_s, \quad \hat{q} = \sum_{s=g,b} \frac{\pi_s}{1+r} \hat{R}_s \quad (11)$$

so that pricing is risk neutral. Since the date 1 payoff of the bond is $(1, 1)$, if $R_g \neq R_b$ or $\hat{R}_g \neq \hat{R}_b$, the bond and equity contracts have linearly independent payoff streams, so that the financial markets are complete and the sequential budget constraints (9) are equivalent to the

single intertemporal (present value) budget constraint

$$m_0^i + \sum_{s=g,b} \frac{\pi_s}{1+r} m_s^i = e_0^i + \frac{e_1^i}{1+r} + \xi^i + \sum_{s=g,b} \frac{\pi_s}{1+r} (w_s \ell_s^i - p_s c_s^i), \quad i = w, c, k \quad (12)$$

where (ξ^i, c^i, ℓ^i) are given by (10). In view of the linearity of the agents' preferences in $m^i = (m_0^i, m_g^i, m_b^i)$ any m^i satisfying (12) is equivalent for agent i , and when the budget constraint (12) is satisfied, the utility of agent i is

$$\begin{aligned} \bullet \quad & e_0^w + \frac{e_1^w}{1+r} + \sum_{s=g,b} \frac{\pi_s}{1+r} (w_s \ell_s - v(\ell_s)) && \text{for a worker} && \text{(a)} \\ \bullet \quad & e_0^c + \frac{e_1^c}{1+r} + \sum_{s=g,b} \frac{\pi_s}{1+r} (u(c_s) - p_s c_s) && \text{for a consumer} && \text{(b)} \\ \bullet \quad & e_0^k + \frac{e_1^k}{1+r} + [q - \gamma(\pi)] \theta_0^i + \hat{q} \hat{\theta}_0^i && \text{for a capitalist} && \text{(c)} \end{aligned} \quad (13)$$

Thus a worker will choose ℓ to maximize 13(a), a consumer will choose c to maximize 13(b) and a capitalist has no other choice than to spend his income on the composite good. If he is among the owners of firm 1, $\hat{\theta}_0^i = 0$, and his utility is maximized when firm 1 maximizes shareholder value $SV = q - \gamma(\pi)$. If he is among the owners of firm 2, $\theta_0^i = 0$, and his utility is maximized when firm 2 maximizes its shareholder value $\widehat{SV} = \hat{q}$.³

Summing the budget equations (12), assuming that (10) holds, gives

$$\sum_{i=w,c,k} m_0^i + \sum_{s=g,b} \frac{\pi_s}{1+r} m_s^i = e_0 + \frac{e_1}{1+r} + q + \hat{q} - \gamma(\pi) + \sum_{s=g,b} \frac{\pi_s}{1+r} (-p_s c_s + w_s \ell_s)$$

If the markets clear for the produced good ($c_s = f_s(l_s) + \hat{f}(\hat{l}_s)$) and labor ($\ell_s = l_s + \hat{l}_s$) then in view of (11) the terms involving the firm's market value cancel giving

$$\sum_{i=w,c,k} m_0^i + \sum_{s=g,b} \frac{\pi_s}{1+r} m_s^i = e_0 + \frac{e_1}{1+r} - \gamma(\pi).$$

Given the indeterminacy in the choice of m^i , we can assume that when agents choose m^i to satisfy (12) they in addition choose money holdings such that

$$\sum_{i=w,c,k} m_0^i + a = e_0, \quad \sum_{i=w,c,k} m_s^i = e_1, \quad s = g, b \quad (14)$$

³Our assumption that the two firms are owned by distinct subsets of shareholders ($\theta_0^i \cdot \hat{\theta}_0^i \equiv 0$) simplifies the analysis. It implies that each capitalist firm maximizes its own shareholder value. Our results would still hold for different ownership structures but the investment choice of firm one would have to take into account the externalities it exerts on firm two.

so that the market for the composite good clears at date 0 and in each outcome s at date 1.

Since our objective is to compare the consumption, labor and investment choices which arise in a market equilibrium with those at the social optimum, we focus directly on a succinct reduced-form definition of an equilibrium involving these three choices: from this reduced-form equilibrium a complete description of the equilibrium on the spot markets for the produced good, money and labor, and on the financial markets for the bond and equity can be easily reconstructed using (9)-(11) and (14).

Definition 1. A (*reduced-form*) *capitalist equilibrium* of the economy \mathcal{E} is a vector of actions and prices $\left((\bar{\ell}, \bar{c}, \bar{\pi}, \bar{l}, \bar{\hat{l}}), (\bar{w}, \bar{p}) \right)$ such that

- (i) the labor choice $\bar{\ell} = (\bar{\ell}_g, \bar{\ell}_b) \geq 0$ maximizes worker's utility 13(a) given \bar{w} ;
- (ii) the consumption choice $\bar{c} = (\bar{c}_g, \bar{c}_b) \geq 0$ maximizes consumer's utility 13(b) given \bar{p} ;
- (iii) firm 1's production plan $(\bar{\pi}, \bar{l}) = (\bar{\pi}, \bar{l}_g, \bar{l}_b) \geq 0$ maximizes shareholder value (7) given (\bar{w}, \bar{p}) ;
- (iv) firm 2's production plan $(\bar{\hat{l}}) = (\bar{\hat{l}}_g, \bar{\hat{l}}_b) \geq 0$ maximizes shareholder value (8) given (\bar{w}, \bar{p}) ;
- (v) the markets clear: $\bar{\ell}_s = \bar{l}_s + \bar{\hat{l}}_s$, $\bar{c}_s = f_s(\bar{l}_s) + \hat{f}(\bar{\hat{l}}_s)$, $s = g, b$.

Let us compare the FOCs for the maximum problems (i)-(iv) of a capitalist equilibrium with the FOCs for a Pareto optimum. In a capitalist equilibrium the optimal labor choice $\bar{\ell}$ for the workers satisfies

$$v'(\bar{\ell}_s) = \bar{w}_s, \quad s = g, b \quad (15)$$

and the consumers' optimal choice \bar{c} satisfies

$$u'(\bar{c}_s) = \bar{p}_s, \quad s = g, b \quad (16)$$

while the firm's profit-maximizing choices of labor $(\bar{l}, \bar{\hat{l}})$ imply that for each outcome at date 1 the real wage equals the marginal product of labor

$$\bar{p}_s f'_s(\bar{l}_s) = \bar{w}_s, \quad \bar{p}_s \hat{f}'(\bar{\hat{l}}_s) = \bar{w}_s \quad s = g, b. \quad (17)$$

Using (15), (16) to eliminate spot prices and adding the market clearing condition (v) gives the equations

$$\begin{aligned} u'(\bar{c}_s) f'_s(\bar{l}_s) &= v'(\bar{\ell}_s), & u'(\bar{c}_s) \hat{f}'(\bar{\hat{l}}_s) &= v'(\bar{\ell}_s), & s &= g, b \\ \bar{c}_s &= f(\bar{l}_s) + \hat{f}(\bar{\hat{l}}), & \bar{\ell}_s &= \bar{l}_s + \bar{\hat{l}}_s, & s &= g, b \end{aligned} \quad (18)$$

which characterize the spot market equilibrium at date 1. Since (18) is identical to (5), which characterizes the maximum of the social welfare, the choice of labor in equilibrium is optimal and

$$(\bar{c}, \bar{\ell}, \bar{l}, \bar{\hat{l}}) = (c^*, \ell^*, l^*, \hat{l}^*).$$

The remaining first-order condition for the choice of investment $\bar{\pi}$ which maximizes shareholder value (7) is

$$\frac{1}{1+r} (\bar{R}_g - \bar{R}_b) = \gamma'(\bar{\pi}) \text{ if } \bar{R}_g > \bar{R}_b, \quad \bar{\pi} = 0 \text{ otherwise,} \quad (19)$$

where \bar{R}_s is the maximized profit of firm 1 in outcome s : this equation has a unique solution since $\gamma'(\pi)$ increases from 0 to ∞ . Comparing (19) with (6) we see that if $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$ then $\bar{\pi} < \pi^*$ since γ' is increasing: the profit criterion underestimates the gain from obtaining the outcome f_g rather than the outcome f_b .

Proposition 1. *There is under-investment in the capitalist equilibrium: $\bar{\pi} < \pi^*$.*

Proof: We want to show that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$. To this end consider the parameterized family of production functions for firm 1

$$f(t, l) = t f_g(l) + (1-t) f_b(l), \quad t \in [0, 1]$$

where the parameter takes the production function continuously from the bad to the good outcome. We associate with each $t \in [0, 1]$ a fictitious ‘t’ spot economy at date 1 with the characteristics $(u, v, f(t, \cdot), \hat{f})$. The maximized social welfare for the t economy is

$$W(t) = \max\{u(c) - v(\ell) \mid c = f(t, l) + \hat{f}(\hat{l}), \ell = l + \hat{l}\}$$

The solution $(c(t), \ell(t), l(t), \hat{l}(t))$ of this maximum problem is characterized by the equations

$$u'(c(t)) f_2(t, l(t)) = v'(\ell(t)), \quad u'(c(t)) \hat{f}'(\hat{l}(t)) = v'(\ell(t)), \quad (20)$$

$$c(t) = f(t, l(t)) + \hat{f}(\hat{l}(t)), \quad \ell(t) = l(t) + \hat{l}(t) \quad (21)$$

and this allocation can be induced by letting agents and firms make their choices on spot markets at prices

$$p(t) = u'(c(t)), \quad w(t) = v'(l(t)).$$

Let $R(t) = p(t)f(t, l(t)) - w(t)l(t)$ denote the (optimized) profit of firm 1 under these spot prices. We show that the function

$$D(t) = W(t) - R(t)$$

is strictly increasing on $[0, 1]$: this will imply that $D(1) = W_g^* - \bar{R}_g > D(0) = W_b^* - \bar{R}_b$ and hence establish the result.

By the envelope theorem

$$W'(t) = u'(c(t))f_1(t, l(t)), \quad R'(t) = p'(t)f(t, l(t)) + p(t)f_1(t, l(t)) - w'(t)l(t).$$

Thus $D'(t) = -p'(t)f(t, l(t)) + w'(t)l(t)$. Since (20) implies that the marginal products of labor are equalized, $f_2(t, l(t)) = \hat{f}'(\hat{l}(t))$, it follows that

$$p'(t) = u''(c(t))[f_1(t, l(t)) + f_2(t, l(t))(l'(t) + \hat{l}'(t))]$$

$$w'(t) = v''(\ell(t))(l'(t) + \hat{l}'(t)).$$

The change in the optimal allocation of labor to the two firms $(l'(t), \hat{l}'(t))$ can be obtained by differentiating the FOCs for the optimal allocation of labor (20). This gives the pair of linear equations

$$u''(f_1 + f_2(l' + \hat{l}'))f_2 + u'(f_{21} + f_{22}l') - v''(l' + \hat{l}') = 0$$

$$u''(f_1 + f_2(l' + \hat{l}'))f_2 + u'\hat{f}''\hat{l}' - v''(l' + \hat{l}') = 0, \quad (22)$$

where the arguments of the functions have been omitted to simplify notation. Solving these equations leads to

$$l' + \hat{l}' = \frac{-u''f_1f_2(f_{22} + \hat{f}''') - u'f_{21}\hat{f}''}{u'\hat{f}''f_{22} + (u''(f_2)^2 - v'')(f_{22} + \hat{f}''')}. \quad (23)$$

The denominator is positive since f_{22}, \hat{f}''', u'' are negative and v'' is positive, while the sign of the numerator is ambiguous. However substituting this expression into $D'(t) = -u''f_1f + (v''l - u''f_2f)(l' + \hat{l}')$ gives

$$D'(t) = \frac{1}{den} \left[u''u'f\hat{f}''(f_{21}f_2 - f_1f_{22}) + u''v''f_1(f_{22} + \hat{f}''')(f - f_2l) - u'v''\hat{f}''f_{21}l \right]$$

where “den” is the positive denominator of $l' + \hat{l}'$. Since by concavity of f , $f - f_2l > 0$, all the terms are positive and $D'(t) > 0$: thus moving toward the good outcome constantly increases the welfare by more than the increase in profit. \square

2.4 General Model

Proposition 1 applies to a setting in which a dominant firm (firm 1) operates on spot markets for labor and output in parallel with a competitive fringe (represented by \hat{f}) in which the

idiosyncratic risks of the small firms cancel out. We now extend this result to the more general setting where there are J firms, each of which invests in a risky technology. In the general case where the firms face different risks and have access to different technologies we can show, by comparing first-order conditions, that a capitalist equilibrium is not Pareto optimal. But it is much harder to obtain an exact generalization of Proposition 1 in which there is a monotone ranking of the firms' investments when comparing a capitalist equilibrium with a Pareto optimal allocation. However when the firms are sufficiently similar—in short when we appeal to symmetry—the under-investment result can be extended to the case of J firms.

To keep notation simple we focus on the case where $J = 2$ and assume that the second firm now has a technology that is exposed to risk. If it invests $\hat{\gamma}$ at date 0 it will operate \hat{f}_g with probability $\hat{\pi}(\hat{\gamma})$ and \hat{f}_b with probability $1 - \hat{\pi}(\hat{\gamma})$. We assume in addition that $(\hat{f}_g, \hat{f}_b) = (f_g, f_b)$ and $\hat{\pi}(\hat{\gamma}) = \pi(\gamma)$ (symmetry assumption). There are now four possible outcomes $\mathcal{S} = \{(g, g), (g, b), (b, g), (b, b)\}$. Any outcome $s \in \mathcal{S}$ can be written $s = (s_1, s_2)$, where $s_1 \in \{g, b\}$ and $s_2 \in \{g, b\}$. We assume that the risks of the firms are independent so that the probability of outcome $s = (s_1, s_2)$ is $\pi_s = \pi_{s_1} \hat{\pi}_{s_2}$. With this change in the definition of the outcome s , finding a Pareto optimal allocation still consists in finding a solution to (2) subject to the resource constraint (1) and can be decomposed into two steps: the first consists in finding the consumption-labor combination $(c_s^*, \ell_s^*, l_s^*, \hat{l}_s^*)$ which for each s maximizes the social welfare $W_s = u(c_s) - v(\ell_s)$; the second consists in finding the optimal investments $(\pi^*, \hat{\pi}^*)$ which maximize the expected discounted welfare net of the cost of investment. The solution of the first problem is characterized by (5) where (\hat{f}, \hat{f}') is replaced by (\hat{f}_s, \hat{f}'_s) . The first-order conditions for the optimal investment choices are

$$\begin{aligned} (W_{gg}^* - W_{bg}^*)\hat{\pi}^* + (W_{gb}^* - W_{bb}^*)(1 - \hat{\pi}^*) &= \frac{1}{\delta}\gamma'(\pi^*) \\ (W_{gg}^* - W_{gb}^*)\pi^* + (W_{bg}^* - W_{bb}^*)(1 - \pi^*) &= \frac{1}{\delta}\hat{\gamma}'(\hat{\pi}^*) \end{aligned} \quad (24)$$

where W_s^* denotes the optimized social welfare in outcome $s \in \mathcal{S}$. (24) is the generalization of (6) to the case where both firms make investment decisions at date 0. When the two firms have the same risks and the same technology the first-order condition for a symmetric Pareto optimal allocation reduces to the single equation

$$(W_{gg}^* - W_{bg}^*)\pi^* + (W_{gb}^* - W_{bb}^*)(1 - \pi^*) = \frac{1}{\delta}\gamma'(\pi^*). \quad (25)$$

The increments in social welfare have the following intuitive submodularity property which serves to establish the uniqueness of the symmetric Pareto optimum.

Lemma 1 $W_{gb}^* - W_{bb}^* > W_{gg}^* - W_{bg}^* > 0$.

Lemma 1, whose proof is given in appendix, asserts that the increment in social welfare when firm 1 has a good rather than a bad outcome is greater when the other firm has the outcome “b” rather than “g”, since firm 1 adds its production to the smaller production by firm 2. The existence and uniqueness of a symmetric Pareto optimum follows at once by noting that the function

$$\phi(\pi) = (W_{gg}^* - W_{bg}^*)\pi + (W_{gb}^* - W_{bb}^*)(1 - \pi) - \frac{1}{\delta}\gamma'(\pi)$$

satisfies $\phi(0) > 0$, $\phi(\pi) \rightarrow -\infty$ as $\pi \rightarrow 1$, and $\phi'(\pi) < 0$ by Lemma 1 and $\gamma'' > 0$. Since ϕ is continuous there is a unique π^* satisfying $\phi(\pi^*) = 0$.

The concept of a (reduced-form) capitalist equilibrium (Definition 1) extends in an obvious way to this new setting where both firms have risks: the maximum problem of firm 2 ((iv) in Definition 1) now involves choosing a probability $\hat{\pi}$ as well as a production plan in each outcome $s \in \mathcal{S}$. As before profit maximization and optimal choices of consumers and workers on spot markets at date 1 lead to an optimal consumption-labor allocation for each outcome $s \in \mathcal{S}$. The first-order conditions for the optimal choices of investment $(\bar{\pi}, \hat{\pi})$ by the firms which maximize shareholder values are given by

$$\begin{aligned} (\bar{R}_{gg}^1 - \bar{R}_{bg}^1)\bar{\pi} + (\bar{R}_{gb}^1 - \bar{R}_{bb}^1)(1 - \bar{\pi}) &\leq \frac{1}{\delta}\gamma'(\bar{\pi}), = \text{ if } \bar{\pi} > 0 \\ (\bar{R}_{gg}^2 - \bar{R}_{gb}^2)\hat{\pi} + (\bar{R}_{bg}^2 - \bar{R}_{bb}^2)(1 - \hat{\pi}) &\leq \frac{1}{\delta}\hat{\gamma}'(\hat{\pi}), = \text{ if } \hat{\pi} > 0 \end{aligned} \quad (26)$$

where \bar{R}_s^1 and \bar{R}_s^2 denote the maximized profit of firms 1 and 2 given the spot prices (\bar{p}_s, \bar{w}_s) . (26) is the generalization of (19) to the setting where both firms make investment decisions at date 0. At a symmetric equilibrium $\bar{R}_{bg}^1 = \bar{R}_{gb}^2$, $\bar{R}_{gg}^1 = \bar{R}_{gg}^2$, $\bar{R}_{bb}^1 = \bar{R}_{bb}^2$ so that the common choice $\bar{\pi}$ of investment satisfies the FOC

$$(\bar{R}_{gg}^1 - \bar{R}_{bg}^1)\bar{\pi} + (\bar{R}_{gb}^1 - \bar{R}_{bb}^1)(1 - \bar{\pi}) \leq \frac{1}{\delta}\gamma'(\bar{\pi}), = \text{ if } \bar{\pi} > 0. \quad (27)$$

As we mentioned, monotone ranking of the solutions of the first-order conditions (24) at a Pareto optimum and at an equilibrium (26) is difficult: however when the firms are similar it is possible to compare the solution of (25) and (27) and this leads to the following generalization of Proposition 1.

Proposition 2. *In any symmetric capitalist equilibrium of an economy with J firms there is under-investment: $\bar{\pi} < \pi^*$.*

Proof: The proof of Proposition 1 consisted in showing that $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$ when firm 2 has a fixed technology. This implies that for any realization of the technology of firm 2

$$W_{gs_2}^* - W_{bs_2}^* > \bar{R}_{gs_2}^1 - \bar{R}_{bs_2}^1, \quad s_2 = g, b \quad (28)$$

We want to prove that $\bar{\pi} < \pi^*$. Suppose by contradiction that $\bar{\pi} \geq \pi^*$. Since π^* is positive, this implies that $\bar{\pi} > 0$, and thus that (27) holds with equality. Then $\gamma'(\bar{\pi}) \geq \gamma'(\pi^*)$ and by (25) and (27)

$$\begin{aligned} (\bar{R}_{gg}^1 - \bar{R}_{bg}^1)\bar{\pi} + (\bar{R}_{gb}^1 - \bar{R}_{bb}^1)(1 - \bar{\pi}) &\geq (W_{gg}^* - W_{gb}^*)\pi^* + (W_{bg}^* - W_{bb}^*)(1 - \pi^*) \\ &\geq (W_{gg}^* - W_{gb}^*)\bar{\pi} + (W_{bg}^* - W_{bb}^*)(1 - \bar{\pi}) \end{aligned}$$

where the second inequality follows from Lemma 1: the convex combination with weights $(\bar{\pi}, 1 - \bar{\pi})$ puts less weight on the larger term $(W_{bg}^* - W_{bb}^*)$ than the convex combination with weights $(\pi^*, 1 - \pi^*)$. But the resulting inequality between expected profit and expected welfare increments contradicts (28). Thus $\bar{\pi} < \pi^*$. The proof is readily extended to the case $J > 2$ and is left to the reader. \square

Propositions 1 and 2 assert that a system of spot and financial markets guides the firms to an inefficient allocation, indeed an allocation with explicit under-investment. This may come as a surprise: after all the agents and firms are price takers and there are thus no distortions on the prices. Furthermore the financial markets for inter-temporal transfers of income are complete—and risk sharing can not be an issue here since agents are risk neutral. The remainder of the paper seeks to explain the source of the inefficiency and to suggest ways of improving on the profit-maximizing equilibrium. However since the Arrow-Debreu (AD) model is the reference model describing the conditions under which markets function well, we first seek to understand how the economy \mathcal{E} and the equilibrium concept just introduced in this section differ from that of an AD model of the same underlying economy. Thus in the next section we present an Arrow-Debreu description of an economy with the same characteristics (preferences, technology and uncertainty) and show that the equilibrium concept is very different from that of a capitalist equilibrium. Since it suffices to convey our message, we revert to the simpler benchmark model where only one firm is exposed to technological risk.

3 Arrow-Debreu Equilibrium

An economy under uncertainty is basically an economy in which some of the characteristics are random variables. The AD model uses the state of nature approach to model these random variables: it is well-known that any random variable can always be based on such a description. An important restriction however is that the probability of occurrence of the states must not be affected by the actions of the economic agents: this may seem a difficult requirement for

describing our economy in which the firms' investment decisions affect the probabilities of the good and bad outcomes but must not affect the probabilities of the states. However as we will see shortly this is not impossible.

The second step of the AD model is to assume that there is a market for contracts contingent on the realization of each state of nature. This is where the difficulties begin. To appreciate what is involved let us try to imagine a real world setting corresponding to the type of environment we have in mind. Consider for example an automobile company that needs to design and implement the production of a new model or improve on the design and production of an existing model. It will hire engineers to design the various components of the car, test the prototypes, and set up a factory to produce and assemble all the components. At the end of the period of design and production, cars get produced which are either "good" (no flaws) or "bad" (have flaws in the functioning of some parts)⁴. It is difficult to pinpoint exactly the circumstances that lead to good or bad cars—one design concept rather than another which comes to the minds of the engineers, the choice of tests for the prototype which may or may not catch the possible malfunctioning of some components, in short all the myriad circumstances which can occur in the design, testing and production of cars. The model must then describe how these exogenous circumstances (states of nature) combine with a given investment expenditure to lead to "good" or "bad" cars: a possible design flaw on the part of an engineer which could lead to a "bad" car may be corrected if the car maker hires two engineers rather than one to do the job, or if the quality control department increases the length or thoroughness of the test of its prototypes.

It should be clear from the above description that the contingencies which condition the outcome of the production process are numerous and difficult to describe and it is hard to identify the contingencies which lead to the good and the bad outcome as the investment expenditure of the firm is changed. Furthermore, whatever the difficulties involved in their enumeration, the contingencies are essentially internal to the firm and, while they may be understood by the firm's manager, they are unlikely to form the basis for tradeable contracts contingent on their occurrence. The latter property is however an essential ingredient of the Arrow-Debreu model since it assumes that firms base their investment decisions on prices associated with these contingencies. It is here that we see the dramatic difference between

⁴This example is set in terms of "quality" instead of "quantity" (number of cars produced) like in our benchmark model. This is because it fits well the real-life problems that the Japanese firm Toyota recently encountered in the gearing system of some of its cars. Our model could easily be adapted to capture this quality dimension.

the Arrow-Debreu model and the model outlined above in which there are just two prices, the price of a “car without flaws” and the price of a “car with flaws”.

Our model, which we call a “probability model” to distinguish it from the state-of-nature model, is of course much less ambitious in its description of the uncertainties faced by the firm: it only attempts to model in a summary way how expenditure on design and production influences the probability of achieving a good outcome through the cost function $\gamma(\pi)$, leaving the states of nature un-modeled in the background.

3.1 Arrow-Debreu Model of the Benchmark Economy

Coming back to the model, an Arrow-Debreu representation of the economy presented above thus hinges on the existence of an underlying probability space $(\Omega, \mathcal{B}, \mathbf{P})$ where Ω denotes the set of possible states of nature. All agents are assumed to know Ω and understand how probabilities \mathbf{P} are assigned to events (elements of \mathcal{B}). For each level of investment γ by firm 1 there is a subset $\Omega(\gamma) \subset \Omega$, with $\mathbf{P}(\Omega(\gamma)) = \pi(\gamma)$, which leads to the good technology f_g , while the complement $\Omega \setminus \Omega(\gamma)$ leads to the bad technology f_b . The map $\gamma \rightarrow \Omega(\gamma)$ is monotonic: $\tilde{\gamma} > \gamma$ implies $\Omega(\tilde{\gamma}) \supset \Omega(\gamma)$ so that $\pi(\tilde{\gamma}) > \pi(\gamma)$. In order that the function $\pi(\gamma)$ be differentiable the probability space must be non-atomic so we assume that Ω is a subset of a space \mathbb{R}^n and \mathbf{P} has a density \mathbf{P}_ω with respect to the Lebesgue measure. Consider all the investment levels that lead to f_g if ω occurs

$$\Gamma(\omega) = \{\gamma \in \mathbb{R}_+ \mid \omega \in \Omega(\gamma)\}$$

Given the monotonicity assumption, $\Gamma(\omega)$ is a half line: if we let $\underline{\gamma}(\omega) = \inf\{\gamma \mid \gamma \in \Gamma(\omega)\}$ (with $\underline{\gamma}(\omega) = \infty$ if $\Gamma(\omega) = \emptyset$) and if we assume that $\Gamma(\omega)$ is closed, then $\Gamma(\omega) = [\underline{\gamma}(\omega), \infty)$. The state-dependent production function⁵ for firm 1 is

$$F_\omega(\gamma, l) = \begin{cases} f_g(l) & \text{if } \gamma \geq \underline{\gamma}(\omega) \\ f_b(l) & \text{if } \gamma < \underline{\gamma}(\omega) \end{cases} \quad (29)$$

Consistent with the form of preferences given above, the workers’ preferences are given by

$$U^w(m, \ell) = m_0 + \delta \int_{\omega \in \Omega} (m_\omega - v(\ell_\omega)) d\mathbf{P}_\omega, \quad (30)$$

the consumers’ preferences by

$$U^c(m, c) = m_0 + \delta \int_{\omega \in \Omega} (m_\omega + u(c_\omega)) d\mathbf{P}_\omega, \quad (31)$$

⁵In order that the production set be closed we could define the production correspondence by (29) if $\gamma \neq \underline{\gamma}(\omega)$ and by $F_\omega(\gamma, l) = \{t f_g(l) + (1-t) f_b(l), t \in [0, 1]\}$ if $\gamma = \underline{\gamma}(\omega)$, but this would not change any of the results in the analysis below.

and the capitalists' preferences by

$$U^k(m) = m_0 + \delta \int_{\omega \in \Omega} m_\omega d\mathbf{P}_\omega. \quad (32)$$

Agents have deterministic endowments (e_0^i, e_1^i) , $i = w, c, k$ and the capitalists own the firms.

A complete set of contingent contracts promising the delivery of one unit of money, or of the consumption good, or of labor at date 0 and in each state of nature are traded at date 0. We normalize the price of money to be 1 at date 0. Given the agents' preferences, the price of a promise to deliver 1 unit of money in state ω must be $\delta \mathbf{P}_\omega$ almost surely so we do not introduce a separate notation for this price. In the same way the price of a promise to deliver one unit of labor and the produced good in state ω are almost surely $\delta \mathbf{P}_\omega w_\omega$ and $\delta \mathbf{P}_\omega p_\omega$ respectively: factoring out $\delta \mathbf{P}_\omega$ from the prices makes it easier to write the equilibrium.

A worker chooses $(m^w, \ell) = (m_\omega^w, \ell_\omega)_{\omega \in \Omega}$ to maximize (30) subject to the budget constraint

$$m_0^w + \delta \int_{\omega \in \Omega} m_\omega^w d\mathbf{P}_\omega = e_0^w + \delta e_1^w + \delta \int_{\omega \in \Omega} w_\omega \ell_\omega d\mathbf{P}_\omega \quad (33)$$

which is equivalent to choosing ℓ to maximize

$$\delta \int_{\omega \in \Omega} (w_\omega \ell_\omega - v(\ell_\omega)) d\mathbf{P}_\omega \quad (34)$$

the choice among money streams then being indeterminate among those satisfying (33). In the same way a consumer chooses $(m^c, c) = (m_\omega^c, c_\omega)_{\omega \in \Omega}$ to maximize (31) subject to the budget constraint

$$m_0^c + \delta \int_{\omega \in \Omega} (m_\omega^c + p_\omega c_\omega) d\mathbf{P}_\omega = e_0^c + \delta e_1^c \quad (35)$$

which is equivalent to choosing c to maximize

$$\delta \int_{\omega \in \Omega} (u(c_\omega) - p_\omega c_\omega) d\mathbf{P}_\omega \quad (36)$$

and the agent is indifferent among the money streams satisfying (35). Finally a capitalist chooses m^k to maximize (32) subject to the budget constraint

$$m_0^k + \delta \int_{\omega \in \Omega} m_\omega^k d\mathbf{P}_\omega = e_0^k + \delta e_1^k + PVP(\gamma, l; w, p) + \widehat{PVP}(\hat{l}; w, p), \quad (37)$$

where PVP and \widehat{PVP} denote the present values of the profits of the two firms. All capitalists agree that firm 1 should choose (γ, l) to maximize the present value of its profit⁶

$$PVP(\gamma, l; w, p) = \delta \int_{\omega \in \Omega} (p_\omega F_\omega(\gamma, l_\omega) - w_\omega l_\omega) d\mathbf{P}_\omega - \gamma, \quad (38)$$

⁶As we show below, this function (and the analogous function for the second firm) actually differ from the Shareholder Value functions defined above. This explains why we use a different notation. This point is rather subtle, and is at the core of the non existence result in Proposition 3.

and that firm 2 should choose \hat{l} to maximize the present value of its profit

$$\widehat{PVP}(\hat{l}; w, p) = \delta \int_{\omega \in \Omega} \left(p_\omega \hat{f}(\hat{l}_\omega) - w_\omega \hat{l}_\omega \right) d\mathbf{P}_\omega. \quad (39)$$

They are indifferent among all money streams satisfying (37). The indeterminacy of agents' money streams implies that if the markets for the produced good and labor clear in every state ω , the money streams can be chosen so that the market for money clears at each date and each state of nature: thus we can omit the markets for money in the description of the equilibrium.

Definition 2. $\left((\tilde{\ell}, \tilde{c}), (\tilde{\pi}, \tilde{l}, \tilde{\hat{l}}), (\tilde{w}, \tilde{p}) \right)$ is a (reduced-form) *Arrow-Debreu (AD) equilibrium* of the economy \mathcal{E} if

- (i) $\tilde{\ell}$ maximizes (34) given \tilde{w}
- (ii) \tilde{c} maximizes (36) given \tilde{p}
- (iii) $(\tilde{\gamma}, \tilde{l})$ maximizes (38) given (\tilde{w}, \tilde{p})
- (iv) $\tilde{\hat{l}}$ maximizes (39) given (\tilde{w}, \tilde{p})
- (v) markets clear: $\tilde{c}_\omega = F_\omega(\tilde{\gamma}, \tilde{l}_\omega) + \hat{f}(\tilde{\hat{l}}_\omega)$, $\tilde{\ell}_\omega = \tilde{l}_\omega + \tilde{\hat{l}}_\omega$, for almost all $\omega \in \Omega$.

In the informal discussion preceding the description of the AD model we expressed reservations on the realism of the AD market structure for this economy based on states of nature. We now show that even if we were to accept the strong assumption that such markets can be put in place, it would not suffice to solve the inefficiency, since this economy has no Arrow-Debreu equilibrium.

Proposition 3. (Non-existence) *The economy \mathcal{E} has no Arrow-Debreu equilibrium.*

Proof: Suppose $\left((\tilde{\ell}, \tilde{c}), (\tilde{\gamma}, \tilde{l}, \tilde{\hat{l}}), (\tilde{w}, \tilde{p}) \right)$ is an AD equilibrium. By the First Theorem of Welfare Economics, the equilibrium is Pareto optimal so that $\tilde{\pi} = \pi(\tilde{\gamma}) = \pi^* > 0$. In all the states $\omega \in \Omega(\tilde{\gamma})$, $F_\omega(\tilde{\gamma}, \cdot) = f_g$ and the demand and supply conditions are the same: thus $(\tilde{w}_\omega, \tilde{p}_\omega) = (\bar{w}_g, \bar{p}_g)$ and $(\tilde{l}_\omega, \tilde{\hat{l}}_\omega) = (\bar{l}_g, \bar{\hat{l}}_g)$ where $(\bar{w}_g, \bar{p}_g, \bar{l}_g, \bar{\hat{l}}_g)$ are the spot prices and firms' labor in the capitalist equilibrium. If $\omega \notin \Omega(\tilde{\gamma})$, then $(\tilde{w}_\omega, \tilde{p}_\omega, \tilde{l}_\omega, \tilde{\hat{l}}_\omega) = (\bar{w}_b, \bar{p}_b, \bar{l}_b, \bar{\hat{l}}_b)$.

By definition of the AD equilibrium, the expected profit of firm 1 must be maximal for $\pi = \tilde{\pi}$. Suppose the firm considers increasing the probability from $\tilde{\pi}$ to $\pi > \tilde{\pi}$ incurring the cost $\gamma(\pi) > \gamma(\tilde{\pi})$, *taking the prices* $(\tilde{w}_\omega, \tilde{p}_\omega)$ *as given*. In the states of the subset $\omega \in \Omega(\gamma(\pi)) \setminus \Omega(\gamma(\tilde{\pi}))$ of measure $\pi - \tilde{\pi}$ the firm would operate f_g facing the prices (\bar{w}_b, \bar{p}_b) leading to a change in (spot) profit

$$\Delta R^+ = \max_{l \geq 0} \{\bar{p}_b f_g(l) - \bar{w}_b l\} - \max_{l \geq 0} \{\bar{p}_b f_b(l) - \bar{w}_b l\}$$

In all other states π and $\tilde{\pi}$ give the same profit. Thus the difference in the present value of the profit net of investment is

$$\delta (\pi - \tilde{\pi}) \Delta R^+ - (\gamma(\pi) - \gamma(\tilde{\pi}))$$

A necessary condition for $\tilde{\pi}$ to be optimal is that the increase in cost be more than the additional profit i.e.

$$\delta \Delta R^+ \leq \frac{\gamma(\pi) - \gamma(\tilde{\pi})}{\pi - \tilde{\pi}}, \quad \forall \pi > \tilde{\pi}$$

which requires that

$$\Delta R^+ \leq (1+r)\gamma'(\tilde{\pi}), \quad (40)$$

where r is the implicit interest rate in equilibrium given by $\delta = \frac{1}{1+r}$.

A similar reasoning for a deviation $\pi < \tilde{\pi}$ shows that the loss in profit

$$\Delta R^- = \max_{l \geq 0} \{\bar{p}_g f_g(l) - \bar{w}_g l\} - \max_{l \geq 0} \{\bar{p}_g f_b(l) - \bar{w}_g l\}$$

in the states $\omega \in \Omega(\tilde{\pi}) \setminus \Omega(\pi)$ where the firm operates f_b and faces prices (\bar{w}_g, \bar{p}_g) must be higher than the saving in the investment cost:

$$\delta (\tilde{\pi} - \pi) \Delta R^- \geq \gamma(\tilde{\pi}) - \gamma(\pi), \quad \forall \pi < \tilde{\pi}$$

which requires that

$$\Delta R^- \geq (1+r)\gamma'(\tilde{\pi}). \quad (41)$$

Let us show that (40) and (41) cannot hold at the same time, because $\Delta R^+ > \Delta R^-$. This will show the value of the probability that maximizes the profit (38) is never equal to $\tilde{\pi}$ and thus that there is no AD equilibrium.

To show this property we use the function $f(t, l) = t f_g(l) + (1-t) f_b(l)$, introduced in the proof of Proposition 1 and the hypothetical equilibrium $(c(t), \ell(t), l(t), \hat{l}(t), p(t), w(t))$ which would hold in a spot t economy with characteristics $(u, v, f(t, \cdot), \hat{f})$. Consider the function

$$R(t, t') = \max_{l \geq 0} \{p(t) f(t', l) - w(t) l\}$$

which gives the profit of firm 1 obtained by operating the technology $f(t', \cdot)$ when prices are those corresponding to the equilibrium with technology $f(t, \cdot)$. We want to show that

$$\Delta R^+ = R(0, 1) - R(0, 0) > R(1, 1) - R(1, 0) = \Delta R^-$$

A sufficient condition for this is that $\frac{\partial^2 R}{\partial t \partial t'}(t, t') < 0$. The following Lemma thus completes the proof of Proposition 3:

Lemma 2. $R_{12}(t, t') = \frac{\partial^2 R}{\partial t \partial t'}(t, t') < 0$ for all $(t, t') \in [0, 1] \times [0, 1]$.

Proof: see the appendix. □

The intuition for the nonexistence of an Arrow-Debreu equilibrium is easy to get in the case where production does not involve labor, i.e. when $f_g(l) = y_g, f_b(l) = y_b, v(l) = 0$ for all $l \geq 0$. The only possible level of investment at an AD equilibrium is the efficient level π^* for which

$$\delta(W_g^* - W_b^*) = \delta(u(y_g) - u(y_b)) = \gamma'(\pi^*) \quad (42)$$

where π^* is positive because $\gamma'(\pi) \rightarrow 0$ when $\pi \rightarrow 0$. In the AD equilibrium the firm is assumed to act as if the spot price was exogenously determined by the state of nature. Thus a marginal increase in investment above π^* will entail an increase in expected revenue of $\Delta R^+ = \delta p_b(y_g - y_b)$, by producing y_g in states where π^* results in y_b , and in which the price is assumed to stay equal to p_b . On the other hand a marginal decrease in investment below π^* will entail a decrease in revenue ΔR^- such that $|\Delta R^-| = \delta p_g(y_g - y_b)$ by producing y_b in states where π^* yields y_g . Since $p_b > p_g$, $\Delta R^+ > \Delta R^-$. If $\Delta R^+ > \gamma'(\pi^*)$ it appears worthwhile to marginally increase the investment since the increased revenue exceeds the additional cost. If $\Delta R^+ \leq \gamma'(\pi^*)$ then $|\Delta R^-| < \gamma'(\pi^*)$ and it appears worthwhile to decrease investment. Thus the profit is not maximal for $\pi = \pi^*$ and the AD equilibrium does not exist⁷.

The main difficulty faced by the Arrow-Debreu model of our simple economy is revealed in the course of proving Proposition 3: it lies in the “price-taking” assumption for the firms. In the probability model (capitalist equilibrium) firm 1 anticipates that the spot prices will be (p_s, w_s) if it produces with technology f_s . In the Arrow-Debreu version of \mathcal{E} the price-taking assumption requires that firm 1 “believes” that prices are determined by the state $\omega \in \Omega$, and thus that they do not depend on the realization of its technology f_s : herein lies the

⁷If $\gamma'(0)$ were not 0 there would be cases where a trivial AD equilibrium with $\pi^* = 0$ would exist for some parameter values.

fundamental cause of the non-existence of an AD equilibrium. This price-taking assumption would be reasonable if the states of nature were economy-wide shocks, but it is no longer plausible when the states of nature refer to circumstances which are internal to the firm.

The simple stochastic two-outcome (or more generally finite-outcome) economy falls into the class of stochastic economies mentioned by Arrow (1971) for which no Arrow-Debreu equilibrium exists due to the inherent non-convexity of the production set when translated to the state-of-nature setting. And yet this success/failure type of uncertainty with the probability of success influenced by some action on the part of the firm is a common and pervasive type of uncertainty, which is handled in a natural way by the basic probability model presented in the previous section.

3.2 Continuum of Firms

There is a way of changing the structure of our economy in which each firm has a finite number of outcomes and the probability of the outcome can be influenced by its investment, to obtain an equilibrium with profit-maximizing firms which is Pareto optimal. In this modified economy there is a continuum of firms identical to firm 1 with i.i.d risks for the outcomes at date 1 so that an appropriate variant of the Law of Large Numbers can be applied.⁸ More precisely consider a modified economy with a continuum of ex-ante similar firms, where each firm makes an investment at date 0 which influences the probability of its outcome f_g or f_b at date 1. If each firm's outcome is independent of the outcomes of the other firms and all firms choose the same probability π , then a proportion π of firms will produce with f_g , a proportion $1 - \pi$ will produce with f_b , and the average output produced and the spot prices are non random. It is easy to show that a symmetric capitalist equilibrium exists in which each firm's investment maximizes the present value of its profit and the equilibrium investment is Pareto optimal (see the Internet Appendix).

However in this modified model, which is elegant and well behaved from a theoretical point of view, a firm has been transformed into an *infinitesimal* entity, far removed from the large corporate firm that we seek to model: the infinitesimal firms that populate this economy aptly fit what Berle and Means (1932) in their classic study described as the small sole proprietorships originally envisaged by Adam Smith. As they argued with great clarity, such firms have little or nothing in common with the large corporate firms whose securities are traded on the stock

⁸Beginning with Prescott and Townsend (1984a and b) models with a continuum have been widely used to explore equilibria with moral hazard. See e.g. Bisin-Gottardi (1999), Citanna-Villanaci (2000), Lisboa (2001), Zame (2007) and Acemoglu-Simsek (2010) for models of this type.

market and which, even in their day, had come to have a significant footprint on the economic landscape.⁹

In our benchmark model the spot prices vary with the outcome f_g or f_b of firm 1, so that this firm has a non-negligible impact on the economy. This provides a more appropriate model of the large corporations traded on the stock market than the perfectly competitive model with a continuum of negligible firms. To focus on the optimal choice of investment in risky projects by a large firm, we abstract from the firm's potential exploitation of its market power in choosing prices or quantities on the spot markets at date 1.

4 Stakeholder Approach

The analysis of the preceding section makes it clear that it is not by altering the market structure to that of an Arrow-Debreu model for the economy \mathcal{E} that we will resolve the inefficiency uncovered in Section 2. Let us try to understand where the inefficiency comes from and what can be done to attenuate or correct it, using the probability model of Section 2. The firm's choice of investment γ at date 0 determines the probabilities $\pi_s(\gamma)$ of the good and the bad outcomes. This investment decision affects not only firm 1's profit but also the expected utilities $\delta \sum_{s=g,b} \pi_s(\gamma)(u(c_s) - p_s c_s)$ and $\delta \sum_{s=g,b} \pi_s(\gamma)[w_s \ell_s - v(\ell_s)]$ of the consumers and the workers: it is this externality that creates the inefficiency. In a capitalist equilibrium, by choosing its investment to maximize its expected profit, the firm ignores the effect of its decision on consumers and workers although its decision directly affects their expected welfare.¹⁰

There is of course nothing new with the idea that firms' actions can have external effects on a much broader array of agents than their shareholders, and that in such a setting it may be appropriate to develop a "stakeholder theory of the firm". Indeed Tirole (2001) defines "corporate governance as the design of institutions that induce or force management to internalize the welfare of stakeholders..." and that such a theory should encompass a broad array of externalities including those exerted "on management and workers who have invested their human capital as well as off-work related capital (housing, spouse employment, schools, social relationships, etc.) in the employment relationship; on suppliers and customers who also have sunk investments in the relationship and foregone alternative opportunities; on communities

⁹ "When Adam Smith talked of "enterprise" he had in mind as the typical unit the small individual firm in which the owner perhaps with the aid of a few ... workers, labored to produce goods for market. ... These units have been supplanted ... by great aggregations in which tens or even hundreds of thousands of workers and property ... belonging to tens or even hundred of thousands of individuals are combined through the corporate mechanism into a single producing organization under unified control", Berle and Means (1932, pp.4 & 303).

¹⁰ Magill-Quinzii (2009) studies a probability model in which the externality is on risk-averse investors.

who suffer from the closure of a plant...". We do not seek to develop a theory for such a broad family of externalities: our contribution is rather to identify a new form of externality that seems to have been overlooked, and to suggest ways to correct it. Such an externality exists as soon as a firm is large and takes actions that influence the probability of its outcomes, even without any frictions in labor and consumption markets.

Three approaches have been proposed for resolving externalities:¹¹ (i) *outside interventions* either by government (through regulation or Pigouvian taxes) or by the judicial system (in the form of civil tort laws implemented by courts); (ii) *internal solutions* such as mergers (integration of all the parties involved in the externality); (iii) *market solutions*, for example by creating Coasian securities i.e. tradeable property rights associated with the externality (Coase, 1960).

There may well be settings closely related to that of our model where a strong case can be made for adopting the outside intervention approach.¹² However government intervention under the form of taxes or regulation always meet informational problems. The nature of the externality which comes from actions that a firm can take to influence the probability of its outcomes is linked to the internal functioning of the firm and its technology on which the firm has privileged information. Moreover the externality affects agents closely related to the operations of the firm—its consumers and its workers—agents who are natural stakeholders of the firm. A strong case can thus be made for exploring an “internal approach” in which the firm merges the interest of all its stakeholders. While it is an idea that has been widely discussed, it has not—as is clear from the discussion in Tirole (2001)—been precisely articulated in the framework of a formalized model. Let us explore how our model suggests formalizing a stakeholder theory and whether such an approach can restore efficiency.

4.1 Single Firm: Stakeholder Equilibrium

We begin with the simplest version of the benchmark model (f_s, \hat{f}) in which $\hat{f} = 0$, i.e. there is a single firm which makes an investment a date 0 and uses labor to produce output at date 1. Let $((\bar{\pi}, \bar{l}), (\bar{w}, \bar{p}))$ denote the capitalist equilibrium in this case. We saw that $\bar{l}_s = l_s^*$, where l_s^* is the labor choice which maximizes the social welfare $W_s^* = \max_{l \geq 0} \{u(f_s(l_s) - v(l_s))\}$: spot markets allocate labor efficiently in each outcome s at date 1 and the social welfare in equilibrium \bar{W}_s is the maximum welfare W_s^* . The inefficiency comes from the investment

¹¹See for example the discussion in Laffont (1989).

¹²For example, Blanchard-Tirole (2001) propose introducing a tax on workers' layoffs aimed at inducing firms to internalize the externalities inflicted on laid-off workers.

choice $\bar{\pi}$ at date 0 which is characterized by the FOC for profit maximization

$$\frac{1}{1+r}(\bar{R}_g - \bar{R}_b) = \gamma'(\bar{\pi})$$

whereas the socially optimal investment π^* is characterized by

$$\frac{1}{1+r}(\bar{W}_g - \bar{W}_b) = \gamma'(\pi^*)$$

Given spot prices (w_s, p_s) the consumer and worker surpluses are defined by

$$CS_s(p_s) = \max_{c_s \geq 0} \{u(c_s) - p_s c_s\} \quad WS_s(w_s) = \max_{\ell_s \geq 0} \{w_s \ell_s - v(\ell_s)\} \quad (43)$$

Since $u(0) = 0$ and $v(0) = 0$, $CS_s(p_s)$ is the net gain in utility for the representative consumer from being able to buy the good at price p_s , while $WS_s(w_s)$ is the net utility gain for the representative worker from being able to sell labor at the wage w_s . When, as in the capitalist equilibrium, agents trade on spot markets at prices (\bar{w}_s, \bar{p}_s) and markets clear ($\bar{l}_s = \bar{\ell}_s$) the social welfare in outcome s can be expressed as

$$\begin{aligned} \bar{W}_s &= u(\bar{c}_s) - v(\bar{\ell}_s) = \left(u(\bar{c}_s) - \bar{p}_s \bar{c}_s\right) + \left(\bar{w}_s \bar{\ell}_s - v(\bar{\ell}_s)\right) + \left(\bar{p}_s \bar{f}_s(\bar{l}_s) - \bar{w}_s \bar{l}_s\right) \\ &= \bar{CS}_s + \bar{WS}_s + \bar{R}_s \end{aligned}$$

namely as the sum of consumer surplus, worker surplus, and shareholder profit. As we saw in Proposition 1 the inefficiency of investment in a capitalist equilibrium comes from the property that $\bar{W}_g - \bar{W}_b > \bar{R}_g - \bar{R}_b$. The difference comes from the sum of the two terms $\bar{CS}_g - \bar{CS}_b$ and $\bar{WS}_g - \bar{WS}_b$, the difference in consumer and worker surplus between the good and the bad outcome, which measures precisely the external effect which is not internalized by the firm when it uses the present value of profit as its criterion for the choice of investment. Thus to obtain a stakeholder criterion for the firm which ensures that the effect of its investment decision is fully internalized, the firm needs to take into account not only its shareholders but also the consumers it serves and the workers it employs since these latter two parties also gain from having the good rather than the bad outcome.

Proposition 4. *If in the concept of equilibrium we replace the criterion of maximizing shareholder value by maximizing the total surplus of the stakeholders*

$$\frac{1}{1+r} \sum_{s=g,b} \pi_s \left(CS(\bar{p}_s) + WS(\bar{w}_s) + R_s(\bar{w}_s, \bar{p}_s) \right) - \gamma(\pi) \quad (44)$$

then we obtain a stakeholder equilibrium $((\pi^*, \bar{l}), (\bar{w}, \bar{p}))$ which is Pareto optimal.

Although criterion (44) provides a precise definition of the stakeholders’ interests, it does not guarantee that the criterion can or will be used as the basis of decision making by the firm’s manager: ways of measuring the “surpluses” CS and WS, as well as incentives for the management to maximize (44), must also exist. Thus the implementation of a stakeholder equilibrium raises three issues¹³:

- *Incentives*: incentives must be given to the firm’s manager to apply the stakeholder criterion.
- *Information* : to apply the stakeholder criterion the manager needs information on the characteristics of the consumers and workers to evaluate their surpluses.
- *Financing*: if the shareholder value at the stakeholder equilibrium is negative, an additional source of funds beyond equity and debt must be found, since otherwise the shareholders would dispose of their ownership shares rather than being forced to finance a project with a negative net present value.

Since markets are typically good at providing both incentives and information, can we imagine a way to use markets that would provide the appropriate incentives and information to maximize the sum of the surpluses in (44), leaving aside for the moment the problem of financing? In the spirit of Coase (1960) we introduce the idea that creating explicit tradeable property rights associated with the externalities created by the firm may provide the extension of the markets required to implement a stakeholder equilibrium.¹⁴

Suppose therefore that at date 0, in addition to the market for equity on which ownership shares are traded, there is a market for “consumer rights”—or more briefly c-rights—on which

¹³See Tirole (2001) for a discussion of these issues.

¹⁴In an economy without classes, i.e. when all agents are identical and simultaneously consumers, workers and shareholders, the externalities can be internalized by giving identical equity shares to all agents, since they will all agree that the firm should maximize the welfare of the representative agent (see Morgan-Tumlinson (2012)) for a development of this idea in the case where the firm creates a standard externality). In our model however consumers, workers and capitalists have different preferences regarding the optimal investment of the firm and just distributing equity shares among all the agents will not yield the correct FOC and thus will not lead to the Pareto optimal investment. In a model with imperfect competition and two distinct classes of agents Demichelis and Ritzberger (2011) show that efficient pricing decisions can be obtained if agents trade equity shares strategically, being aware that their ability to influence the firm’s decision, taken by majority voting, depends on the magnitude of their ownership share. Analyzing such strategic behavior on the stock market is difficult and we do not know what the result of such an analysis would be for our model. We adopt a simpler and more direct approach by assuming that firms issue distinct rights for distinct groups of stakeholders—w-rights for workers, c-rights for consumers, equity for shareholders—and that managers maximize the total value of these rights. This approach implicitly assumes that a right of any type is associated with a voting right and that unanimity with possibility of transfers is necessary to overturn an investment decision by the management.

agents exchange the right to buy the good produced by the firm at date 1 at the spot price $p = (p_g, p_b)$. In addition there is a market for “worker rights”—or more briefly w-rights—on which agents exchange the right to sell labor to the firm at date 1 at the spot price $w = (w_g, w_b)$. Suppose every consumer has an endowment of one c-right and every worker as an endowment of one w-right. To understand how the market values these rights we need to create some scarcity by assuming that only a mass $1 - \varepsilon$ of consumers and workers is endowed with rights and then let ε go to zero.

A worker with no initial w-right who observes the investment decision $\gamma(\pi)$ and anticipates a date 1 wage $w = (w_g, w_b)$ would be willing to pay up to

$$WV(\pi, w) = \delta[\pi WS_g(w_g) + (1 - \pi)WS_b(w_b)] \quad (45)$$

to obtain the right to work for the firm, where $WS(w_s)$ defined by (43) is the surplus utility that a worker derives from selling labor at the wage w_s : $WV(\pi, w)$ is the date 0 “worker value” of being employed by the firm. A worker who owns a w-right will accept to sell it if its price is equal to or exceeds (45). Thus if $\varepsilon > 0$, equilibrium on the market for w-rights occurs at the price

$$q_w(\pi, w) = WV(\pi, w) \quad (46)$$

If $\varepsilon = 0$ and every worker is endowed with a w-right, then no worker needs to buy a right, so that any price between 0 and $q_w(\pi, w)$ (at which every worker wants to keep the initial w-right) is an equilibrium price. To keep the symmetry of the model we assume that every worker is endowed with a w-right and that the market price of a w-right is given by (46), since any scarcity, no matter how small, will immediately force the price to $q_w(\pi, w)$. By a similar argument, the market price $q_c(\pi, p)$ of a c-right is taken to be the discounted expected surplus utility derived by a consumer from buying the produced good at price p from the firm, namely the “consumer value” $CV(\pi, p)$

$$q_c(\pi, p) = CV(\pi, p) = \delta\left(\pi CS_g(p_g) + (1 - \pi)CS_b(p_b)\right) \quad (47)$$

With the market valuations (46) and (47) in hand we now have a way of implementing a stakeholder equilibrium. If the firm’s manager makes the labor choice which maximizes the date 1 profit $R_s(\hat{p}_s, \hat{w}_s)$ ¹⁵ and chooses the probability π to maximize the total market value of the rights of its stakeholders

$$q_w(\pi, \hat{w}) + q_c(\pi, \hat{p}) + q_e(\pi, \hat{w}, \hat{p}) - \gamma(\pi) \quad (48)$$

¹⁵Choosing (l_g, l_b) to maximize total value or to maximize profit leads to the same choice (\bar{l}_g, \bar{l}_b) .

net of the cost of investment, then the firm’s criterion for choosing investment coincides with the net surplus criterion (44) of a stakeholder equilibrium and leads to the socially optimal investment decision π^* .

The advantage of having an explicit market for w-rights and c-rights in addition to equity is that the firm’s manager maximizes an objective, observable market value rather than an unobservable surplus. However to provide the manager with the incentive to maximize the stakeholder value (48), workers and consumers must be able to influence the investment decision of the firm. The reform of capitalism that we have in mind requires that when w-rights and c-rights are issued by the firm, the owners of these rights acquire legal voting rights in the decision process for investment. If unanimity is required to approve a change of management, then the management will maximize the net stakeholder value (48) or be replaced: for if a manager fails to maximize (48), a “raider” could choose an investment with a higher stakeholder value and in the process transfer enough value to workers, consumers and shareholders to buy their votes. In addition to providing the manager with incentives to apply the stakeholder criterion, the existence of markets for w-rights and c-rights provides the required information on the worker and consumer surpluses: knowledge of the price functions $q_w(\pi, \hat{w})$ and $q_c(\pi, \hat{p})$, which may be acquired from repeated observations of market prices, is sufficient information to be able to maximize the total surplus in the economy.¹⁶

In the above analysis we assumed that the w-rights and c-rights had already been issued. Thus neither consumers nor workers contribute to the funding of the firm’s investment which

¹⁶Our model can be generalized to incorporate the possibility of moral hazard on the part of the manager. Suppose for example that the realized investment is not perfectly observable by the stakeholders so that the manager can secretly divert funds: 1 dollar diverted from investment allows the manager to consume λ dollars (with $\lambda \leq 1$) while $1 - \lambda$ dollars are dissipated. In this simple set-up the optimal level of investment can be implemented by promising a bonus B to the manager if the good outcome occurs, and zero otherwise. The level of B must be such that the manager does not find it optimal to divert funds and invests the total amount $\gamma(\pi^*)$ provided by the shareholders: $\operatorname{argmax}_{\pi \leq \pi^*} \{\delta\pi B + \lambda(\gamma(\pi^*) - \gamma(\pi))\} = \pi^*$. This condition is satisfied whenever

$$\delta B \geq \lambda \gamma'(\pi^*)$$

Since π^* is characterized by

$$\delta (W_g^* - W_b^*) = \gamma'(\pi^*)$$

the level of the bonus must be such that

$$B \geq \lambda (W_g^* - W_b^*)$$

Since $W_g^* - W_b^* > \bar{R}_g - \bar{R}_b$, the bonus promised to the manager in a stakeholder firm must be higher than in a profit maximizing firm, since it must incorporate the increase in social surplus—and not only the increase in profit—associated with $s = g$ rather than $s = b$. This suggests that corporate governance issues may become more acute in a stakeholder firm. Since the pledgeable income (in the sense of Tirole (2001)) is reduced by the necessity of paying higher bonuses to the manager, the firm may have more difficulty financing its investment, unless consumers and/or workers participate in the financing (see next footnote).

must be paid by the shareholders, either directly as assumed in Section 2, or indirectly through the issue of bonds, which is equivalent. Such financing is possible only if $q_e(\pi^*, \hat{w}, \hat{p}) \geq \gamma(\pi^*)$. Otherwise the shareholders will prefer to dispose of their equity shares rather than finance a project with a negative net present value. If $q_e(\pi^*, \hat{w}, \hat{p}) < \gamma(\pi^*)$, the stakeholder equilibrium can still be implemented through stakeholder value maximization, provided that the model is taken at the stage where the firm issues the rights. Since by assumption the optimal expected total surplus is positive

$$\pi^* W_g^* + (1 - \pi^*) W_b^* - \gamma(\pi^*) > 0,$$

the net market value of these surpluses is positive

$$q_w(\pi^*, \hat{w}) + q_c(\pi^*, \hat{p}) + q_e(\pi^*, \hat{w}, \hat{p}) - \gamma(\pi^*) > 0. \quad (49)$$

If the firm issues the rights and chooses π^* to maximize the market value of the rights plus the net profit, then the proceeds $q_w(\pi^*, \hat{w}) + q_c(\pi^*, \hat{p})$ from the sale of the rights is sufficient to ensure that the shareholder value is positive since (49) can be written as

$$q_e(\pi^*, \hat{w}, \hat{p}) - \left(\gamma(\pi^*) - q_w(\pi^*, \hat{w}) + q_c(\pi^*, \hat{p}) \right) > 0$$

Thus the issue of rights can resolve the problem of financing when the net expected profit at the optimal investment is negative¹⁷

4.2 Multi-firm: Improving on Capitalist Equilibrium

Let us see how the above analysis can be extended to the benchmark model (f, \hat{f}) with $\hat{f} \neq 0$, where the firm which has the risky investment must compete with other firms on the labor and product markets. This simple setting suffices to illustrate the difficulties with extending a stakeholder theory to the multi-firm case.

As before labor is allocated efficiently when each firm maximizes its profit on the spot markets. The efficient level of investment is obtained if firm 1 chooses π to maximize the social welfare $\delta(\pi W_g^* + (1 - \pi) W_b^*) - \gamma(\pi) = \delta(\pi \overline{W}_g + (1 - \pi) \overline{W}_b) - \gamma(\pi)$, where $\overline{W}_s = u(f_s(\bar{l}_s) + \hat{f}(\bar{\tilde{l}}_s)) - v(\bar{l}_s + \bar{\tilde{l}}_s)$, and where $\bar{l}_s, \bar{\tilde{l}}_s$ are the profit maximizing choices of labor at the

¹⁷This is corroborated by Michelacci and Quadrini (2005, 2009), who argue that employees sometimes participate in the financing of their firms. They provide empirical evidence that some firms pay their employees below the market wage during the first years of employment and above market wages after some years. They interpret this finding along the lines suggested here: credit constrained firms may find it optimal to borrow from their employees.

price $\bar{p}_s = u'(f_s(\bar{l}_s) + \hat{f}(\bar{l}_s))$ and wage $\bar{w}_s = v'(\bar{l}_s + \bar{l}_s)$. Using the notation $\bar{y}_s = f_s(\bar{l}_s)$ and $\bar{\hat{y}}_s = \hat{f}(\bar{l}_s)$ the social welfare in outcome s can be decomposed as

$$\begin{aligned}\bar{W}_s &= \left(u(\bar{y}_s + \bar{\hat{y}}_s) - \bar{p}_s(\bar{y}_s + \bar{\hat{y}}_s) \right) + \left(\bar{w}_s(\bar{l}_s + \bar{l}_s) - v(\bar{l}_s + \bar{l}_s) \right) + \left(\bar{p}_s(\bar{y}_s + \bar{\hat{y}}_s) - \bar{w}_s(\bar{l}_s + \bar{l}_s) \right) \\ &= \bar{CS}_s + \bar{WS}_s + \bar{R}_s + \bar{\hat{R}}_s,\end{aligned}$$

where the surplus terms can be further decomposed as

$$\begin{aligned}\bar{CS}_s &= \left([u(\bar{y}_s + \bar{\hat{y}}_s) - u(\bar{y}_s)] - \bar{p}_s \bar{y}_s \right) + \left(u(\bar{\hat{y}}_s) - \bar{p}_s \bar{\hat{y}}_s \right) \\ \bar{WS}_s &= \left(\bar{w}_s(\bar{l}_s - [v(\bar{l}_s + \bar{l}_s) - v(\bar{l}_s)]) \right) + \left(\bar{w}_s \bar{l}_s - v(\bar{l}_s) \right).\end{aligned}$$

To be an “ideal” stakeholder firm, firm 1 would need to choose investment to maximize $\delta \sum_s \pi_s (\bar{CS}_s + \bar{WS}_s + \bar{R}_s + \bar{\hat{R}}_s) - \gamma(\pi_s)$: this requires taking into account not only the difference between the good and the bad outcome for the profit of its shareholders and the surplus it generates for its consumers and workers, but also for the consumer and worker surpluses created by the other firms, as well as the profit of the other firms’ shareholders. This is indeed an encompassing vision of who the stakeholders of the firm are, which is difficult to reconcile with competition between firms on the product and labor markets.

Realistically the most that can be expected of a corporation is that it take into account the interests of its own stakeholders—its shareholders, the consumers it serves and the workers it employs. Building on the notion of “value” of firm 1 for consumers and workers which we introduced in section 4.2, we can define the *consumer and worker values*

$$\begin{aligned}CV_s(y_s, \hat{y}_s, p_s) &= u(y_s + \hat{y}_s) - u(\hat{y}_s) - p_s y_s \\ WV_s(l_s, \hat{l}_s, w_s) &= w_s l_s - [v(l_s + \hat{l}_s) - v(\hat{l}_s)]\end{aligned}\tag{50}$$

CV_s and WV_s are the money equivalent of the increase in utility attributable to the ability to buy from firm 1 for the consumers, and to work for firm 1 for the workers, taking the decisions of other firms as given. The consumer and worker values are firm 1’s contribution to the total consumer and worker surpluses—but are not equal to the total surpluses.

It is difficult to describe a market structure on which these values are elicited using a model where the two firms produce a homogeneous good. The value CV_s needs to be understood as the limit of the price of a c-right in a model with differentiated goods, when the goods become very close substitutes. If the goods produced by firm 1 and 2 were differentiated, the representative consumer would be willing to pay $u(y_s, \hat{y}_s) - u(0, \hat{y}_s) - p_s y_s$ for the right to buy from firm 1, when the other firm produces \hat{y}_s (per capita) and p_s is the price of good sold by firm 1. A

model with differentiated goods is certainly natural for large firms, but outside the scope of this paper. We thus study the property of a stakeholder value equilibrium in which firm 1's manager is instructed to maximize the total value that the firm creates for its stakeholders, leaving the study of the implementation of the equilibrium for further research.

Definition 3. A *stakeholder equilibrium* of the economy \mathcal{E} is a pair of actions and prices $\left((\bar{\ell}, \bar{c}, \pi^{stv}, \bar{l}, \bar{\tilde{l}}), (\bar{w}, \bar{p}) \right)$ such that (i), (ii), (iv),(v) of Definition 1 hold, and (iii) is replaced by (iii') (\bar{l}, π^{stv}) maximizes the total value of firm 1 net of the investment cost

$$TV(\bar{w}, \bar{p}) = \frac{1}{1+r} \sum_{s=g,b} \left(CV_s(y_s, \bar{p}_s) + WV_s(l_s, \bar{w}_s) + R(l_s, \bar{w}_s, \bar{p}_s) \right) - \gamma(\pi)$$

It is easy to see that the first-order conditions for the choice of labor are the same as those of a capitalist equilibrium, so that the labor choices $(\bar{l}, \bar{\tilde{l}})$ and the spot prices (\bar{w}, \bar{p}) are identical: we have thus kept the same notation. It is also easy to see that maximization of profit or maximization of stakeholder value for firm 2 gives the same labor choice, so we have retained profit maximization for firm 2. The change in the criterion for firm 1 changes the FOC for the choice of investment which becomes

$$\frac{1}{1+r} \left[\left(\overline{CV}_g - \overline{CV}_b \right) + \left(\overline{WV}_g - \overline{WV}_b \right) + \left(\overline{R}_g - \overline{R}_b \right) \right] = \gamma'(\pi^{stv})$$

where the values are calculated at the spot market equilibrium. Adding the difference in consumer and worker values between the good and bad outcomes to the difference in profit, which is taken into account in the capitalist equilibrium, increases the perceived benefit by firm 1 to achieving a good outcome, thus leading to an increase in investment. To compare π^{stv} with the optimal investment π^* , note that

$$\overline{TV}_s = u(\bar{y}_s + \bar{\tilde{y}}_s) - u(\bar{\tilde{y}}_s) - \bar{p}_s \bar{y}_s + \bar{w}_s \bar{l}_s - (v(\bar{l}_s + \bar{\tilde{l}}_s) - v(\bar{\tilde{l}}_s)) + \bar{p}_s \bar{y}_s - \bar{w}_s \bar{l}_s$$

so that

$$\overline{TV}_s = \overline{W}_s - \widehat{W}_s, \quad \text{where } \widehat{W}_s = u(\bar{\tilde{y}}_s) - v(\bar{\tilde{l}}_s).$$

\widehat{W}_s is the social welfare that can be attributed to firm 2 in the thought experiment in which firm 1 is absent from the market, and the total value of firm 1 is the difference between the total social welfare and that attributable to firm 2. The FOC for optimal investment in a stakeholder value equilibrium is

$$\frac{1}{1+r} \left[\left(\overline{W}_g - \overline{W}_b \right) - \left(\widehat{W}_g - \widehat{W}_b \right) \right] = \gamma'(\pi^{stv}) \quad (51)$$

while π^* is defined by $\frac{1}{1+r}(\overline{W}_g - \overline{W}_b) = \gamma'(\pi^*)$. It is intuitive that firm 2 will “fill in” for firm 1 when firm 1 has a bad outcome: as a result firm 2 should produce more and create more surplus in outcome b than in outcome g . Let us show that this is indeed the case, so that (51) implies that there is over-investment at a stakeholder value equilibrium

Proposition 5. *In a stakeholder equilibrium of the benchmark model (f, \hat{f}) with $\hat{f} \neq 0$ there is over-investment: $\pi^{stv} > \pi^*$.*

Proof: In view of (51) it remains to show that $\widehat{W}_g < \widehat{W}_b$. Firm 2’s surplus function $\widehat{W}(\hat{l}) \equiv u(\hat{f}(\hat{l})) - v(\hat{l})$ is concave, satisfies $\widehat{W}(0) = 0$ and has a maximum for \hat{l}^m defined by

$$\widehat{W}'(\hat{l}^m) = u'(\hat{f}(\hat{l}^m))\hat{f}'(\hat{l}^m) - v'(\hat{l}^m) = 0$$

For $\hat{l} < \hat{l}^m$, $\widehat{W}(\hat{l})$ is increasing. Thus if we show that (i) $\hat{l}_g < \hat{l}_b$ and (ii) $\hat{l}_b \leq \hat{l}^m$, then it follows that $\widehat{W}_g < \widehat{W}_b$. (i) can be deduced from the proof of Proposition 1 as shown in the appendix.

Lemma 3. $\hat{l}_g < \hat{l}_b$.

To show (ii) first suppose that $f_b \equiv 0$, i.e. in the bad outcome firm 1 is bankrupt and does not produce. Then firm 2 is the only producer on the market and, assuming price taking behavior, chooses \tilde{l}_b so that $\bar{p}_b \hat{f}'(\tilde{l}_b) = \bar{w}_b$. Since $\bar{p}_b = u'(\hat{f}(\tilde{l}_b))$ and $\bar{w}_b = v'(\tilde{l}_b)$ it follows that $u'(\hat{f}(\tilde{l}_b))\hat{f}'(\tilde{l}_b) - v'(\tilde{l}_b) = 0$, so that $\tilde{l}_b = \hat{l}^m$. Since, by Lemma 3, $\hat{l}_g < \hat{l}_b$, it follows that $\widehat{W}_g < \widehat{W}_b$. To extend the result to the case where $f_b > 0$, consider a related economy $\tilde{\mathcal{E}}$ for which $\tilde{f}_g = f_b$ and $\tilde{f}_b \equiv 0$. Applying the above reasoning to $\tilde{\mathcal{E}}$, we find $\tilde{l}_b = \hat{l}^m$ and $\tilde{l}_g = \tilde{l}_b < \hat{l}^m$. Thus (ii) is again satisfied and since (i) holds by Lemma 3, $\widehat{W}_g < \widehat{W}_b$. \square

The stakeholder value criterion asks firm 1 to bear in mind the increased surplus that will accrue to its workers and consumers if it succeeds in obtaining the good outcome. However the optic that the criterion induces fails to take into account the response of firm 2. When firm 1 has a good outcome, firm 2 faces stiffer competition and a lower output price and produces less than in outcome b , thereby creating a smaller surplus. Since the surpluses of the two firms move in opposite directions, an investment decision based solely on the surplus created by firm 1 exaggerates the gain in outcome g and leads to over-investment.

Proposition 1 asserts that pure profit underestimates the benefit of investment, while Proposition 5 asserts that surplus value overestimates it. This suggests that a reform of capitalism in which the pure profit criterion is replaced by one which assigns some weight to consumers

and workers, but not as much as in the stakeholder value equilibrium, may improve on the capitalist equilibrium.

We say that firm 1 is *stakeholder oriented* if it uses a criterion of the form

$$V(\pi; \theta) = \frac{1}{1+r} \sum_{s=g,b} \pi_s \left[R_s(\bar{w}_s, \bar{p}_s) + \theta \left(CV_s(\bar{p}_s) + WV_s(\bar{w}_s) \right) \right] - \gamma(\pi)$$

to choose its investment for some $0 < \theta < 1$. An equilibrium with a stakeholder oriented firm 1 is the same as a stakeholder value equilibrium with the sole difference that the criterion of choice of investment in Definition 3 (iii)' is replaced by the criterion $V(\pi; \theta)$. The improvement obtained by replacing the profit criterion by $V(\pi; \theta)$ can be formalized as follows.

Proposition 6. (Reform of Capitalism) *There exists $\theta^* \in (0, 1)$ such that (i) if firm 1 uses the criterion $V(\pi; \theta)$ with $0 < \theta \leq \theta^*$ then the stakeholder oriented equilibrium improves on the capitalist equilibrium; (ii) if $\theta = \theta^*$ the equilibrium is Pareto optimal.*

Proof: For any $0 \leq \theta \leq 1$ the equilibrium with criterion $V(\pi; \theta)$ leads to the same spot prices (\bar{w}, \bar{p}) and the same labor choices (\bar{l}, \bar{l}) as in the capitalist equilibrium. The choice of investment $\pi(\theta)$ which maximizes $V(\pi; \theta)$ is defined by the first-order condition

$$V'(\pi, \theta) = \frac{1}{1+r} \left[\bar{R}_g + \theta \left(\overline{CV}_g + \overline{WV}_g \right) \right] - \left[\bar{R}_b + \theta \left(\overline{CV}_b + \overline{WV}_b \right) \right] - \gamma'(\pi(\theta)) = 0$$

which, when \overline{CV}_s and \overline{WV}_s are replaced by their expressions in (50), can be written as

$$\frac{1}{1+r} \left[(1-\theta) \left(\bar{R}_g - \bar{R}_b \right) + \theta \left((\bar{W}_g - \bar{W}_b) - (\widehat{W}_g - \widehat{W}_b) \right) \right] - \gamma'(\pi(\theta)) = 0. \quad (52)$$

Differentiating (52) gives

$$\frac{1}{1+r} \left[- \left(\bar{R}_g - \bar{R}_b \right) + \left((\bar{W}_g - \bar{W}_b) - (\widehat{W}_g - \widehat{W}_b) \right) \right] = \gamma''(\pi(\theta)) \pi'(\theta).$$

Proposition 1 implies $\left((\bar{W}_g - \bar{W}_b) - (\widehat{W}_g - \widehat{W}_b) \right) > 0$, and Proposition 4 implies $(\widehat{W}_g - \widehat{W}_b) < 0$. Since $g'' > 0$, $\pi'(\theta) > 0$.

Let $W(\theta) = \delta \sum_{s=g,b} \pi_s(\theta) \overline{W}_s - \gamma(\pi(\theta))$ denote the discounted expected social welfare induced by the investment $\pi(\theta)$, with derivative $W'(\theta) = \pi'(\theta) \left(\delta \left((\bar{W}_g - \bar{W}_b) - \gamma'(\pi(\theta)) \right) \right)$. Since $g' > 0$ and $\pi' > 0$, $\gamma'(\pi(\theta))$ is increasing. Since by Proposition 1, $\delta(\bar{W}_g - \bar{W}_b) > \gamma'(\pi(0))$ and, by Proposition 4, $\delta(\bar{W}_g - \bar{W}_b) < \gamma'(\pi(1))$, there exist θ^* such that $\gamma'(\pi(\theta^*)) = \delta(\bar{W}_g - \bar{W}_b) = \gamma'(\pi^*)$, and $W'(\theta)$ is positive for $\theta < \theta^*$, negative for $\theta > \theta^*$. Thus the social welfare increase on $[0, \theta^*)$, and $\pi(\theta^*) = \pi^*$, i.e. the investment is socially optimal for θ^* , which proves the proposition. \square

The interesting part of Proposition 6 is the qualitative result that placing some weight on consumer and worker surpluses in making the investment decision improves on the capitalist outcome. The existence of a critical value θ^* which gives the ideal weight to attach to different stakeholders is less important since in practice θ^* will be difficult to determine. The criterion $V(\pi; \theta)$, which improves on capitalism if θ is not too large, places some weight on the *sum* of the consumer and worker surpluses, not just on the surplus of the workers.¹⁸ In the proof of Lemma 2 it is shown that $p_g < p_b$, so that consumers are always better off in outcome g . However workers are not always better off: in the proof of Proposition 1 we show that without further conditions the sign of $l'(t) + \hat{l}'(t)$ in (23) is ambiguous, so that it is possible that $l_g + \hat{l}_g < l_b + \hat{l}_b$ and $\bar{w}_g < \bar{w}_b$. Whether or not this inequality holds depends on the elasticity of demand. If the utility function u is linear, the price of the output does not change ($\bar{p}_g = \bar{p}_b$) and the consumers have no surplus: all the improvement in technology goes toward increasing the wages of the workers. As the elasticity of demand increases, the consumer surplus increases so that the consumers benefit more from the new technology, while the worker surplus decreases and may end up being negative. In this case transfers between consumers and workers would be necessary to improve the welfare of both groups of stakeholders.

5 Conclusion

The orthodox view of economists regarding the objectives of firms is based on a faith in the universal applicability of the invisible hand: a stakeholder theory in which interests of stakeholders other than shareholders are taken into account has no place in their pantheon of ideas. The arguments are essentially those of the certainty setting in which competitive markets and profit maximization ensure that actions by firms are taken in the best interest of the whole economy, and this is extended to uncertainty by invoking the states of nature and contingent contracts of the Arrow-Debreu theory. This is cold comfort, for as far as we have shown, states of nature and contingent contracts on states do not provide an apt representation of the uncertainty setting in which firms take decisions on risky investments. A less ambitious and more realistic description of the uncertainty setting is that markets are based on the outcomes of firms and firms' actions (investments) influence the probabilities of these outcomes. This approach however implies that firms' actions can potentially have external effects on consumers

¹⁸In countries like Germany and Japan in which the stakeholder view of the corporation is prevalent, representatives of the workers are typically involved in the strategic decisions of firms while consumers are not represented on the corporate boards.

and workers. It is true that if there is a continuum of independent firms—each firm being akin to the infinitesimal enterprise of Adam Smith—then the orthodox view can indeed be carried over to a world of uncertainty, since the external effects are negligible and profit maximization leads to Pareto optimality. However if a firm is a large corporate enterprise of the type studied by Berle-Means (1932) then its external effects on consumers and workers must be taken into account to achieve a Pareto optimal outcome: in a world of large corporate enterprises in which a firm can have a significant footprint on the economic landscape, the orthodox view of profit maximization is no longer valid.

In the setting that we study, firms motivated by profit maximization are led to insufficient investment, since they fail to take into account the benefits of these investments for their consumers and workers. Since the uncertainty which lies behind the externality, namely a firm’s ability to influence the probability of its outcome, depends inherently on the internal functioning and the technology of the firm, government intervention in the form of investment subsidies would present both informational and incentive problems. We are thus led to explore the possibility of internalizing the externality within the firm, by explicitly including the benefits of consumers and workers in addition to those of the shareholders in the firm’s objective function.

A valid *theoretical* foundation for a stakeholder theory of the firm requires two preconditions: (1) decisions taken by the firms must have an external effect on stakeholders (2) these externalities must not be readily resolved by government intervention (regulation or taxation). To obtain an *operational* stakeholder theory, three additional conditions must be satisfied: it must be possible to

- (i) assign well-defined benefits for each group of stakeholders,
- (ii) exhibit a way of assigning relative weights to the benefits of the different groups in (i) to obtain a well-defined objective for a firm,
- (iii) provide incentives to the firm’s manager to maximize this objective.

Jensen (2001) argues forcefully that a stakeholder theory¹⁹ does not provide a solution to (i) and (ii). Without using an explicit model of the economy, Tirole (2001) argues that measuring consumer and worker surpluses may be difficult since there are no liquid markets on which they can be evaluated akin to the stock market for the firms’ profits. If (i) can not be

¹⁹The management literature defines a stakeholder firm as one which “pursues multiple objectives of parties with different interests” (Kochan-Rubinstein (2001)).

solved then there is no solution to (ii), so that there is no well-defined criterion for evaluating a manager’s performance. Like Jensen, Tirole argues that any attempt to take into account the interests of the different stakeholders leaves the firm open to manipulation by the management: “Management can almost always rationalize any action by invoking its impact on the welfare of *some* stakeholder” (Tirole (2001)); “Stakeholder theory plays into the hands of managers by allowing them to pursue their own interests at the expense of the firm’s financial claimants and society at large. It allows managers and directors to devote the firm’s resources to their own favorite causes—the environment, art, cities, medical research. . . . By expanding the power of managers in this unproductive way, stakeholder theory increases the agency costs in the economic system” (Jensen (2001)).

Our analysis offers a first step to the solution of (i) and (ii): under the assumption of quasi-linearity of agents’ preferences, profit measures the benefits of shareholders, while consumer and worker surpluses measure the benefits accruing to consumers and workers. In the idealized case of an economy with a single firm the stakeholder objective, which leads to the social optimum, is to maximize the expected sum of these three benefits, i.e. it puts equal weight on each of the benefits in (i). However this theoretical result, while formally answering (i) and (ii), does not respond to Tirole’s concern that consumer and worker surpluses may be difficult to evaluate in practice. We propose a solution to this difficulty by drawing on the Coasian idea of creating property rights for externalities: if the firm can issue consumer and worker rights, and if these rights can be traded on reasonably liquid markets, then their market prices will reveal the benefits that consumers and workers derive from being stakeholders of the firm. In effect our proposal would lead to reforming corporate accounting, by introducing new assets—employee and consumer surpluses—and corresponding liabilities—employee rights and consumer rights—in a spirit close to the proposal of Cornell-Shapiro (1987)²⁰.

If the elements of a stakeholder theory seem to fall into place in the idealized case of an economy with a single firm, extending the theory to the more general setting where several firms compete on the product and labor markets presents new difficulties. For in this setting, to achieve the social optimum each firm would need to take into account the effect of its investment on the expected utilities of all agents in the economy, including the consumers, workers

²⁰There is a theoretical literature that uses incomplete contracts models to explain why we see other forms of corporations than for profits: non-profit (Glaeser and Shleifer 2001), government ownership (Hart et al. 1997), cooperatives (Hart and Moore 1998, Rey and Tirole 2000). There is also an early literature on labor managed firms. However this paper and the contemporaneous paper of Allen et al.(2011) are the only formal models of stakeholder oriented firms (viewed as hybrids between for profit, consumers cooperatives and labor managed firms) that we are aware of.

and shareholders of the other firms. Placing the welfare of the stakeholders of competing firms directly into the objective function of a firm is not however a realistic proposal since it would come into conflict with competition of the spot markets, which is required for efficiency. Our analysis shows however that the optimal investment, or at least an investment that improves on the capitalist outcome, is obtained if the firm's objective includes a positive, perhaps small, weight on just the surpluses of its *own* consumers and workers. Thus a straightforward modification of the pure profit criterion can lead to an improvement on capitalism. If full weight were placed on the surpluses of its own consumers and workers, then the firm would exaggerate the benefit of achieving a good outcome since it would neglect the fact that its competitors produce more and create more surplus for the economy when it is less productive. Modifying the stakeholder criterion by decreasing the weight placed on the surpluses of the firm's consumers and workers implicitly takes into account the offsetting surpluses created by the other firms.

There remain the informational and incentive problems of evaluating the surpluses and ensuring that they are in some measure taken into account by a firm's manager. These are problems which are not easily addressed with the simple model of this paper in which firms produce homogeneous goods using homogeneous labor. Extending the Coasian idea of creating consumer and worker rights requires that firms produce differentiated products and use different types of labor or in different locations. Since in a setting with heterogeneous firms, consumers, and workers, the price of a right will not reveal the full surplus, only the surplus of the marginal buyer, maximizing the total value of rights seems commensurate with the theoretical result that only a part a firm's consumer and worker surpluses should be taken into account. More research is needed to find robust and practical ways of introducing markets for consumer and worker rights, thereby enabling corporations to simultaneously take the interests of their stakeholders into account, while retaining an objective market-based criterion for measuring management performance.

APPENDIX: Proofs

Proof of Lemma 1. Consider the (t, θ) economy in which the production functions of the two firms are

$$f(t, l) = tf_g(l) + (1-t)f_b(l), \quad \hat{f}(\theta, \hat{l}) = \theta\hat{f}_g(\hat{l}) + (1-\theta)\hat{f}_b(\hat{l}),$$

and the consumers and workers have the characteristics (u, v) . The maximum social welfare in the (t, θ) economy is

$$W(t, \theta) = \max\{u(c) - v(\ell) \mid c = f(t, l) + \hat{f}(\theta, \hat{l}), \ell = l + \hat{l}\} \quad (53)$$

We show that $\frac{\partial^2 W}{\partial t \partial \theta} < 0$, which proves the lemma since it implies $W(1, 1) - W(0, 1) < W(1, 1) - W(0, 1) \iff W_{gg}^* - W_{bg}^* < W_{gb}^* - W_{bb}^*$.

$$\frac{\partial W}{\partial \theta} = u'(c(t, \theta))\hat{f}_1(\theta, \hat{l}(t, \theta)), \quad \frac{\partial^2 W}{\partial t \partial \theta} = u'' \left(f_1 + f_2 \left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t} \right) \right) \hat{f}_1 + u' \hat{f}_{12} \frac{\partial \hat{l}}{\partial t}$$

where the arguments of the function in the second derivative have been omitted to simplify the expression. As in the proof of Proposition 1 $\frac{\partial l}{\partial t}$ and $\frac{\partial \hat{l}}{\partial t}$ can be calculated by differentiating the FOCs of the maximum problem (53). Calculations similar to those in the proof of Proposition 1 lead to

$$u'' \hat{f}_1 \left(f_1 + f_2 \left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t} \right) \right) = u'' \hat{f}_1 \frac{u' f_1 f_{22} \hat{f}_{22} - v'' f_1 (f_{22} + \hat{f}_{22}) - u' f_2 f_{21} \hat{f}_{22}}{u' f_{22} \hat{f}_{22} + (u'' (f_2)^2 - v'')(f_{22} + \hat{f}_{22})} \quad (54)$$

which is negative since the numerator and the denominator of the fraction on the right side are positive. From the calculation in the proof of Proposition 1 we also deduce

$$\frac{\partial \hat{l}}{\partial t} = \frac{1}{u' \hat{f}_{22}} \left((v'' - u'' (f_2)^2) \left(\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t} \right) - u'' f_1 f_2 \right)$$

which after substituting the value of $\frac{\partial l}{\partial t} + \frac{\partial \hat{l}}{\partial t}$ gives

$$\frac{\partial \hat{l}}{\partial t} = \frac{-u' f_{21} \hat{f}_{22} (v'' - u'' (f_2)^2) - u' u'' f_1 f_2 f_{22} \hat{f}_{22}}{u' \hat{f}_{22} \text{den}}$$

where “den” is the positive denominator in (54). The numerator of the fraction is positive, den is positive and since $\hat{f}_{22} < 0$, $\frac{\partial \hat{l}}{\partial t} < 0$. This property is intuitive: if the productivity of firm 1 increases the amount of labor used by firm 2 in the efficient allocation decreases. Thus the two terms in $\frac{\partial^2 W}{\partial t \partial \theta}$ are negative and the result follows. \square

Proof of Lemma 2. Let $L(t, t')$ denote the optimal labor choice that solves $R(t, t') = \max\{p(t)f(t', l) - w(t)l\}$. It is defined by the first-order condition

$$p(t)f_2(t', L(t, t')) = w(t)$$

By the envelope theorem

$$R_2(t, t') = p(t)f_1(t', L(t, t'))$$

so that

$$R_{21}(t, t') = p'(t)f_1(t', L(t, t')) + p(t)f_{12}(t', L(t, t'))L_1(t, t')$$

Since $f_1 > 0, p > 0, f_{12} > 0$, showing that $R_{21} < 0$ amounts to showing that (i) $p'(t) < 0$, and (ii) $L_1(t, t') < 0$. In proving (i) and (ii) we often omit the arguments of the functions in order to simplify notation.

(i) We have seen in the proof of Proposition 1 that $p' = u''(f_1 + f_2(l' + \hat{l}'))$. Inserting the value of $l' + \hat{l}'$ calculated in (23) leads to

$$p' = u'' \frac{u' \hat{f}_{22}(f_1 f_{22} - f_2 f_{21}) - v' f_1 (f_{22} + \hat{f}_{22})}{den}$$

where den is the positive denominator of $l' + \hat{l}'$ in (23). The numerator of the fraction is positive and $u'' < 0$, so that $p' < 0$. A better technology decreases the equilibrium price of the output.

(ii) Let $\rho(t) = \frac{w(t)}{p(t)}$ be the relative price of labor with respect to output in the 't' equilibrium. The FOC defining L can be written as

$$f_2(t', L(t, t')) = \rho(t) \implies f_{22}(t', L(t, t')) L_1(t, t') = \rho'(t)$$

Since $f_{22} < 0$, the proof of (ii) consists in showing that $\rho'(t) > 0$: when the technology improves the price of labor relative to output increases.

$$\rho'(t) = \frac{d}{dt} \left(\frac{v'(l(t) + \hat{l}(t))}{u'(f(t, l(t)) + \hat{f}(\hat{l}(t)))} \right) = \frac{u'v''(l' + \hat{l}') - v'u''(f_1 + f_2(l' + \hat{l}'))}{(u'^2)} = \frac{u'v''(l' + \hat{l}') - v'p'}{(u'^2)}$$

Inserting the value of $l' + \hat{l}'$ calculated in (23) and the value of p' calculated above leads to

$$\rho' = \frac{1}{(u'^2 den)} \left(u''v''f_1(-u'f_2 + v')(f_{22} + \hat{f}''') - (u'^2v''f_{21}\hat{f}'' - u'u''v'\hat{f}''(f_1f_{22} - f_2f_{21})) \right)$$

D is negative and after simplification

$$N = v'u'''(f_{21}f_2 - f_1f_{22}) - (u'^2v''f_{21} + v''u''f_1(v' - u'f_2))$$

The term $(-u'f_2 + v')$ is equal to 0 by the first-order condition for the choice of $l(t)$. All other terms are positive, so that $\rho' > 0$, which completes the proof of Lemma 2. \square

Proof of Lemma 3. To prove $\hat{l}_b > \hat{l}_g$ it is sufficient to prove that $\hat{l}'(t) < 0$, where $\hat{l}(t)$ is the optimal choice of labor by firm 2 in the artificial t economy introduced in the proof of Proposition 1. It follows from (22) that

$$\hat{l}' = \frac{(v'' - u''(f_2)^2)(l' + \hat{l}') - u''f_1f_2}{u'\hat{f}''}$$

Inserting the value of $l' + \hat{l}'$ in (23) leads to

$$\hat{l}' = \frac{-u'f_{21}\hat{f}''(v'' - u''(f_2)^2) - u'u''f_1f_2\hat{f}''f_{22}}{u'\hat{f}''den} < 0,$$

where den denotes the positive denominator of (23). Thus $\hat{l}(1) = \hat{l}_g < \hat{l}(0) = \hat{l}_b$, which proves the Lemma. \square

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