Costs

Lecture 5

Reading: Perloff Chapter 7

July 2017

Introduction

- Last lecture, we discussed how firms turn inputs into outputs.
- But exactly how much will a firm wish to produce?

Introduction

- How much a firm wishes to produce depends on the cost function.
- The firm's first step is to find the production process that is *technically efficient.*
- Technical efficiency is a necessary condition for profit maximization, but it is not sufficient.
- The firm's second step is to find the technologically efficient production process that is also **economically efficient.**
- A firm is economically efficient if it minimizes the cost of producing a specified amount of output.

Outline

- **Measuring Costs** - How exactly do economists measure costs?
- **Short-Run Costs** - What does a firm's cost function look like when some inputs are fixed?
- **Long-Run Costs** - What does a firm's cost function look like when all inputs are variable? What is the firm's optimal input combination?
- **Lower Costs in the Long Run** - Firm has more flexibility in the long run, which implies lower costs.
- **Cost of Producing Multiple Goods** - Does producing multiple goods in the same factory make sense?

Measuring Costs

- To find the economically efficient level of output, we need to know how to measure costs.
- It is easy to measure **explicit costs**.
- Paying a worker £7 an hour is an explicit cost.
- But we must look at all costs, including the **implicit** ones.
- A cost is implicit if it reflects forgone opportunity rather than current expenditure.

Measuring Costs

- **Opportunity cost** is an implicit cost.
- The opportunity cost is the value of the next best alternative.
- The opportunity cost of me being in school is $\$25,000$ salary.
- Opportunity cost is important when a firm purchases capital, because it durable.

Measuring Costs

- Opportunity costs *should* influence the firms current decisions, but **sunk costs** *should not*.
- A sunk cost is an expenditure that cannot be recovered.
- A non-refundable movie ticket is an example of a sunk cost.

EXAMPLE

- You can play tennis inside or outside. You can book the inside court for a non-refundable £20 fee in advance.
- Playing outside is free.
- You prefer to play inside if it is rainy but outside if it is sunny.
- You booked the inside tennis court in advance and it turns out it is sunny.
- Do you play inside or outside?

Short-Run Costs

- To maximize profit, the firm needs to know how costs vary with output.
- A cost function $C(q)$ tells us how much it will cost to produce various levels of output. All points on the cost function are economically efficient.

EXAMPLE

- Lets graph the cost function $C(q) = 10q + 10$.
- If the firm wants to produce 10 units of output, could it do so at a cost of £100? At a cost of £200?

Short-Run Costs

- Remember in the short run, at least one input is fixed.
- The cost of producing 10 units in the short-run is not always the same as the cost of producing 10 units in the long-run.
- We will first look at the firm's cost function in the short-run, then the cost function in the long-run.

Short-Run Costs

- It is useful to break up our costs into different types.
- One type of cost is a **fixed cost** (F)**.**
- Fixed costs do not vary with the level of output.
- For example, it costs £10,000 to heat a factory no matter how much you produce.

Short-Run Costs

- Fixed costs might be sunk or non-sunk.
- It is sunk if it cannot be recovered by shutting down.
- If you own a factory that has no alternative uses upon shutting down (you can't sell it), that is a sunk fixed cost.

Short-Run Costs

- A **Variable cost** (VC) is the production expense that does change with quantity produced.
- The cost of dough is a variable cost for a bakery.
- **Total cost** (C) is the sum of fixed and variable cost.

$$C = VC + F$$

EXAMPLE

- If our cost-function looks like $C(q) = 100q + 10$, what are the variable costs and what are the fixed costs?

Short-Run Costs

- **Marginal cost** is the amount by which the total cost changes when we add more output.

$$MC = \frac{dC(q)}{dq}$$

Short-Run Costs

- **Average fixed cost** is the fixed cost divided by the amount produced q.

$$AFC = \frac{F}{q}$$

- It declines with output because the fixed cost is spread over more units.
- **Average variable cost** is the variable cost per each unit produced.

$$AVC = \frac{V}{q}$$

Short-Run Costs

- **Average cost** is the sum of these.

$$AC = \frac{C}{q} = \frac{VC}{q} + \frac{F}{q}$$

EXAMPLE

- Suppose our cost function looks like.

$$C = q^2 - 100q + 1000$$

- What is the variable cost, fixed cost, marginal cost, average variable cost, average fixed cost and average cost?

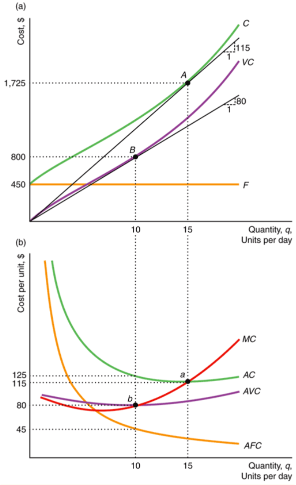
Short-Run Costs

- What do all these cost curves look like graphically?
- Fixed cost does not vary with output, so it is a straight line.
- Average fixed cost falls as output decreases.
- Average cost is the vertical sum of average fixed cost and average variable cost.

Short-Run Costs

- Average cost slopes downward at first because average fixed cost declines.
- Average cost begins to slope upward because of diminishing marginal returns.
- Marginal cost intersects average cost at the minimum of average variable cost. **Why?**

Short-Run Costs



Short-Run Costs

- The production function we saw Ch. 6 and the cost function are basically mirror images of each other.
- We can find the cost function from the production function and *vice versa*.
- For example, the production function tells us we need 10 units of labour to produce 6 units of output. The cost of one unit of labour is £5.
- The cost of producing 6 units of output is then £5 *10 = £50.

Short-Run Costs

- Suppose we have the following short-run production function.

$$q = f(L, \overline{K}) = g(L)$$

- We are in the short-run, so capital is fixed. Labour is the variable cost and capital is the fixed cost.

$$VC = wL$$

Short-Run Costs

- If we invert the production function we can find the amount of labor needed to produce any amount of output

$$L = g^{-1}(q)$$

- Plugging this in we can see our cost function is now

$$C(q) = V(q) + F = wg^{-1}(q) + F$$

Short-Run Costs

- Suppose our production function is as follows:

$$q = L^{.5}K^{.5}$$

- Capital is stuck at 16 units in the short-run so the short-run production function can be written as:

$$q = 4L^{.5}$$

- Suppose the price of capital is $\$1$ per unit of capital.

Short-Run Costs

- Solve this for L

$$L = \frac{q^2}{16}$$

- This tells us how much labour we will use to produce each amount of output.
- If we want to produce 4 units of output, we must use 1 unit of labour.

Short-Run Costs

- We can express the short-run cost function as

$$C\left(q\right) = w\frac{q^2}{16} + 16$$

EXAMPLE

- Suppose capital is fixed at 19 units and the price of capital is $2 per unit.
- The production function is

$$q = LK + L$$

- What is the short-run cost function?

Short-Run Costs

- If we know marginal product of labour, we can easily find the marginal cost.
- Recall that variable cost in the short run is $V(q) = wL$

$$MC = \frac{dV(q)}{dq} = \frac{d(wL)}{dq} = w\frac{dL}{dq}$$

- We know the marginal product of labour is $\frac{dq}{dL}$ so we can write the relationship as

$$MC = w\frac{1}{MP_L}$$

Short-Run Costs

- If we know the average product of labour, we can easily find average variable cost.
- Remember $AVC = \frac{VC}{q} = \frac{wL}{q}$
- The average product of labour is $\frac{q}{L}$, so we can write the relationship as

$$AVC = \frac{w}{AP_L}$$

EXAMPLE

- Suppose your short-run cost function is

$$C(q) = q + q^2 + 10$$

- The wage rate is £1. What is the average product of labour and the marginal product of labour when $q = 4$?

Short-Run Costs

- The government can affect a firm's cost curves through various forms of taxation.
- Different types of taxes affect the cost curves in different ways.
- A specific tax will shift the firm's variable and marginal costs up, but won't affect the fixed costs.
- A franchise tax will affect the firm's fixed costs.

Long-Run Costs

- Now let's turn to the long-run.
- Firms can vary everything in the long run.
- There are no fixed costs (technically they can have avoidable fixed costs in the long run but we assume they don't).
- Long run cost is just

$$C(q) = VC$$

Long-Run Costs

- Now that both inputs are free to vary, what input combination should the firm select?
- Remember that isoquants show us all the technologically efficient input combinations.
- The firm must pick the technically efficient input combination that is the cheapest (economically efficient).

Long-Run Costs

- The **Isocost line** shows all the input combinations that cost exactly the same.
- You hire L units of labour at a price of w and K units of capital at a price of r.
- We can write the isocost line as

$$C = wL + rK$$

- The isocost is a lot like the budget line in consumer theory, the difference being that the firm has many isocosts and the consumer has only one budget line.

EXAMPLE

- What is the equation for an isocost if $w = 4$, $r = 5$ and we want to spend $\$1,000$?
- What is the slope of the isocost line?
- What happens if we want to spend $\$2,000$?
- What happens if the wage increases?

Long-Run Costs

- Suppose the firm wants to produce $\overline{Q}$ units. How does the firm find the cheapest input combination?
- There are three **equivalent** ways the firm can find this out.

Lowest Isocost Rule

- pick the isocost closest to the origin that touches the isoquant.

Tangency Rule

- Assuming we have an interior optimum, the lowest isocost is where the isocost is tangent to the isoquant.
- The slope of the isocost is $-\frac{w}{r}$.
- The slope of the isoquant is the $MRTS = -\frac{MP_L}{MP_K}$.
- The optimal input combination occurs where

$$\frac{w}{r} = \frac{MP_L}{MP_K}$$

Last Dollar Rule

- We can rearrange the tangency condition

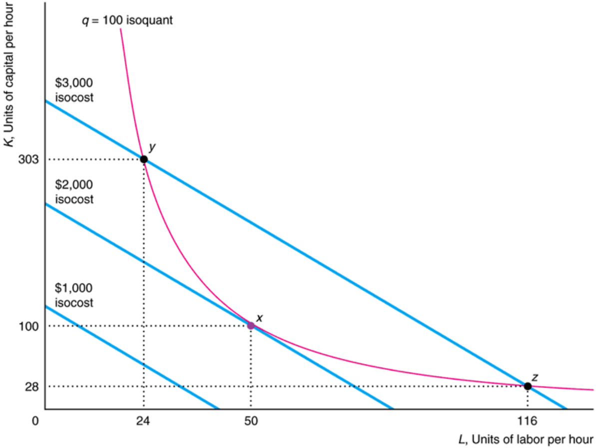
$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

- cost is minimized when the last dollar spent on labour adds as much extra output on the last dollar spent on capital.

EXAMPLE

- If $w = 2$, $MP_L = 10$, $r = 10$, $MP_K = 10$, what should we do to lower costs?

Long-Run Costs



EXAMPLE

- Suppose $w = 5$ and $r = 20$.
- If our production function is $q = K^{\frac{1}{2}} L^{\frac{1}{2}}$, what is the optimal input combination?
- If we want to produce 10 units, how much labour and capital will we use?

Long-Run Costs

- We can use math to get the same tangency condition.
- We want to minimize costs $wL + rK$ such that we produce $\overline{q} = f(L, K)$

$$\min \mathcal{L} = wL + rK + \lambda[\overline{q} - f(L, K)]$$

- Lets prove together that this results in $\frac{MP_L}{MP_K} = \frac{w}{r}$.

Long-Run Costs

- Rather than minimize costs for a desired level of output, what if we want to maximize output for some given cost?
- Our Lagrangian becomes

$$\max \mathcal{L} = f(L, K) - \lambda(wL + rK - \overline{C})$$

- If you do this, you get exactly the same result that $\frac{MP_L}{MP_K} = \frac{w}{r}$
- This is a **"dual" problem**... two sides of the same coin.

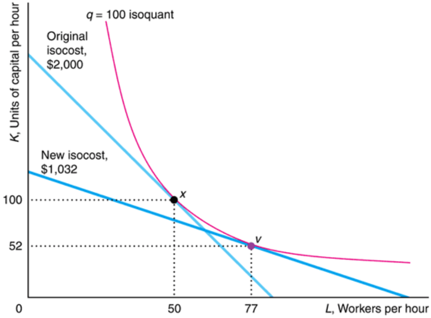
EXAMPLE

- Suppose your production function is:

$$q = KL$$

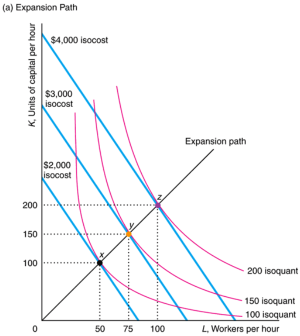
- The wage rate is $w = 1$ and the rental rate is $r = 1$.
- Use the Lagrangian method to find the cheapest way of producing 25 units of output.

- What happens when one factor becomes relatively cheaper?
- The slope of the isocost is $-\frac{w}{r}$, so the slope changes and we have a new cost minimizing combination.



Long-Run Costs

- How does the firm's cost change when we increase output?
- The **expansion path** shows us all the tangency points for each level of output.



Long-Run Costs

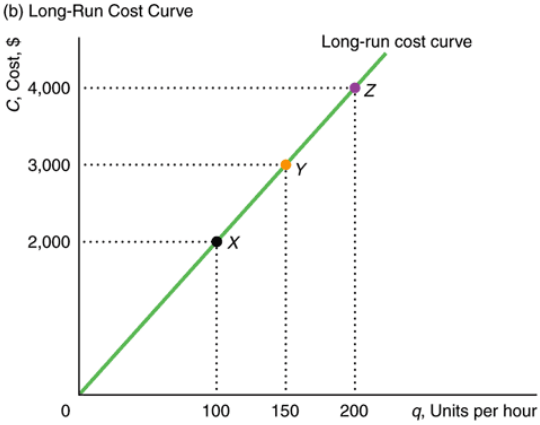
- The expansion path tells us the same thing as the long-run cost function essentially.
- When you produce q_o units, use K_o and L_o units of capital and labour.
- When you produce q_1 units, use K_1 and L_1 units of capital and labour.

Long-Run Costs

- Suppose we found from our expansion path that $K = q$ and $L = \frac{q}{2}$ is the optimal input combination.
- We can plot the long run cost of producing different levels of q.. Suppose the wage is 24 and capital costs 8.

$$C(q) = wL + rK = w\frac{q}{2} + rq = 20q$$

Long Run Cost Functions



Long-Run Costs

- Remember the *short-run* cost curve is U shaped.
- It slopes downward at first because average fixed cost declines.
- It slopes upward because of diminishing marginal returns.
- There are no fixed costs and diminishing marginal returns in the long run.

Long-Run Costs

- If the LRAC curve is downward sloping, the cost function exhibits **economies of scale.**
  - The average cost falls as you produce more.
- If the LRAC curve is upward sloping, the cost function exhibits **diseconomies of scale.**
  - The average cost increases as you produce more.
- If it is flat there are *no economies of scale.*

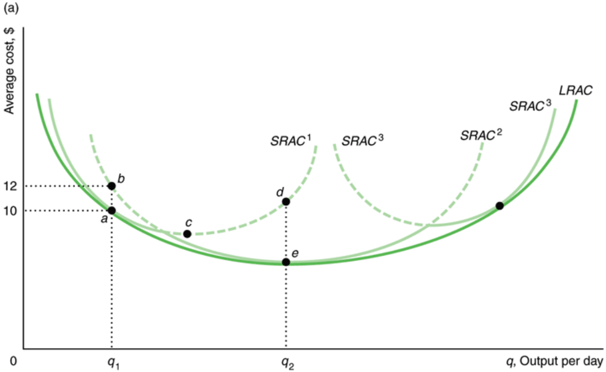
Long-Run Costs

- Recall that returns to scale refers to how much your output will change when you scale up your inputs.
- Do economies of scale imply returns to scale?
- What do you think?

Lower Costs in the Long-Run

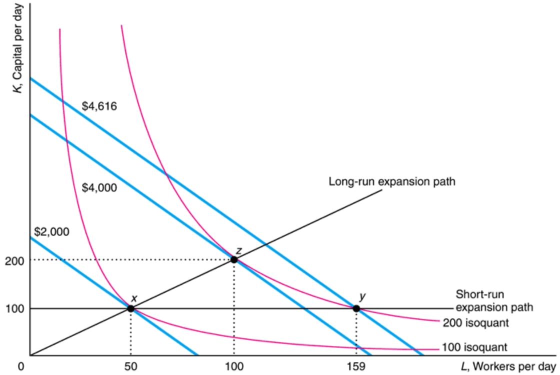
- Firms have more degrees of freedom in the long run.
- Suppose the optimal combination for some level of output is $K = 10$ and $L = 5$, if we are in the short run K might be stuck at the "wrong" level that is not cost minimizing.
- Costs in the short-run are always at least as high as in the long-run.
- In the long run, we can change K to be where we want.

Lower Costs in the Long-Run



Lower Costs in the Long-Run

- We can further illustrate this by comparing the short-run and long-run expansion paths.

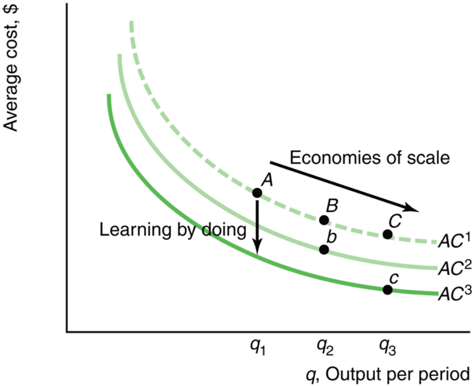


Lower Costs in the Long-Run

- Another reason why costs are lower in the long is **learning by doing.**
- As workers gain experience and managers learn to organize, the average cost tends to fall over time.

Lower Costs in the Long-Run

(b) Economies of Scale and Learning by Doing

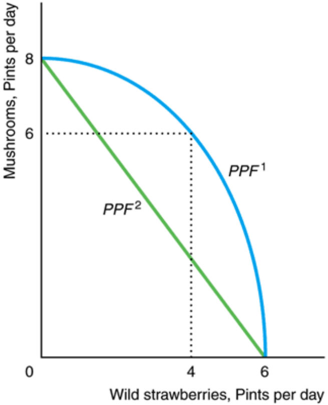


Cost of Producing Multiple Goods

- If firms produce multiple goods, the cost of one good might depend on the output of another.
- It is less expensive to produce poultry and eggs together than separately.
- If it is less expensive to produce goods jointly, the firm enjoys **economies of scope.**
- If it is more expensive to produce goods jointly, the firm experiences **diseconomies of scope.**

Cost of Producing Multiple Goods

- We can illustrate this by a **production possibilities frontier**



Summary

- What is an opportunity cost?
- What are fixed and variable costs?
- What are sunk costs?
- What are avoidable costs?

Summary

- How does a firm find the cost minimizing input combination in the long run?
- What are economies of scale?
- What are economies of scope?
- What is learning by doing?