

THE UNIVERSITY OF EDINBURGH

SCHOOL OF ECONOMICS

MATHEMATICS

SGPE SUMMER SCHOOL 2019

Exam Date: **29 July 2019**

From and To: **09:30-11:30**

Please read full instructions before commencing writing

Exam paper information

- Total number of pages: **8 (not including this cover page)**
- Number of questions: **8**

Special instructions

- Students should answer ALL questions.
- Answers that do not show procedure will not be considered.

Special items

- None.

Examiner(s): **Dr Tatiana Kornienko** (Chair)
Professor Martin Sefton (External)

This examination will be marked anonymously

UoE - SGPE Economics Summer School (2019)
Mathematics Examination

July 2019

Question 1

Let $f(g)$:

$$f(g) = \ln(g^3) \times e^g$$

- Compute $f'(g)$ and $f''(g)$ [5 pt]

Question 2

$$G(x) = \frac{32x^2 + 40\ln(e^x)}{x^{-1}}$$

- Find the critical values for $G(x)$ [5pt]

Question 3

(a) $\lim_{x \rightarrow 1} \frac{\ln(x^x)}{\sqrt{x+8}}$ [4pt]

(b) $\lim_{x \rightarrow \infty} \frac{x^2 + 18x + 81}{x + 9}$ [3pt]

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 8}{x - 64}$ [3pt]

Question 4

(a) Consider the following matrix A :

$$A_{(m \times n)} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Find: A^T [3pt], where A^T is the transpose of the matrix A . Also, find $(A^T)^T$ [2pt].

(Hint: You don't need to write every element in the matrix, a matrix with a number of explicit elements similar to that of matrix A , will suffice.)

(b) Consider the following matrices:

$$Q_{(2 \times 3)} = \begin{bmatrix} 1 & 0 & 3 \\ 8 & 4 & -1 \end{bmatrix}$$

$$L_{(3 \times 2)} = \begin{bmatrix} 3 & 4 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Find: QL [3pt], where QL is the product of both matrices. Also, find $(QL)^{-1}$, where $(QL)^{-1}$ is the inverse of QL [2pt].

Question 5

(a) Consider the following production function for a manufacturing company:

$$Q = 5L^{0.5}K - L$$

Where, Q represents units produced, L represents labour and K represents capital.

If capital is fixed at 10 units, find the number of employees that maximizes production [5pt].

[**Hint:** Production is maximised when the marginal product of labour (MPL) is equal to zero. The MPL is found by calculating $\frac{\partial Q}{\partial L}$]

(b) The company will follow your advice and hire an additional 100 employees to reach the target specified for production maximization. The company is short on cash and requires a loan of 10,000£. The company requests a loan of 10,000 pounds today and the interest rate for the loan is r . Find the interest rate r if the total amount owed after ($n=10$) years is equal to 200,000 pounds (Note: The sum was continuously compounded for n years.). [5pt]

Question 6

Solve the following linear system of equations [10pt]:

$$3x - 3y + 2z = -21$$

$$21x - y = -7$$

$$7x + 4y - 3z = 10$$

$$30x - 10y + 6z = -70$$

Question 7

(a) Graph the following equation:

$$y = -0.2x + 7 \text{ [3pt]:}$$

(b) Determine which of the following relations are functions. Specify the domain and range. Also mention if the function is continuous.

(1) $y^2 = x + 2$ [2pt]:

(2) $y = \ln(x)$ [2pt]:

(c) Calculate the following integral [3pt]:

$$\int x(x+1)^{\frac{3}{2}} dx$$

Question 8

Consider $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{1}{\sqrt{x}}$.

Find the general expression for the k^{th} derivative $f^{(k)}(x)$ [7pt]. Also, find the 5^{th} derivative $f^{(5)}(x)$ [3pt]:

[**Hint:** Write the general expression first and use it to compute the 5^{th} derivative. The notation \prod is used to represent the product of a sequence, for example: $\prod_{i=0}^n (x - i) = (x - 0) \times (x - 1) \times \dots \times (x - n)$]

[End of examination]