Firms and Production

Lecture 4

Reading: Perloff Chapter 6

July 2017

Introduction

- In this lecture we look at firms and production.
- It is the first step in deriving the supply curve we say in the first lecture.

Outline

- **Ownership and Management of Firms** - What exactly is a "firm?"
- **Production** - How a firm makes output from their set of inputs.
- **Short-Run Production** - Look at production when the firm has a fixed input.
- **Long-Run Production** - Look at production when there are no fixed inputs.
- **Returns to Scale** - How the size of a firm affects how much it produces.
- **Productivity and Technical Change** - The most output you can get for your inputs varies across firms and across time.

Ownership and Management of Firms

- A **firm** is simply some organization that takes inputs and turns it into outputs.
- We can roughly divide these firms into
  - **private sector**
  - public
  - non-profit firms

Ownership and Management of Firms

- Sole proprietorship
  - owned by an individual who is responsible for all debts.
  - Example: A freelance writer or a bookkeeper.

Ownership and Management of Firms

- General partnership
  - Jointly owned by multiple people who are together responsible for debts.
  - Example: Law office with multiple partners.

Ownership and Management of Firms

- Corporations
  - Owned by shareholders in proportion to the amount of stock they own.
  - limited liability.
  - Example: Microsoft, recently Facebook.

Ownership and Management of Firms

- Private firms have the single goal of maximizing profits.
- **Profit** (π) is defined as **total revenue** (TR) minus **total cost** (TC).

$$\begin{aligned} \pi &= TR - TC \\ TR &= p * q \end{aligned}$$

- A firm can only maximize profit if it achieves **technical efficiency**.
- Technical efficiency means they get the most output they possibly can from their set of inputs and technology.

Production

- A firm takes inputs (**factors of production**) and turns it into output according to its technology.
- We can broadly classify these inputs into capital (**K**), labour (**L**) and materials (**M**) (which we usually ignore for simplicity).

Production

- The **production function** summarizes this process, and tells us exactly how much output the firm can get from their inputs.
- For example suppose our production function is

$$q = f(L, K) = 2 * L * K$$

- If the firm employs two units labour and 4 units of capital it gets 16 units of output (it could produce less, but that would not be efficient).

EXAMPLE

- Suppose a firm produces output using only labour according to the production function $q = L^2$.
- Sketch this production function and identify two production plans, one that is technically efficient and one that is not.

Production

- The **short run** is the period of time that at least one factor of production cannot be changed.
- For example, dominoes can decide how many delivery drivers it hires in a month, but can't decide how many stores to build in this time frame.

Production

- The **long run** is a period of time in which all inputs can be varied.
- The difference between short and long run varies by industry.
- We call inputs that can't be changed **fixed inputs**, and ones that can be changed **variable inputs.**

Short-Run Production

- Lets say capital is fixed in the short run, our production function is then
$$q = f(\overline{K}, L)$$
- Suppose our production function is $q = 2KL$, but capital is fixed at $K = 4$ in the short-run.
- Our short-run production function becomes $q = 8L$.
- We can summarize the relationship between output and the amount of labour used by the **total product of labour**, **the average product of labour** and the **marginal product of labour.**

Short-Run Production

- **Total product of labour** is the amount of output that a given amount of labour can produce holding other inputs fixed
- **Marginal product of labour** is the extra output you get from increasing labour by some infinitesimally small amount

$$MP_L = \frac{\partial q}{\partial L}$$

- **Average product of labour** is the amount of output produced per worker

$$AP_L = \frac{q}{L}$$

EXAMPLE

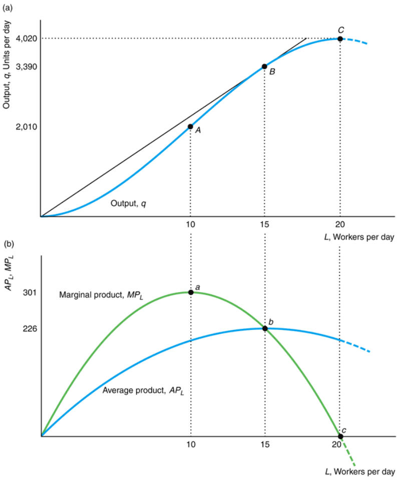
- Suppose our production function is

$$Q = K * L^2$$

- What is the total product of labour, marginal product of labour and average product of labour if capital is fixed at 50?

Short-Run Production

- If our short run production function is $q = L + 30L^2 - L^3$



Short-Run Production

- We assume the firm can hire fractions of workers (which is why this is smooth)
- The MP_L is the slope of the production function
- The AP_L is the slope of the chord from the origin

Short-Run Production

- The previous production function is typical
- AP_L initially rises because of gains from specialization, but it declines because capital is held constant.

Short-Run Production

- The MP_L always intersects the AP_L at the maximum of the AP_L curve.
- When $MP_L > AP_L$, the average is pulled up.
- When $MP_L < AP_L$, the average is pulled down.

EXAMPLE

- Let's prove together that $MP_L = AP_L$ at the maximum of AP_L for the production function $q = L + 30L^2 - L^3$.

Short-Run Production

- The **law of diminishing marginal returns** is huge in economics.
- As you increase one input, holding all other inputs and technology constant, the marginal returns to that input will decrease eventually.
- The second derivative will become negative.
- *You can't grow the world's food supply in a flower pot.*
- This is why Malthus predicted mass starvation (one input -land- is fixed).

Short-Run Production

- For the production function

$$\begin{aligned} q &= L + 30L^2 - L^3 \\ MP_L &= \frac{dq}{dL} = 1 + 60L - 3L^2 \end{aligned}$$

Short-Run Production

- The marginal product of labour is increasing for low levels of output because of gains from specialization.
- Eventually, the marginal product of labour starts to decrease.

$$\begin{aligned} \frac{dMP_L}{dL} &= 60 - 6L \\ L &< 10 \ MP_L \ \textit{increases} \\ L &> 10 \ MP_L \ \textit{decreases} \end{aligned}$$

EXAMPLE

- Suppose the world's supply of food is determined by the production function

$$F = M^{\frac{1}{2}} L^{\frac{1}{2}}$$

- M is land and is fixed at 3600.
- L is labour.
- At what point does this production function exhibit diminishing marginal returns to labour?

Long-Run Production

- In the long run, the firm is free to select as much of any input... nothing is fixed.
- Lets suppose our production function is Cobb-Douglas

$$q = 3L^{\frac{1}{2}}K^{\frac{1}{2}}$$

- Try to graph this.

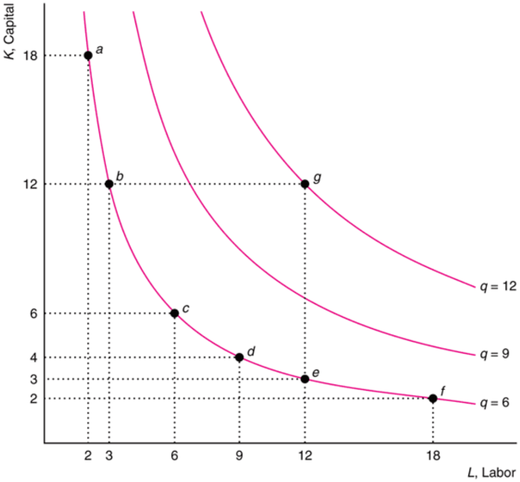
Long-Run Production

- Now that we have multiple variable inputs, our production function has multiple dimensions.
- Can't really deal with it graphically.
- We can summarize this production function in 2 dimensions.

Long-Run Production

- An **isoquant** is a curve that shows the *efficient* combinations of inputs that produce a single level of output
- This has a very similar interpretation to indifference curves.
- Every combination of labour and capital on the same isoquant will produce the same amount of output.

Long-Run Production



Long-Run Production

- Say our production function is

$$Q = 10 * K * L$$

- Draw the isoquant for $Q = 10$ and $Q = 20$.

Long-Run Production

- Lets now discuss the properties of isoquants.

1. Further from the origin, the greater the output

- The more inputs you use, the more output you get if you are producing efficiently

Long-Run Production

2. They cannot cross

- Suppose the isoquant where $Q = 20$ and $Q = 15$ cross.
- The firm could produce 15 or 20 units of output for the same input combination.
- The firm would not be efficient if it produced 15 for that input combination.

Long-Run Production

3. They slope downward

- If they sloped upward, the firm could produce the same level of output with fewer inputs.

Long-Run Production

4. They are thin

- If they were thick, the firm could decrease its input use and get the same level of output

Long-Run Production

5. UNLIKE indifference curves, isoquants are a cardinal measure.

- Output is objective, it is not some abstract thing like utility.

Long-Run Production

- The curvature of the isoquant tells us how substitutable/complementary the inputs are in the production process.
- The more "curvy" the isoquant, the greater degree of complementarity there is between inputs

Long-Run Production

- If they are straight lines, the inputs are perfectly susbstitutable (apples from Oregon or Washington). This would come from a *linear production function*

$$q = x + y$$

- If they are right angles, inputs must be used in fixed proportions (one secretary per phone). this comes from a *fixed-proportions production function*

$$q = \min\{x, y\}$$

Long-Run Production

Long-Run Production

- The slope of the isoquant shows the firm's ability to replace one input with another *holding output constant.*
- This is called the **marginal rate of technical substitution** $(MRTS)$.
- How much K can we give up for another unit of L holding output constant.

Long-Run Production

- Remember that the slope of an indifference curve is the negative ratio of marginal utilities.
- The slope of the isoquant $(MRTS)$ is the ratio of marginal products.

$$MRTS = \frac{\frac{dq}{dL}}{\frac{dq}{dK}} = -\frac{MP_L}{MP_K}$$

EXAMPLE

- Lets find the *MRTS* of

$$q = AL^{\alpha}K^{1-\alpha}$$

- Is the *MRTS* constant for this production function?

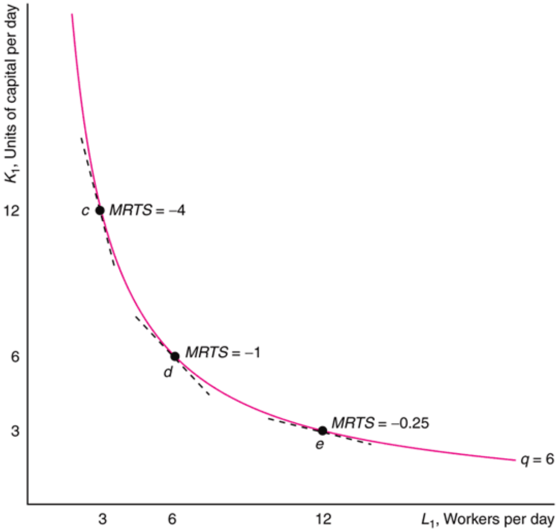
Long-Run Production

- If we have normal convex isoquants, we have a diminishing marginal rate of substitution
- That is, when a lot of capital and little labour is used, the $MRTS$ is really high. When a lot of labour and a little capital is used, it is really low.

Long-Run Production

- Suppose we have a factory with 1,000,000 workers and only one machine.
- To keep output constant, we could trade one machine for a ton of workers (because workers need machines)
- As we get more machines, the number of workers we can trade for one more machine will decrease.

Long-Run Production



Long-Run Production

- For most isoquants, the *MRTS* is not constant.
- As we increase capital and decrease labour, at what rate does the *MRTS* change?
- This is the **elasticity of substitution**... A measure of how "curvy" our isoquants are

Long-Run Production

- Elasticity of substitution (σ) is the percentage change in the capital labour ratio w.r.t a percentage change in the *MRTS*

$$\sigma = \frac{\frac{d(K/L)}{K/L}}{\frac{dMRTS}{MRTS}} = \frac{d(K/L)}{dMRTS}\frac{MRTS}{K/L}$$

- If σ is really high, that means a tiny change in the MRTS results in a big change in K/L, the isoquant is pretty flat.

EXAMPLE

- What is the elasticity of substitution for the following Cobb-Douglas production function?

$$Q = AL^{\alpha}K^{\beta}$$

- What is the elasticity of substitution for a linear production function?
- What does that mean?

Returns to Scale

- If we increase our inputs proportionately, what happens to our output?
- **This is called returns to scale**
- We can have increasing, decreasing or constant returns to scale.

Returns to Scale

- If doubling our inputs leads to exactly double the output, we have **constant returns to scale**

$$2f(L, K) = f(2L, 2K)$$

Returns to Scale

- If doubling our inputs leads to more than double the output, we have **increasing returns to scale**

$$2f(L, K) < f(2L, 2K)$$

- Could be caused by greater specialization

Returns to Scale

- If doubling our inputs leads to less than double the output, we have **decreasing returns to scale**

$$2f(L, K) > f(2L, 2K)$$

- Could be caused by management or organizational problems

Returns to Scale

- What industries do you think have constant, increasing or decreasing returns to scale?
- What are some factors that determine returns to scale?

Returns to Scale

- Generally speaking, our production function is homogenous of degree γ when
$$f(xL, xK) = x^{\gamma} f(L, k)$$

EXAMPLE

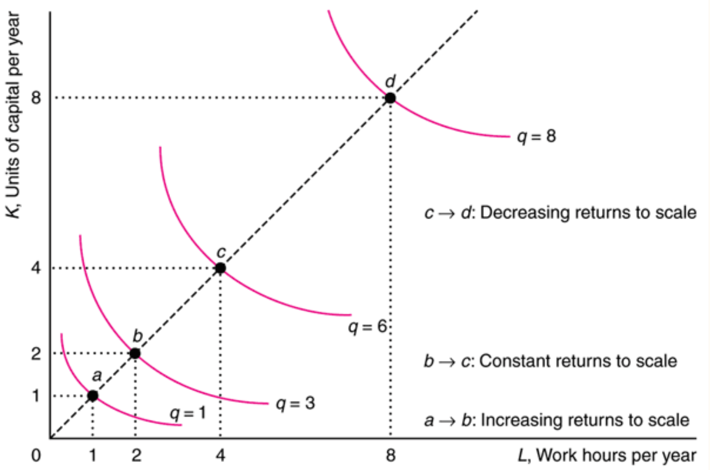
- The following production function is homogenous of degree what?

$$Q = K^{\frac{1}{2}} L^{\frac{1}{4}}$$

Returns to Scale

- It is possible for production functions to have varying returns to scale.
- Could have increasing returns to scale for low levels of production, and decreasing returns to scale for high levels of production.
- For low levels of production, you have gains to specialization and for large levels of production, you run into management problems.

Returns to Scale



Productivity and Technical Change

- Even if two firms are producing efficiency, it is possible that they are not equally as productive.
- We can express a firm's relative productivity by the ratio of the firm's output q to the amount of output the *most productive firm* in the industry could have produced from the same inputs q^*

$$\rho = \frac{q}{q^*} * 100$$

- If you are the most productive firm in the industry, what is ρ?
- Estimated that the average productivity of manufacturing firms in the US is 63% to 99%.

Productivity and Technical Change

- It is possible that one firm can produce more today from a given amount of inputs than it could in the past.
- An advance in knowledge that allows more output to be produced from the same level of inputs is called **technical progress.**
- Can be neutral or non-neutral.

Productivity and Technical Change

- **Neutral technical** change means the firm can produce more output using the same ratio of inputs.

$$q = A(t)f(L, K)$$

- For example

$$\begin{aligned} q_1 &= 10 * K^{.5}L^{.5} \\ q_2 &= 100 * K^{.5}L^{.5} \end{aligned}$$

Productivity and Technical Change

- Or it can be **non-neutral** in which innovations alter the proportion of input used.

$$
\begin{aligned}
q_1 &= 10 * K^{.5} L^{.5} \\
q_2 &= 10 * K^{.5} L^{.8}
\end{aligned}
$$

- If a machine is invented that requires only one person to operate it rather than two, this is a non-neutral labour saving technical change.

Summary

- What is technical efficiency?
- What does a production function show you?
- What is the difference between the short and long run?

Summary

- What causes diminishing marginal returns?
- What is an isoquant
- What is the marginal rate of substitution and the elasticity of substitution?
- What are returns to scale?
- What is the difference between a neutral and non-neutral technical change?