Uncertainty

Lecture 9

Reading: Perloff Chapter 17

August 2017

Introduction

- Everything is uncertain.
- We don't know what *state of nature* we will be in tomorrow.
- We looked at consumer choice over consumption bundles.
- Now we will look at consumer choice over a variety of risky alternatives.

Introduction

- How much insurance should I buy?
- What is the optimal portfolio given my attitude towards risk?
- Should I cross the street when the light is red?

Outline

- **Probability Overview** - How can we measure risk?
- **Decision making under uncertainty** - The decisions people make depend on their attitudes towards risk.
- **Avoiding risk** - People can take steps to avoid risk altogether.
- **Behavioral economics of risk** - Do people behave like we have just described?

Probability Overview

- We don't know what state of nature will be in tomorrow.
- But we might know the relative likelihood of different states of nature.
- Tomorrow is a roll of the dice, it is a random outcome.

- The gender of the next person you meet, your grade on an exam, the number of times your computer crashes all have an element of randomness.
- An **outcome** is a mutually exclusive result of the random process.
- Your computer might crash once, it might never crash... these are outcomes.

Probability Overview

- The **probability** of an outcome is the proportion of time that outcome occurs in the long run
- The probability of rolling a 4 is $\frac{1}{6}$. If we rolled a dice 1,000,000,000 times, we would get 4 $\frac{1}{6}$ of the time
- We could also express it as the probability of event X is

$$\Pr[X] = \frac{\textit{all events favorable to } X}{\textit{all possible events}}$$

Probability Overview

- A **random variable** is a numerical summary of a random outcome.
- If my computer crashes the variable takes a 1, if not it takes a 0 for example.
- The roll you get from a dice is a random variable.

Probability Overview

- A **discrete random variable** takes on only a discrete set of values, like 0, 1, 2, 3.
- A **continuous random variable** takes on a continuum of possible values (any point on some measure of the real line).
- Roll of the dice is a discrete random variable, the amount of rain is a continuous random variable.

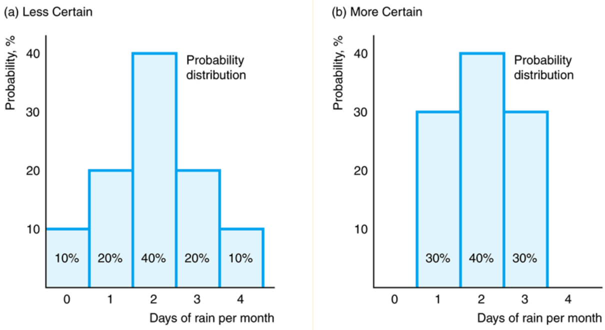
Probability Overview

- A **probability distribution** is the list of all possible values of the random variable and the probability they will occur.
- A **cumulative probability distribution** is the probability that the random variable is less than or equal to some value.

EXAMPLE

- What is the probability distribution and cumulative probability distribution for a dice role?

Probability Overview



Probability Overview

- The **expected value** of a random variable Y is the long run average value the random variable will take over many repeated trials
- It is the weighted average of all possible values (also called expectation or mean)
- If Y is discrete

$$E[Y] = \sum_{i=1}^{\infty} p_i x_i$$

- If Y is continuous

$$E[Y] = \int_{-\infty}^{\infty} x f(x) dx$$

Probability Overview

- Lets say X is the roll of a die, which has 6 possible outcomes all equally likely

$$E[X] = \frac{1}{6}1 + \frac{1}{6}2 + \frac{1}{6}3 + \frac{1}{6}4 + \frac{1}{6}5 + \frac{1}{6}6 = 3.5$$

- If you were to roll a fair die 1,000,000,000,000 times, on average you would get a value of 3.5

EXAMPLE

- I give you £100 if you flip a heads and £0 if you flip a tails. What is your expected payoff from this game?

Probability Overview

- The **variance** of some random variable X is a measure of how spread out the distribution is. If the variance is high You are less likely to get a value close to the mean

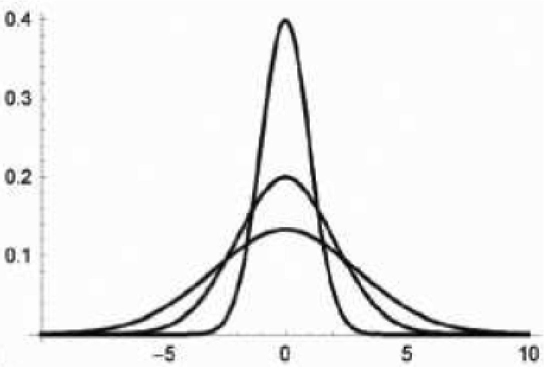
$$
\begin{aligned}
Var[X] &= \sum_{i=1}^{n} p_i (x_i - E[X])^2 \\
Var[X] &= E[X^2] - (E[X])^2
\end{aligned}
$$

- Or if continuous

$$Var[X] = \int (x - E[X])^2 f(x) dx$$

- The **standard deviation** of X is just the square root of the variance $S.D.[X] = \sigma_x = \sqrt{\sigma_x^2}$

Probability Overview



Decision Making Under Uncertainty

- Remember in chapter 3 we learned about preferences over bundles of goods
- People now have preferences over lotteries
- People will pick the lottery that gives them the highest utility.
- You don't know exactly what utility you will get, however.
- People maximize their **expected utilities**
- The **expected utility** of some action is the probability weighted sum of utilities you get associated with each possible outcome.

Decision Making Under Uncertainty

- Suppose with you the following lottery.
- With probability $\frac{1}{2}$, you win £10 and with probability $\frac{1}{2}$ you win £1
- Your utility function is $U = 10 * w$ where w is wealth.

- Your expected utility is

$$
\begin{aligned}
EU &= \Pr[heads] * U(heads) + \Pr[tails] * U(tails) \\
EU &= \frac{1}{2} * 10(10) + \frac{1}{2} * 10(1) = 55
\end{aligned}
$$

EXAMPLE

- Suppose you have a dice roll and you win the value of the dice roll (if you roll a 6 you get $\$6$).
- Write down an equation for expected utility of an individual who has a utility function $U = \ln w$ (don't actually calculate the number).

Decision Making Under Uncertainty

- Two people might face the same lottery, but they will get a different expected utility.
- I buy lottery tickets, many people don't.
- The shape of your utility function determines your attitude over risk.

- A **fair bet** is a wager with an expected value of 0.
- You pay \$10 to play the lottery and the expected value of the ticket is \$10. This is a fair bet. On expectation you do not lose money.

EXAMPLE

- You win the value of the dice roll (roll a 6 get $6 for example)
- If you have to pay £4 to play this game, is that a fair bet?

Decision Making Under Uncertainty

- Suppose I offer you a **fair bet**.
- People who are **risk neutral** are indifferent between taking this bet or not
- People who are **risk loving** will always take this bet
- People who are **risk averse** will not.

Decision Making Under Uncertainty

- Let's look at the risk averse people first.
- If somebody is **risk averse**, they have a diminishing marginal utility to wealth.
- Their utility function is concave.
- The extra utility they get from one more dollar is less than they got from the last.

Decision Making Under Uncertainty

- Suppose somebody's utility function is $U = W^{\frac{1}{2}}$.
- This person can play the following game.
  - You win \$2 with probability $\frac{1}{2}$ or you win *\$0* with probability $\frac{1}{2}$
  - The expected value of this game is \$1

Decision Making Under Uncertainty

- The person's expected utility from this lottery is

$$EU = .5 * (2)^{\frac{1}{2}} + .5 * (0)^{\frac{1}{2}} = .7$$

- The person's utility of having the \$1 for sure is $(1)^{\frac{1}{2}} = 1$.
- This person gets a higher utility from \$1 for sure than a bet whose expected payoff is \$1.
- This person is risk averse.

Decision Making Under Uncertainty

- The **risk premium** is the amount that a risk averse person will pay to avoid taking a risk.
- In the previous example, we know the lottery gives us an expected utility of .7.
- To find the risk premium, we need to find the amount of money we would be willing to give up to eliminate risk altogether.
- The lottery in this case has an expected value of \$1, how much of that \$1 would I give up?

Decision Making Under Uncertainty

$$\begin{aligned} (X)^{\frac{1}{2}} &= .7 \\ X &= .49 \end{aligned}$$

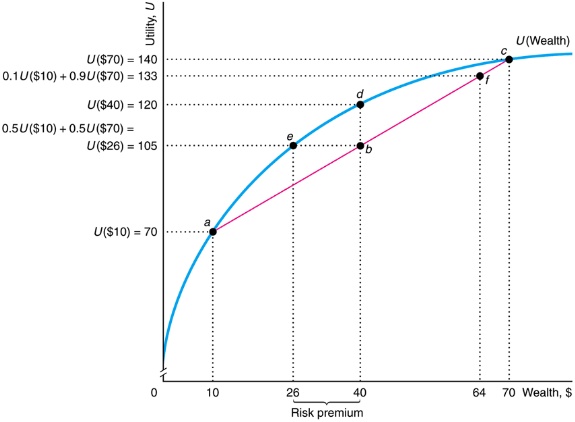
- I am indifferent between playing the lottery with an expected value of \$1 and having \$0.49 cents for sure.
- The risk premium is \$0.51. I would give up \$0.51 to avoid risk altogether.

EXAMPLE

- With probability $\frac{1}{4}$ the car turns out to be bad and is worth \$400.
- With probability $\frac{3}{4}$ the car turns out to be good and is worth \$1600.
- What is the expected **payoff** from buying this car?
- If this person has a utility function $U = W^{\frac{1}{2}}$, what is the expected utility of buying this car?
- What is the risk premium? What is the most this person would be willing to pay for this car?

Decision Making Under Uncertainty

- This is what a risk-averse person's utility function looks like.



- This person would give up 14 dollars to have no risk at all.

Decision Making Under Uncertainty

- Now let's look at a risk neutral person.
- A risk-neutral person has a constant marginal utility over wealth, her utility function is a straight line.
- One extra dollar is worth just as much to her no matter how much money she has.

Decision Making Under Uncertainty

- The utility she gets from the expected value of the bet is exactly equal to the expected utility.
- A risk neutral person will take the bet with the highest expected value.

Decision Making Under Uncertainty

- Suppose somebody has a utility function $U = W$.
- This person can play the following game.
  - You win \$2 with probability $\frac{1}{2}$ or you win \$0 with probability $\frac{1}{2}$
  - The expected value of this game is \$1

Decision Making Under Uncertainty

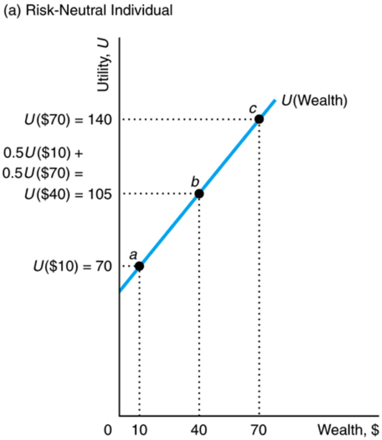
- Will this person take the bet?
- The expected utility of the bet is

$$\frac{1}{2}2 + \frac{1}{2}0 = 1$$

- The utility of having \$1 for sure is exactly the same.
- The presence of risk does nothing.

Decision Making Under Uncertainty

- Now let's look at a risk loving person.
- A risk loving person has an increasing marginal utility of wealth, they will always take a fair bet.
- Their utility function is convex.
- This person has a negative risk premium.
- The expected utility of some bet is higher than the utility she gets from the utility of the expected value.


(a) Risk-Neutral Individual
Utility, U
U(Wealth)
U($70) = 140
0.5U($10) +
0.5U($70) =
U($40) = 105
U($10) = 70
a
b
c
0
10
40
70
Wealth, $
(b) Risk-Preferring Individual
Utility, U
U(Wealth)
U($70) = 140
0.5U($10) +
0.5U($70) = 105
U($40) = 82
U($10) = 70
a
b
c
d
e
0
10
40
58
70
Wealth, $

- To summarize
- Somebody is risk averse if $U(E[X]) > E[U(X)]$
- Somebody is risk averse if $U(E[X]) = E[U(X)]$
- Somebody is risk loving if $U(E[X]) < E[U(X)]$

EXAMPLE

- If you flip a heads I give you £10, if you flip a tails I give you £5.
- What is the most each of the following people would be willing to pay for this game?

1. $U = (W)^{\frac{1}{2}}$
2. $U = W$
3. $U = (W)^2$

- Draw each person's utility function showing the expected value, expected utility and risk premia.

Decision Making Under Uncertainty

- The curvature of the utility function is what determines somebody's risk aversion
- One commonly used measure of this curvature is the **Arrow-Pratt measure of risk aversion**

$$\rho(W) = -\frac{d^2 U(W)/dW^2}{dU(W)/d(W)}$$

- The numerator of this expression is the second derivative.
- If utility is concave, this will be negative
- ρ is positive if somebody is risk averse and it is negative if they are risk loving.

EXAMPLE

- What is the Arrow-Pratt measure of risk aversion for the following utility functions?

$$
\begin{aligned}
U &= W^{\frac{1}{2}} \\
U &= 2W \\
U &= W^2
\end{aligned}
$$

Avoiding Risk

- There are a lot of ways people can avoid or decrease risk
- One such way is through **diversification**.
- Diversification simply means putting your eggs into different baskets.
- The probability that all of the baskets break at the same time is low.

Avoiding Risk

- Another way to deal with risk is through **insurance**.
- Insurance is just contingent consumption.
- You can move some of your payoff from the good state of the world to the bad state of the world.
- Make the bad outcome less bad.

Avoiding Risk

- If your house doesn't burn you keep £1,000,000
- If your house burns down you get $0
- Insurance would be something like buying a plan that gives you £900,000 if your house doesn't burn down and £100,000 if your house does burn down.

Avoiding Risk

- Suppose that for every \$1 of insurance you buy, you get \$4 if your house burns down.
- The probability your house burning down is .1 and the probability it doesn't is .9.
- By being able to purchase insurance, your expected utility changes from the first expression to the second expression.

$$
\begin{aligned}
EU &= 0.9U(1,000,000) + 0.1(0) \\
EU &= 0.9 * U(1,000,000 - x) + 0.1 * U(4x - x)
\end{aligned}
$$

- To find out exactly how much insurance you would by, you would maximize this with respect to x.

Avoiding Risk

- If our utility function were $U = \ln W$

$$EU = 0.9 * \ln(1,000,000 - x) + 0.1 * \ln(4x - x)$$

- maximise this with respect to x and solve for x.

Avoiding Risk

$$
\begin{aligned}
\frac{dEU}{dx} &= \frac{0.9}{x - 1,000,000} + \frac{0.1}{x} = 0 \\
\frac{0.1}{x} &= \frac{0.9}{1,000,000 - x} \\
0.9x &= 100,000 - 0.1x \\
x &= 100,000
\end{aligned}
$$

EXAMPLE

- You can insure your bike worth \$500 against theft.
- The probability your bike gets stolen is $\frac{1}{2}$.
- For every \$1 of insurance you buy you get \$2 if your bike is stolen.
- If your utility function is $U = W^{1/2}$, how much insurance should you buy?
- What is the intuition behind this result?

Avoiding Risk

- In the previous example, insurance was **actuarially fair**.
- A policy is actuarially fair if the insurance company does not make any profit.
- Expected profit for the insurance company in the previous example for every \$1 of insurance bought is

$$E(\pi) = \frac{1}{2}(\$1) - \frac{1}{2}(\$1 - \$2) = 0$$

EXAMPLE

- Your house burns down with probability 1%.
- For every £1 of insurance you buy the insurance company gives you £50 back.
- Is this actuarially fair?

Behavioral Economics of Risk

- Is this how people actually behave towards risk though?
- Many people make decisions that are inconsistent with expected utility theory.

Behavioral Economics of Risk

- One common fallacy is the **gambler's fallacy**
- People believe that past events affect current, independent outcomes.
- If you flip a fair coin 100 times and they all come up heads, the probability the next coin is heads is still .5.
- But many people believe that tails is more likely because it is "due."

Behavioral Economics of Risk

- People often overestimate their probability of winning.
- In one survey, gamblers estimated their chance of winning a bet is 45% when the objective probability was 20%.

Behavioral Economics of Risk

- Many people put excessive weight on outcomes they consider to be certain.
- A group of subjects were asked to choose between two options in an experiment.

Behavioral Economics of Risk

Option A: You receive \$4,000 with probability .8 and \$0 with probability .2

Option B: You receive \$3,000 with certainty

- 80% of people choose option B

- They were then given a separate set of options

Option C: You receive \$4,000 with probability .2 and \$0 with probability .8
Option D: You receive \$3,000 with probability .25 and \$0 with probability .75

- 65% of people choose C

- These choices are inconsistent
- Choosing B over A implies the expected utility of B is greater than A
- $U(3,000) > .08U(4000)$
- Or $\frac{U(3{,}000)}{U(4000)} > .08$

Behavioral Economics of Risk

- Choosing C over D implies that $.2U(4000) > .25U(3,000)$
- This implies $\frac{U(3{,}000)}{U(4000)} < .08$
- This is inconsistent with expected utility theory

Behavioral Economics of Risk

- The way you present the same lottery can change people's preferences.
- The Avian Flu will kill 600 people, the government is proposing two programs to combat it

Program A: 200 people will be saved
Program B: There is a $1/3$ probability that 600 people will be saved and a $2/3$ probability nobody will be saved.

- 72% opted for program A

Behavioral Economics of Risk

- The Avian Flu will kill 600 people, the government is proposing two programs to combat it

Program C: 400 people will die
Program D: There is a $1/3$ probability that nobody dies and a $2/3$ probability that 600 people will die.

- 78% opted for program D

Behavioral Economics of Risk

- A and C are the same and B and D are the same!
- Framing is a huge issue in economics.

Behavioral Economics of Risk

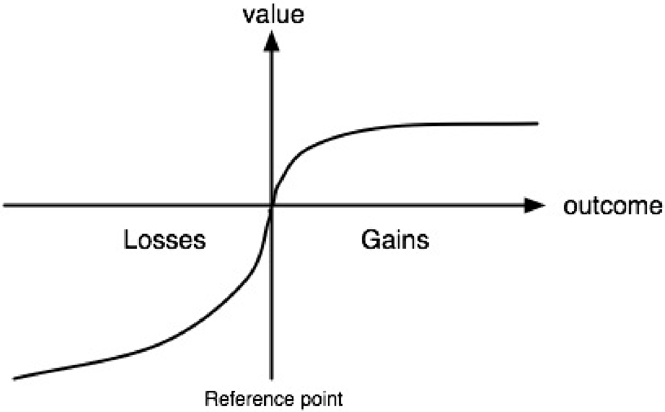
- **Prospect theory** is an alternative approach to decision making under uncertainty.
- Empirically, it is a better match of people's behaviors.
- People treat gains and losses differently in prospect theory.
- Small changes are evaluated differently than big changes.

Behavioral Economics of Risk

- The basic tenants of prospect theory are

1. Everything is evaluated according to some reference point.
2. People are more sensitive to small gains than to large gains. For example the difference between \$1 and \$2 is larger than \$1001 and \$1002.
3. People hate taking losses more than they like taking gains.

Behavioral Economics of Risk



Summary

- What is the expected value of a random variable?
- What is expected utility?
- What are the different attitudes towards risk and who will take a fair bet?
- What is the risk premium?

Summary

- What is fair insurance?
- What are inconsistencies with expected utility theory in the real world?
- What is prospect theory?