Consumer Theory

Lecture 2

Reading: Perloff Chapter 3

July 2017

Introduction

- In this chapter, we formalize the concept of consumer choice.
- We see how an individual, with certain preferences, makes decisions when faced with a constraint.
- It is the foundation of the demand curve we saw yesterday.

Outline

- **Preferences** - Assumptions we make about how individuals rank different options.
- **Utility** - A convenient way to summarize preferences.
- **Budget Constraint** - What can people afford?
- **Constrained Consumer Choice** - Given income, prices and preferences, what is the best choice I can make?
- **Behavioral Economics** - Do individuals behave in reality how we have assumed?

Preferences

- Individuals prefer certain sets of goods to others
- In order to formalize this abstract concept in a useful way, we need to make a few assumptions about these preferences

Preferences

- But first lets go over some notation.
- a **bundle** (or basket) is simply of a combination of goods and services.
- bundle a is two haircuts and a can of Pepsi.
- bundle b is four cans of Pepsi and no haircuts.

Preferences

- Consumers rank bundles in terms of desirability.
- $a \succsim b$. I like bundle a *at least* as much as b. I *weakly prefer* a to b
- $a \succ b$. I definitely like a more than b. I *strictly prefer* a to b
- $a \sim b$. I like the bundles equally. I am *indifferent* between a and b.

Preferences

- We make three assumptions about preferences. If the first two are met, an individual is considered to be rational.

Completeness

- When facing bundles a and b, the consumer must be able to say either $a \succsim b$, $b \succsim a$, or $a \sim b$.
- In other words, the consumer can rank any two bundles. Rules out the possibility that the consumer can't make a decision.

Preferences

Transitivity

- Here we just mean that preferences are logically consistent.
- If $a \succsim b$ and $b \succsim c$, by the transitive property $a \succsim c$

Preferences

More is Better (Non-Satiation)

- Just like it sounds.
- We are talking about *goods* rather than *bads* and I will always prefer more to less.
- People prefer bundles with strictly more of both goods.
- This really is just here to simplify the thing.

Utility

- **Utility** is a set of numerical values that reflect the ranking of various bundles.
- If basket a yields a higher utility than basket b, we strictly prefer a to b.
- We can summarize preferences using a **utility function.**
- we assign a numerical value to all possible bundles of goods reflecting the consumers rankings of the bundles.

Utility

EXAMPLE

- For two goods q_1 and q_2, lets say our utility function takes the form.

$$U = (q_1 q_2)^{\frac{1}{2}}$$

- Do we prefer a basket with 10 q_1 and 10 q_2 or a basket of 48 q_1 and 3 q_2?

Utility

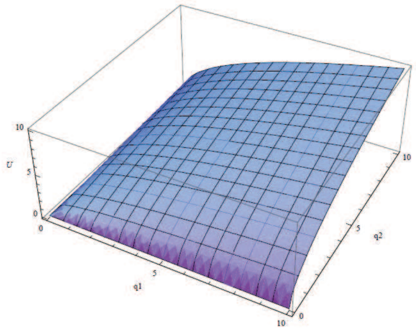
- In practice, it is impossible to quantify these things
- We almost always talk about **ordinal** rather than **cardinal** utility

Utility

- For ordinal, the ordering is all that matters.
- For cardinal, the order AND the magnitude matter.
    - Bob did better on Sam on the test is an ordinal ranking
    - Bob got a 97% and Sam got a 15% is a cardinal ranking.

Utility

- Lets graph $U = (q_1 q_2)^{\frac{1}{2}}$



- This is not incredibly useful. But we can turn this into two dimensions.

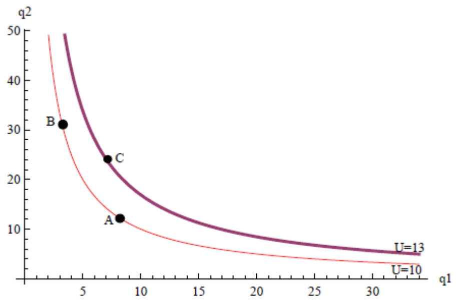
Utility

- An **indifference curve** shows all the bundles that the consumer sees as equally desirable.
- Lets fix a level of utility at $\overline{U}$ and draw all the combinations of of q_1 and q_2 that give the same level of utility.
- For $\overline{U} = 10$, the equation for the indifference curve would be $q_2 = \frac{100}{q_1}$.

Utility

- Any combination on this indifference curve will give us a utility of 10. We are indifferent between any basket on this curve.
- There is an indifference curve associated with each level of utility.
- An **indifference map** shows all the indifference curves.
- As we move from the origin, utility increases.

Utility



- Here, baskets A and B give us the same level of utility because they are on the same curve.
- Basket C gives us a higher utility because it is on a higher indifference curve.
- $c \succ a, c \succ b, b \sim a.$

Utility

EXAMPLE

- Suppose our utility function is

$$U = 2q_1 + 3q_2$$

- Lets draw an indifference curve for $U = 2$ and $U = 3$.

Utility

Properties of Indifference Curves

1. Bundles on indifferent curves farther from the origin are preferred

- This comes from the more is better principle

Utility

Properties of Indifference Curves

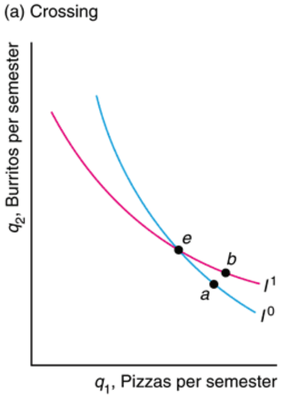
2. There is an indifference curve through every possible bundle

- This comes from the completeness property
- The consumer is able to rank any combination of bundles

Utility

Properties of Indifference Curves

3. They cannot cross

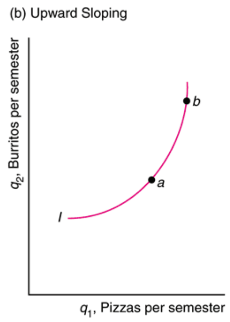


Utility

Properties of Indifference Curves

4. Indifference curves slope downward

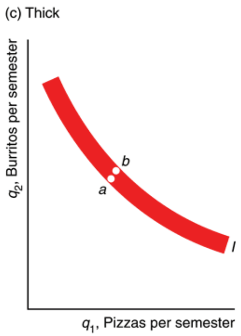
- Otherwise the person would be indifferent between a bundle with more of both goods and less of both goods



Utility

Properties of Indifference Curves

5. They can't be thick
   - violates more is better



Utility

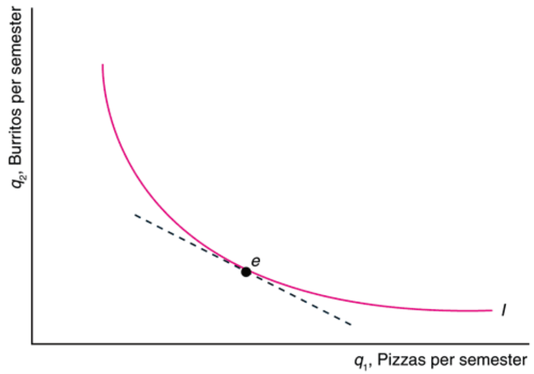
- Of course the bundles you choose between don't have just two goods.
- You can treat one good as a composite good... or spending on everything else.

EXAMPLE

- What would an indifference curve look like for two goods where you get satiated? That is, you only want to consume so much of the good then you start getting sick of it.

- It is useful to know how much of one good the consumer is willing to trade for another.
- This is captured by the slope of the indifference curve... and this is called the **marginal rate of substitution** (MRS)

- The MRS tells us the maximum amount of one good a consumer will sacrifice to get one more of the other
- If your MRS is -2 you would be willing to give up two burritos for one piece of pizza.
- It is simply the slope of the indifference curve.



Utility

- To find the MRS, we need to understand **marginal utility.**
- The marginal utility is the extra utility you get from increasing something by an infinitesimally small amount.

Utility

- The marginal utility of q_1 is just $\frac{\partial U}{\partial q_1}$.
- The marginal rate of substitution is the negative ratio of marginal utilities

$$MRS = -\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}}$$

- What is the intuition behind this?

Utility

EXAMPLE

- What is the MRS for this utility function?

$$U = (q_1 q_2)^{\frac{1}{2}}$$

- What does this mean?
- If you eat 6 units of q_1 and 3 units of q_2, how many q_1 would you give up for a q_2?

Utility

- Most indifference curves are convex to the origin.
- This means that people prefer averages to extremes
- For example...
  - if you have 100 pairs of trousers and no shirts, you would be willing to give up a lot of trousers for one shirt.
  - If you have 100 shirts and no trousers, you would be willing to give up a lot of shirts for one pair of trousers.

Utility

- But they can be other shapes.
- For example, if goods are **perfect substitutes** they have a constant MRS
- You have no desire to consume perfect substitutes together... They do not compliment each other in any way.
- For Coke and Pepsi our utility function might look like $U = 2q_1 + 2q_2$
- They are just straight parallel lines in this case.

Utility

- If goods are **perfect complements**, the indifference curves are right angles
- You only want the goods in fixed proportions
- For right shoes and left shoes it might look like

$$U = \min\{q_1, q_2\}$$

- If I have 1,000,000 right shoes and 1 left shoe, I am no better off than having 1 right shoe and 1 left shoe

Utility

Utility

- They can also be quasi-linear
- If they are linear in one argument
- A quasi-linear utility function would look something like $U = (q_1)^{\frac{1}{2}} + q_2$

Utility

EXAMPLE

- Draw the indifference curves for the following utility functions

$$
\begin{aligned}
U &= x_1 + x_2 \\
U &= x_1 x_2 \\
U &= \min\{x_1, 3x_2\} \\
U &= \max\{x_1, x_2\}
\end{aligned}
$$

Budget Constraint

- Preferences make zero reference to income constraints.
- Given our assumptions on preferences, consumers would like an infinite amount of both goods.
- Obviously, consumers are constrained by the amount of money they have.

Budget Constraint

- We can depict this constraint using a **budget line.**
- The budget line tells us all the combinations of goods a consumer can buy if she spends all her money.

Budget Constraint

- If Y is the consumer's income and p_1 and p_2 are the prices of q_1 and q_2 respectively, the budget line will take the form.

$$
\begin{aligned}
Y &= p_1 q_1 + p_2 q_2 \\
q_2 &= \frac{Y - p_1 q_1}{p_2}
\end{aligned}
$$

- Of course people save and borrow, but we can easily incorporate this by talking about consumption today and consumption tomorrow as two different goods.

Budget Constraint

- We call the **opportunity set** the set of all goods the consumer can possibly buy.

$$Y \geq p_1 q_1 + p_2 q_2$$

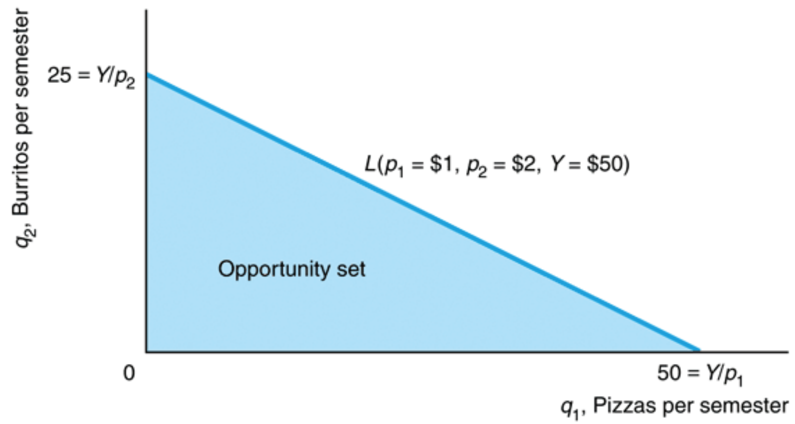
- The budget line is the frontier of the opportunity set.

Budget Constraint

- The slope of the budget line is the **marginal rate of transformation** (MRT)
  - This is how many units of one good the consumer must give up in order to purchase on more of the other.
  - This is just equal to the price ratio

$$MRT = \frac{dq_2}{dq_1} = -\frac{p_1}{p_2}$$

Budget Constraint



- Can you think of a case in which the MRT is not constant?

Budget Constraint

EXAMPLE

- You make $\$100{,}000$ a year
- The price of pickles is $\$5$ and the price of cell phones is $\$100$.
- Draw the budget set, what is the marginal rate of transformation?

Budget Constraint

EXAMPLE

- What happens if the price of pickles doubles to $10?
- What happens if your income doubles to $200,000?

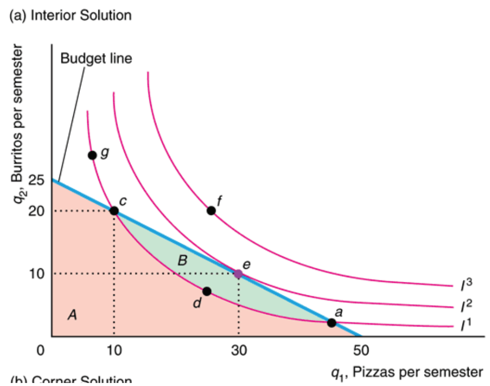
Constrained Consumer Choice

- We can rank baskets by the indifference curves, and we know what we can afford from the opportunity set
- All that is left to do is to find the basket the consumer likes the most that is contained within the budget set.
- We will see how this is done graphically first, then use math.

Constrained Consumer Choice

- First lets show that the optimal bundle lies on the budget constraint.
- If it is outside the budget constraint, the consumer cannot afford it.
- If it is within the opportunity set, the consumer can be made better off by consuming more of both goods (more is better).
- The optimal choice is the bundle on the indifference curve just touching the budget constraint.

Constrained Consumer Choice



Constrained Consumer Choice

- It is possible to have an **interior solution** or a **corner solution**
- An interior solution is one in which the optimal bundle has positive quantities of both goods
- A corner solution is when the optimal bundle contains none of one of the goods

Constrained Consumer Choice

- If there is an interior solution, the indifference curve is *tangent* to the budget constraint at the optimal bundle.
- This means that the indifference curve and the budget line have the same slope.
- The slope of the budget constraint is the price ratio, and the slope of the indifference curve is the marginal rate of substitution.
- The optimal bundle is the one such that.

$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2} = MRT$$

Constrained Consumer Choice

- To get some intuition for why this is the optimal bundle, lets rearrange the thing

$$-\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$$
$$\frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

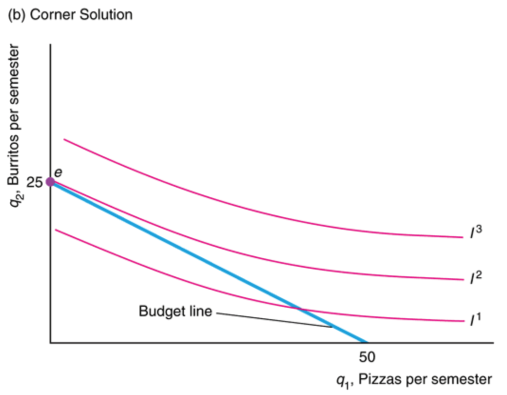
- Think of this as "bang for your buck."
- The utility you get per dollar spent is the same for both goods

Constrained Consumer Choice

- Suppose the marginal utility from pizza is 5 and pizza costs \$1. The marginal utility from burritos is 6 and the price of a burrito is \$3
- We are not at an optimum because $\frac{MU_1}{p_1} > \frac{MU_2}{p_2}$. If I buy 1 less burrito and 3 more pizzas, I will lose 6 units of utility, but I will gain 15 units.

Constrained Consumer Choice

- Sometimes, the highest indifference curve attainable does not occur when the budget line and indifference curve are tangent.
- We have a **corner solution** in this case, and the consumer will only buy one of the goods and none of the other.



Constrained Consumer Choice

EXAMPLES

- Graphically, what will happen to your optimal bundle if the price of a good changes or income changes?
- Graphically show the optimal consumption bundle when the goods are perfect substitutes and when they are perfect complements.

Constrained Consumer Choice

- We can also use math, and we will get the same result.
- Just trying to maximize something subject to a constraint.
- The problem we are trying to solve is.

$$\max_{q_1, q_2} U(q_1, q_2)$$
$$s.t.\ Y = p_1 q_1 + p_2 q_2$$

- That is, we need to pick the q_1 and q_2 that give use the biggest U while not violating the constraint

Constrained Consumer Choice

- If we know that there is an interior solution, we can use two different methods.
- the **substitution method** and the **Lagrangian method.**

Constrained Consumer Choice

- The substitution method entails just plugging the constraint into our objective function.
- We can do this by rearranging the constraint for one of our control variables

$$
\begin{aligned}
Y &= p_1 q_1 + p_2 q_2 \\
q_1 &= \frac{Y - p_2 q_2}{p_1}
\end{aligned}
$$

Constrained Consumer Choice

- Plug this into the utility function. This is now an unconstrained maximization problem

$$
\begin{aligned}
\max_{q_2} U &= \left(\frac{Y - p_2 q_2}{p_1}, q_2 \right) \\
\frac{dU}{dq_2} &= \frac{\partial U}{\partial q_1} \frac{dq_1}{dq_2} + \frac{\partial U}{\partial q_2} = \left(-\frac{p_2}{p_1} \right) \frac{\partial U}{\partial q_1} + \frac{\partial U}{\partial q_2} = 0
\end{aligned}
$$

Constrained Consumer Choice

- Rearranging we get the same condition that

$$
-\frac{\frac{\partial U}{\partial q_1}}{\frac{\partial U}{\partial q_2}} = -\frac{p_1}{p_2}
$$

- To be thorough you would also check the second order condition

Constrained Consumer Choice

- The Langrangian method is similar, but works in a wider variety of cases and this is how you should do things
- We once again turn the constrained maximization problem into an unconstrained one

Constrained Consumer Choice

- The first step is to multiply the constraint by a constant λ and then add it or subtract it from the objective function

$$\max_{q_1,q_2,\lambda} \mathcal{L} = U(q_1, q_2) - \lambda\left(q_1 p_1 + q_2 p_2 - Y\right)$$

- If we were minimizing, we would add the constraint rather than subtract.
- λ is the Lagrangian multiplier - it penalizes us for breaking the constraint

Constrained Consumer Choice

- Take the first order condition with respect to our controls, and then solve the system to get the solution

$$\frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = \frac{\partial U}{\partial q_2} - \lambda p_2 = 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y - q_1 p_1 + q_2 p_2 = 0 \quad (3)$$

Constrained Consumer Choice

- Solve equation 1 for λ

$$\lambda = \frac{\partial U}{\partial q_1} * \frac{1}{p_1}$$

- Plug this into equation two

$$\frac{\partial U}{\partial q_2} - \frac{\partial U}{\partial q_1} * \frac{1}{p_1} p_2 = 0$$

Constrained Consumer Choice

- If we rearrange terms we will get the exact same solution

$$\frac{\frac{\partial U}{\partial q_1}}{p_1} = \frac{\frac{\partial U}{\partial q_2}}{p_2}$$

- This is just the $MRS = MRT$

Constrained Consumer Choice

EXAMPLE

- Use the Langrangian method to find the optimal consumption bundles for the following utility functions

$$U = (q_1 q_2)^{\frac{1}{2}}$$
$$U = ((q_1)^{\rho} + (q_2)^{\rho})^{\frac{1}{\rho}}$$

- (In practice, we know the tangency condition so we can find it right away and plug it into the constraint)

Constrained Consumer Choice

- What if we want to minimize expenditure for some fixed level of utility?
- Rather than fixing the budget line and finding the highest indifference curve, we fix the indifference curve and find the lowest budget line.
- This is the exact same thing... two sides of the same coin
- It can be useful, though because we can observe expenditure but not utility

Constrained Consumer Choice

EXAMPLE

- Show that expenditure minimization gives you the same solution as utility maximization for $U(q_1, q_2)$, and income Y and prices p_1 and p_2.

Behavioral Economics

- We made a number of assumptions on how people behave.
- Behavioral economics looks at some of the departures from our rationality assumptions.
- Experimental Economics (how we see people behave) + Economic Theory = Behavioral Economics

Behavioral Economics

- Transitivity
- Endowment Effect
- Salience
- Bounded Rationality

Summary

- What are the assumptions we make about preferences?
- What is utility? What is a utility function?
- What do indifference curves tell us?
- What is the marginal rate of substitution?
- What is the slope of the budget line?
- How do we find the optimal bundle in general?
- What is behavioral economics?