

**THE UNIVERSITY OF EDINBURGH**

**SCHOOL OF ECONOMICS**

**MATHEMATICS**

**SUMMER SCHOOL**

Exam Date: **31<sup>st</sup> July 2017**

From and To: **09:30-11:30**

**Please read full instructions before commencing writing**

**Exam paper information**

- Total number of pages: **6 (including this cover page)**
- Number of questions: **9**

**Special instructions**

- Please ensure you correctly fill out the front page of the answer book and indicate which questions you have chosen.

**Special items**

- None.

**Examiner(s):** **Dr Tatiana Kornienko** (Chair)  
**Professor Martin Sefton** (External)

This examination will be marked anonymously

**Question 1**

Let  $G(x)$ :

$$G(x) = \frac{3x^2 - 27\ln(e^x)}{x^{-2}}$$

- Compute  $G'(x)$  and  $G''(x)$  [6 pt]
- Find the critical values of  $G(x)$ . [4pt]

**Question 2**

Evaluate the following limits:

(a)  $\lim_{x \rightarrow 1} \frac{x^2 - 8x + 15}{x - 3}$  [2pt]

(b)  $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$  [3pt]

(c)  $\lim_{x \rightarrow 2} \frac{\ln(e^{x-2})}{x^2 - 4}$  [3pt]

(d)  $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$  [3pt]

**Question 3**

(a) Find the solution for the quadratic expression [3pt]:

$$x^2 - 8x = 1$$

(b) Solve the system of equations [5pt]:

$$q - 2m + 3z = 10$$

$$2q + m + z = 7.5$$

$$-3q + 2m - 2z = -20$$

(C) Find the derivative [4pt]:

$$\frac{\partial}{\partial x}[\sqrt{4x^5 - x^2}]$$

**Question 4**

Calculate the following:

(a)  $\int_2^4 x^2 dx$  [3pt]

(b) Graph the integral (area under the function) in part (a) on the X,Y plane. [5pt]

(c)  $\int_{-6}^6 (13x^3 + 10\sqrt{x} + 12) dx$  [4pt]

### Question 5

A box manufacturer, hires you to help them figure out the optimal cost of their new product. The product is a rectangular container with an open top that requires a volume capacity of  $20m^3$ .

The length of the base is twice the width of the box and the materials for the base cost 20 pounds per square meter. The materials for the sides cost 12 pounds per square meter.

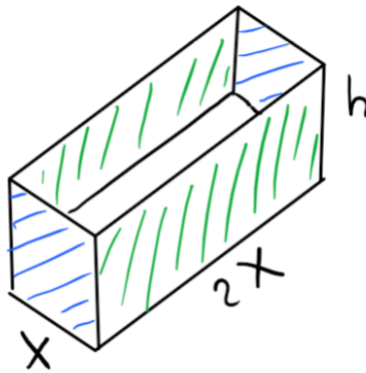


Figure 1: Open Box

- Find the cost function in terms of  $(x)$  [7pt]
- Find if the function has a minimum or a maximum [3pts]
- Find the optimal cost of materials for the new product development [5pts]

**Question 6**

**Find:**  $\iint_D (4xy - y^3) dA$ , **D** is the region bounded by the functions  $y = \sqrt{x}$  and  $y = x^2$  [10pt]:

**Question 7**

**Consider the following matrix A:**

$$A = \begin{bmatrix} 7 & 5 \\ 6 & 14 \end{bmatrix}$$

**Find:**  $A^{-1}$  [5pt];  $\det(A^{-1})$  [2pt];  $A^{-1} * I_2$  [3pt], where  $I_2$  is the identity matrix of size  $[2 \times 2]$ .

**Question 8**

You ask for a loan of 100 pounds today. The interest rate for the loan is  $r$ . Find  $r$  if the total amount owed after ( $n=10$ ) years is equal to 2000 pounds (Note: The sum was continuously compounded for  $n$  years.). [10pt]

**Question 9**

A factory in China produces smart phones according to the following revenue and cost functions:

$$R(x) = 25x$$

$$C(x) = x^3 - 9x^2 + 35x$$

Build the profit function ( $\pi$ ) and find the optimal amount that the firm should produce in order to maximize their profit.[pt 10]

—————END OF THE EXAM—————