Unemployment and Endogenous Reallocation over the Business Cycle[*†]

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Abstract

This paper builds an analytically and computationally tractable stochastic equilibrium model of unemployment with heterogeneous labor markets. Unemployment is caused and affected by search frictions within markets and reallocation frictions across markets. We use this model to study quantitatively the relation between heterogeneity in labor market conditions across occupations, the cyclical patterns of unemployed workers' occupational (im)mobility, and overall aggregate fluctuations in unemployment. Empirically, using the 1986-2008 SIPP panels, we document the occupational mobility patterns of the unemployed, finding notably that occupational change of unemployed workers is procyclical. Theoretically, we find also the latter to be the constrained efficient pattern. Calibrating the heterogeneous-market model yields highly volatile countercyclical unemployment, and is simultaneously consistent with procyclical reallocation, countercyclical separations, a clear Beveridge curve, and unemployment duration dependence. Due to the model's tractability, we can derive many of these results analytically. We decompose unemployment into the underlying search, reallocation and rest components.

Keywords: Unemployment, Business Cycle, Search, Reallocation.

JEL: E24, E30, J62, J63, J64.

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1 Introduction

The Great Recession has revived an important debate about the extent and nature of the misallocation of unemployed workers across labor markets. Central to this debate is the notion that some labor markets offer better employment prospects than others and that some unemployed workers would benefit if they were to reallocate. In this paper we investigate to what extent unemployed workers are able to change markets in response to changes in aggregate or local conditions. Do they gain by doing so? How much does this reallocation affects aggregate unemployment fluctuations? Do the observed reallocation patterns coincide with, or are close to, those preferred by society?

To answer these questions we build a tractable business cycle model of heterogeneous labor markets where unemployed workers change markets endogenously. Aggregate unemployment can be decomposed into search, reallocation and rest unemployment. Search unemployment arises as it takes time to find suitable jobs in a given labor market. Rest unemployment occurs when there are no jobs available, but workers wait in their labor markets for conditions to improve and jobs to arrive. Reallocation unemployment arises as workers transit between labor markets in the hope for better job opportunities. It is a novelty of the paper to explicitly consider, in an equilibrium framework with aggregate shocks, workers' decisions to reallocate and search for jobs in a different labor market or stay searching for jobs in their current labor market. We show that our model is able to reproduce several important features that characterise the US labor market. Namely, procyclical worker reallocation through unemployment, countercyclical job separations, a strong negative correlation between unemployment and vacancies, and a high cyclical volatility of unemployment and vacancies.

Our approach is motivated by new evidence for the US economy, showing that the cyclical behaviour of aggregate unemployment and the job finding rate is strongly influenced by the reallocation of workers across labor markets, which we operationalize by occupations. We use the Survey of Income Program Participation (SIPP) for the period 1986-2010 to document the extent and the cyclical properties of occupational mobility *through unemployment*. Our findings show that the monthly job finding rate with "major" occupational change accounts for more than 50 percent of the aggregate job finding rate. Although the monthly probability of finding a job in a different occupation is slightly lower than that of finding a job in the same occupation (occupational mobility takes relatively longer), on average 54.5 percent of unemployed workers will be re-hired in a different "major" occupational group.[1] Unemployed workers who change occupation will also have higher re-employment wages, experience longer employment spells and have a 35 percent chance of changing occupation once again if they experience a subsequent unemployment to employment transition. We also show that unemployed workers find it more profitable to change occupations in economic upswings, when jobs are plentiful and the probability of finding a job in a different occupation is higher.

In line with this evidence our theory combines the ideas originally set out by Lucas and Prescott (1974), and more recently by Alvarez and Veracierto (1999), and Mortensen and Pissarides (1994).

[1]The importance of occupational mobility through unemployment is also stressed in Longhi and Taylor (2011) for the UK. Using the Labour Force Survey for the period 2001Q2-2010Q1, they find that on average 57 percent of unemployed job seekers found a job involving a "major" occupational change.

It considers an island economy in which individual islands are subject to idiosyncratic and aggregate productivity shocks each period. In each island unemployed workers can decide to (i) search and apply for existing job opportunities, (ii) become rest unemployed and wait for jobs to arrive to the island or (iii) to reallocate to another island.[2] Employed workers can decide to separate from their employers and become unemployed. Within each island search frictions are modelled through a matching function that governs the meeting process of workers and firms. Across islands reallocation frictions are modelled through a time consuming and costly reallocation process. Workers who move islands pay an explicit reallocation cost and remain unemployed during reallocation. Further, we assume that the new island is a random draw from a subset of islands across the economy (see also Alvarez and Veracierto, 1999).

Though our economy is subject to both aggregate shocks and island-specific shocks, the model stays tractable and is easily computed, because the equilibrium in the labor market studied has a block recursive structure. This means that equilibrium mobility decisions, i.e. vacancy posting decisions which determine job finding probabilities, and separation and reallocation decisions, are only dependent on the aggregate and idiosyncratic productivity states.[3] We show existence and uniqueness of an equilibrium which such properties. We also show that this equilibrium is efficient when the social planner faces the same search and reallocation frictions as agents in the decentralised economy. As a result, the aforementioned equilibrium decisions can be derived using a simple contraction mapping. Without this structure, computation becomes very involved, and as a result studies with heterogeneous frictional markets have mostly been confined to steady state analysis (Lkhavasuren, 2011), or have abstracted from endogenous mobility between markets (Shimer, 2007, and Mortensen, 2009).

The model's parsimony and block recursive property also allow for the analytical derivations of implications in terms of two functions of aggregate productivity. This allows us to gain additional insight into the forces at work in the model. For each aggregate productivity, there is a reservation island productivity below which workers decide to reallocate, z^r, and a reservation island productivity below which workers decide to separate from their jobs, z^s. When $z^s > z^r$, rest unemployment occurs along side search and reallocation unemployment. Search unemployment occurs in islands with idiosyncratic productivities above z^s. Rest unemployment occurs in islands with idiosyncratic productivities between z^r and z^s. In these islands the state of the labor market is sufficiently depressed for firms not to post vacancies but is not bad enough for workers to decide to reallocate. Workers decide to stay in their islands and wait for conditions to improve and jobs to arrive. Worker reallocation occurs from islands with productivity below z^r. When $z^s < z^r$, however, rest unemployment does not arise. Unemployment in this case is only due to search and reallocation.

[2]In our model rest unemployment is very similar to the type of unemployment that arises in stock-flow matching models. See for example, Coles and Smith (1998) and Ebrahimy and Shimer (2010). In the stock-flow literature workers that have not matched with the inflow of vacancies wait until new vacancies appear in their labor market.

[3]Our model becomes very tractable as we do not have to keep track of the distribution of employed and unemployed workers across islands to determine wages and employment probabilities. Instead, we can first solve for decisions, and then use these decision rules to update the distribution of employed and unemployed workers, block recursiveness. Menzio and Shi (2011) were the first to formally apply this concept to solve a directed search model of the business cycle that considers on-the-job search with an infinite dimensional state space.

The co-existence of rest, search and reallocation unemployment is crucial for the model to generate three empirical business-cycle regularities: (i) procyclical worker reallocation through unemployment, (ii) countercyclical job separation into unemployment, and (iii) countercyclical unemployment with a high cyclical volatility. The cyclical patterns of search, rest and reallocation unemployment follow in part from the behaviour of the reservation productivities as a function of aggregate productivity $z^s(p), z^r(p)$, and their position relative to each other, as explained above. The other part is formed by the distribution of employed and unemployed workers over islands, which summarizes the history of past shocks and previous decisions characterized by $z^s(p), z^s(p)$. As a result, we gain a lot of insight by studying the forces affecting the position and slopes of these two lines: e.g. procyclical reallocation is closely related to an upward-sloping $z^r(p)$, while downward-sloping $z^s(p)$ relates very closely to countercyclical separations.

We show that search frictions within each island alter workers' reallocation decisions in response to aggregate productivity shocks. Search frictions within each island always induces more procyclical reallocation of workers than under perfect competition. In particular, without technological complementarity between aggregate and island-specific productivity, reallocation can still be procyclical as a result. Further, our theory shows that the procyclicality of worker reallocation is driven by procyclical movements in two dimensions: wages and job finding rates rise in good times. The wage gain, importantly, is proportional to the competitive markets benchmark, but additionally workers enjoy a benefit in faster job arrival rates in desirable markets. This adds an additional dimension of gains to reallocation in good times, and makes the procyclicality of reallocation stronger. It illustrates that instead of comparing instantaneous production flows, to perhaps conclude that recessions are a good time to reallocate because no market production is lost, one should (also) compare job arrival rates for the unemployed. For modular or supermodular production functions, we find that good times are also better times to reallocate. This arises because job finding rates in better islands go up more than in islands close to the reallocation margin. Loosely, in good times it will be easier to find a job on a 'marginal' island, but much, much easier to find a job in better islands.

The interaction between reallocation and search frictions also has implications for the cyclical properties of job separations. We highlight the tension that exists between generating procyclical reallocations and countercyclical separations. When rest unemployment occurs job separations are countercyclical as workers in islands with productivity $z = z^s > z^r$ do not consider reallocating when making separation decisions. However, when $z^s < z^r$, a positive aggregate productivity shock can induce procyclical separations if the benefits of reallocation are sufficiently high. We show that the latter occurs when the production function exhibits a sufficiently high degree of supermodularity.[4]

When aggregate unemployment has the search, rest and reallocation components, shifts in its composition over the business cycle increase its cyclical volatility. In bad times a higher proportion of islands have no vacancies and hence a zero labor market tightness. In these islands unemployed

[4]The interaction between reallocation and separation decisions relates to the discussion about the "cleansing" and "sullying" effects of recessions. See Mortensen and Pissarides (1994), Caballero and Hammour (1994) and Barlevy (2002). Although in this model we do not consider on-the-job search, procyclicality of workers flows across islands would help resource reallocation in expansions, very much in the same way as job-to-job transition do in Barlevy (2002).

workers switch from searching for jobs and reallocating to waiting for jobs to arrive; while those employed workers who lost their jobs also become rest unemployed. In good times the opposite happens, firms post vacancies in a higher proportion in these islands and unemployed workers move from waiting to searching and reallocating; and those newly unemployed are less likely to become rest unemployed. These shifts amplify the response of aggregate unemployment to productivity changes compared to the canonical search and matching model because firms are now able to choose in which markets to post vacancies depending on the state of the economy.

To quantitatively evaluate our model's implications we calibrating it to match long run features of the US labor market based on the SIPP for the period 1986-2010.[5] The calibrated model fits the data well on several dimensions. First, the model generates procyclical reallocations and a countercyclical aggregate unemployment. This feature is important as in the Lucas and Prescott framework both series would have the same cyclical patterns. Second, the model is also able to reproduce a Beveridge curve that resemble the empirical one quite well and at the same time generate a countercyclical separation rate. Further, it accounts for a significant proportion of the volatility of unemployment, vacancies and labor market tightness. These features are also important since most of the extensions to the canonical search and matching model, as described in Pissarides (2001), are able to reproduce some but not all of these features at the same time.

Our framework also provides a simple decomposition of unemployment into its search, rest and reallocation components and allows us to construct an index similar to that of Jackman and Roper (1987) to measure the extend of mismatch unemployment. We apply our unemployment decompositions to the calibrated model. This exercise shows that most occupations experience search unemployment. However, rest unemployment is more prominent among workers with high levels of occupational human capital, while reallocation unemployment is more prominent among workers with low levels of occupational human capital. We also show that rest unemployment decreases faster than search and reallocation unemployment when the economy expands. Our mismatch measure, on the other hand, compares the per period difference between unemployment in each island to a long run measure of unemployment based on the ergodic distribution of island specific shocks. This unemployment rate is consistent with the one generated by the canonical search and matching model. The index shows that mismatch unemployment is countercyclical.

The rest of the paper is organised as follows. After a brief review of related literature, we present evidence on worker reallocation through unemployment in Section 2. In Section 3 we set out the model and describe the decision problems of workers and firms considering the complete state space. In Section 4 we define and analyse block recursive equilibria. Here we show existence, uniqueness and the efficiency properties of such an equilibrium. Section 5 discusses the implications of the model. Section 6 presents the extension based on occupation-specific human capital. Section 7 considers the quantitative analysis of the model. Section 8 concludes. All proofs are relegated to the Appendix.

[5]A novel feature of our calibration strategy is to use the aggregate unemployment duration survival function observed in the data to recover the parameters that govern the island specific productivity process. Since in our model there is a negative relation between the island specific productivity and its unemployment duration, it provides a tight link between island specific productivity process and the aggregate unemployment duration survival function.

1.1 Related Literature

The coexistence of unemployment and vacancies has at least two widely accepted explanations. On the one hand, search frictions prevent unfilled jobs and unemployed workers from finding each other. On the other hand, reallocation frictions hinder the free movement of unemployed workers and unfilled jobs across labor markets leading to mismatch. These two lines of explanation have been investigated mostly in isolation. The Diamond-Mortensen-Pissarides framework, for example, only considers unemployment that arises from search frictions within a single (aggregate) labor market (Pissarides, 2001). Alternatively, island models *á* la Lucas and Prescott (1974) study unemployment patterns induced by reallocations across labor markets and by the resting behaviour of workers.[6] The present paper combines these two frameworks to analyse the behaviour of aggregate unemployment, its search, rest and reallocation components and of mismatch unemployment over the business cycle.

In the tradition of the Lucas and Prescott (1974) framework, our paper is closest to Veracierto (2008) which considers a business cycle version of Lucas and Prescott (1974) with random reallocation across islands. A crucial feature of this framework is to assume that the labor market within an island is competitive and reallocation frictions are the only source of market imperfection. Further, as the worker becomes unemployed during the reallocation process, this friction is the only driving force behind aggregate unemployment. Lucas and Prescott (1974), and others using their framework, refer to the latter as search unemployment. Here, however, we make the distinction between unemployment due to frictions within and across islands. Furthermore, Veracierto (2008) shows that, under reasonable parameter values, a real business cycle model that only considers reallocation frictions generates procyclical unemployment, a counterfactual implication. By introducing search frictions within islands, a worker who decides to reallocate to a desirable island will not find a job immediately. This adds an additional margin that helps towards generating countercyclical unemployment. Indeed, our calibration shows that reallocations across islands are procyclical *and* unemployment is countercyclical as observed in the data.

Gouge and King (1997) also point out the inability of the Lucas and Prescott framework to generate countercyclical unemployment (see also Jovanovic, 1987). They consider the Lucas and Prescott model with a two state aggregate and idiosyncratic productivity shock process and introduce rest unemployment within islands. They show that their model can generate procyclical reallocations, while also countercyclical unemployment flows. There are some important difference between our papers. Although Gouge and King only hint about what would happen if each island's labor markets exhibited search frictions, they do not provide a full analysis of its implications as we do in this paper. Further, to preserve tractability, these authors only consider a very simple productivity shock processes. We are able to show existence and uniqueness of equilibrium and prove its efficiency by requiring both productivity process to be Markovian and the island productivity shock process to show some persistence in the form of stochastic dominance. Finally, we provide a quantitative evaluation of the model, while Gouge and King only consider the qualitative properties.

Alvarez and Shimer (2011) extend in an elegant way the Lucas and Prescott framework to study

[6] Using this framework Shimer (2007) develops his model of mismatch. Jovanovic (1987), Hamilton (1988), Gouge and King (1997), Albrecht, Storesletten and Vroman (1998) and Alvarez and Shimer (2011) introduce rest unemployment within island, where workers decide not to search, but wait until the state of their labor market improves.

rest and reallocation unemployment in a steady state environment.[7] There are several important difference with our paper. The focus of the two papers is different. Alvarez and Shimer (2011) stress the link between the dynamics between industry level wages and steady state unemployment, while we focus on the behaviour of unemployment over the business cycles. Further, we consider a model that encompasses three types of unemployment simultaneously: search, rest and reallocation unemployment. As we show adding search unemployment is important both theoretically and quantitatively in explaining the cyclical properties of aggregate unemployment.

Lkhagvasuren (2011) also considers the interaction between reallocation and search frictions in a similar setup as ours. His analysis is focused on explaining the coexistence of large difference in the unemployment rates across US states (the operationalisation of islands in his model) and large reallocation flows between them. His model is a steady state model, the specifics of his setup do not allow the type of easily computable equilibrium that we show to exist in this paper, and hence computational concerns do not allow for an investigation of the behaviour of local labor markets over the business cycle.

Kambourov and Manovskii (2009b) consider steady state model based on the Lucas and Prescott framework with random reallocation and competitive labor markets to analyse the effects of occupational mobility on the increase in wage inequality in the US.[8] In particular, they document the importance of occupation specific human capital in explaining such an increase. Motivated by these authors we extend our model to incorporate accumulation of occupation specific human capital. The main difference between our papers, however, is three-fold: (i) we allow for frictions within the labor market attached to an occupation; (ii) we consider a business cycle analysis; and (iii) our emphasis is on the behaviour of the unemployment rate.

In the tradition of the Diamond-Mortensen-Pissarides framework, Shimer (2005) and Costain and Reiter (2008) have shown the inability of the canonical search and matching model to reproduce the observed volatility of unemployment and vacancies at business cycle frequency. Further, Shimer (2005) also show that the model is unable to generate a strong negative relation between unemployment and vacancies when separations are countercyclical. Since these contributions, there has been a large literature that extends the canonical model to try to reconcile it with the data. However, most of these papers have been unable to reproduce all these features at the same time.[9] Our analysis shows that by dropping the assumption of a single labor market and considering rest and reallocation unemployment along side with search unemployment, the model is consistent with a high volatility of unemployment and vacancies, countercylical separations and a strong negative relation between unemployment and vacancies.

[7] Alvarez and Shimer (2011) label reallocation unemployment as search unemployment, following the Lucas and Prescott tradition.

[8] See also Wong (2011), who extends Kambourov and Manovskii (2009b) analysis and consider the connection between occupational mobility, wage inequality and aggregate shocks. In his model there is no human capital accumulation, but the cost of reallocating to a new occupation is a function of the skills required by different occupations and is increasing with skill differentials.

[9] For a recent exception see Coles and Moghaddasi (2011), who show how doing away with the free-entry condition allows the model to be consistent with the above empirical features, albeit they assume a stochastic exogenous job separation process.

2 Patterns of Worker Reallocation Through Unemployment

In this section we present evidence on the extent and cyclical behaviour of the occupational reallocation of the unemployed. For this purpose we use the Survey of Income Program Participation (SIPP) for the period 1986 - 2010. The SIPP, administrated by US Census Bureau, provides demographic data on reasonably large number of individuals of all ages at a moment in time, and follows them typically for 2.5 or 4 years, depending on the panel, while keeping track of the individuals' labor market status, including workers' occupations and matches with firms. From this sample, we consider all workers between 16 and 65 years of age, not taking into account spells of self-employment, in government employment or in the armed forces. In Appendix D we provide further details of the data used, and its construction; here we present the main results.

2.1 Unemployment Transitions and Occupational Mobility

Let us first look at the patterns of unemployment transitions with and without occupational changes, averaged over the entire sample period. We divide unemployment spells that are completed within the sample into two groups: those that end with employment in the same occupation as before, and those that end with a transition to employment in a different occupation. After becoming unemployed, on average 54.5% will be rehired in a different major occupational group (two-digit occupation). If one makes finer distinctions between occupational categories, the corresponding numbers are 63.6% in a different minor occupational group (three-digit occupation), and 71.6% of workers in a different broad (four-digit) occupation (from the Census' Occupational Classification).[10]

The proportion of unemployed who start new employment in a different occupational category than their previous employment declines with age. For brevity, we will simply refer to these unemployed as 'occupational movers', and their counterparts as 'occupational stayers'. In table **??** we see that the decline in the proportion of occupational movers occurs for two-, three- and four-digit occupational categories. Similarly, we see that this patterns is preserved for subgroups of the population, when we e.g. stratify our sample by education and gender. In general, patterns in these subgroups are very similar to the aggregate patterns, though it is interesting to note that the decline with age in the proportion of occupational movers is slightly less pronounced for women and the college educated.

The monthly outflow to employment from the subset of unemployed who (eventually) are occupational movers is lower by 4%-5% percentage points than the corresponding outflow from the subset of occupational stayers; this pattern is again robust across occupational categorization **??**.

Interestingly, the difference between the outflow probability of the subset of unemployed occu-

[10]The SIPP uses the Standard Occupational Code (SOC); there are about 20 "major groups" of occupations, about 100 "minor groups", and about 500 "broad occupations", which the SIPP aggregates into somewhat more coarse categories. The panels from 1986 up to 2001 use the 1980 or 1990 SOC classifications, which differ only slightly between them. The 2004 and 2008 panels use the 2000 SOC classification, which differs more substantially from the previous classifications. Since we find there to be continuity in both the levels and cyclical patterns, we consider the full 1986-2010 period as our benchmark. At each step, we calculate a separate set of statistics spanning the 1986-2001 panels for robustness purposes, but have not find substantial differences, unless explicitly noted. Additional information about these classifications can be found at `http://www.census.gov/hhes/www/ioindex/faqs.html`.

Broad occupations (4digit)	all	male	female	high school	college
age $\leq$ 30y	0.769	0.770	0.768	0.778	0.765
30 < age $\leq$ 50	0.664	0.637	0.707	0.680	0.685
50 < age $\leq$ 60	0.646	0.620	0.687	0.654	0.724
all working ages	0.716	0.703	0.736	0.729	0.706

Minor Occupational Groups (3digit)	all	male	female	high school	college
age $\leq$ 30y	0.682	0.688	0.672	0.695	0.692
30 < age $\leq$ 50	0.594	0.565	0.640	0.608	0.617
50 < age $\leq$ 60	0.570	0.554	0.589	0.580	0.661
all working ages	0.636	0.626	0.652	0.65	0.627

Major Occupational Groups (2digit)	all	male	female	high school	college
age $\leq$ 30y	0.593	0.609	0.568	0.606	0.594
30 < age $\leq$ 50	0.500	0.492	0.510	0.515	0.510
50 < age $\leq$ 60	0.476	0.480	0.470	0.489	0.520
all working ages	0.545	0.551	0.536	0.56	0.536

Table 1: Proportion of completed unemployment spells ending with an occupation change

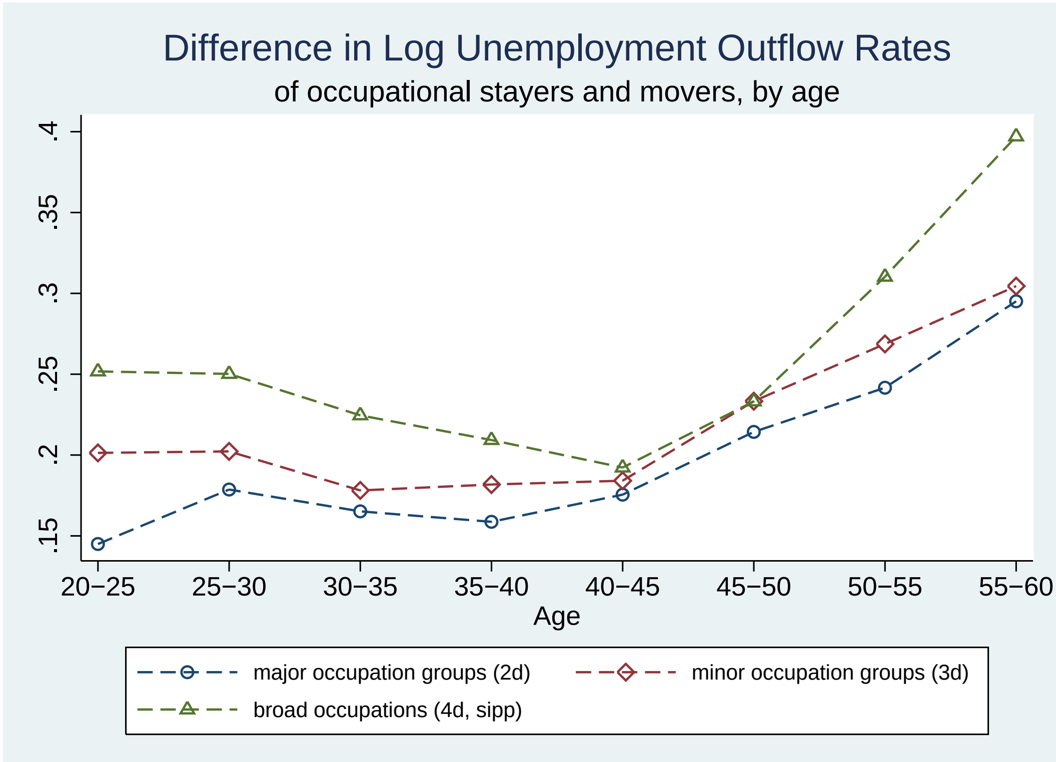
	movers	stayers
major occ. groups	0.264	0.305
minor occ. groups	0.265	0.315
broad occupations	0.266	0.327

Table 2: Monthly outflow rate relative to stock of unemployed occupational changers, resp. stayers

pational stayers (into employment) and the outflow probability of unemployed occupational movers tends to be higher at higher age. At the two- and three-digit level in particular, this difference starts rising after age 30, with, overall, a mild U-shape over the entire working life.

Post-reallocation labor market outcomes Workers who ended their unemployment spell with a transition to employment in a different occupation have different outcomes than those who remain in the same occupation. We document three such differences: in the proportion that will end a subsequent unemployment with or without an(other) occupational transition; in the outflow rate of their respective unemployment category when they re-enter unemployment, and in the observed wage difference conditional on occupational moving or staying.

First, focussing exclusively on the major occupation (two-digit SOC) groups, we find that of those unemployed who end their spell with an transition to a job in the same occupation group cover 46.5% of all unemployment spells with occupational information. Workers who are observed to re-enter unemployment after an unemployment spell which ended with a job in the same occupation as the job before their first unemployment spell, are observed with a job in the same occupation



after the second unemployment spell in 71.8% of the cases. Likewise, workers who have exited their first observed unemployment spell to a different (two-digit) occupation, and have re-entered unemployment subsequently, are observed to end their second unemployment spell with a transition to a job in the same occupation as the one immediately preceding the second unemployment spell in 64.5% of the cases. Thus, both occupational mobility and the lack thereof seem to hand in hand with *subsequent* persistence.

Secondly, the outflow rates of unemployment vary with occupational mobility at the end of the previous unemployment spell. In table **??**, we document that the unemployment outflow rate is higher for the category of unemployed worker who will accept a job in the same occupation, and who previously also stayed in the same occupation; or, especially when younger, when previously ending penultimate unemployment spell with an occupational change.

	all	young	prime	old
occ. stay after occ. stay	0.317	0.344	0.316	0.259
occ. stay after occ. move	0.332	0.370	0.295	0.287
occ. move after occ. move	0.306	0.316	0.293	0.281
occ. move after occ. stay	0.286	0.323	0.259	0.236

Table 3: Re-employment rates of the repeat unemployed, by initial and subsequent occupational moving and staying

We can similarly investigate wage patterns after unemployment spells that end with an occupational change (relative to their previous job) and those that do not. Table **??** considers the average of the monthly median re-employment wage changes for workers that experienced an unemployment spell leading to either an occupational or a non-occupational change.[11] The monthly median re-employment wage growth is overall negative. However, young workers (those with ages between 16-30) have a positive re-employment wage growth. Further, re-employment wage growth is higher when the workers ended their unemployment spell with an occupational change than when they stayed in the same occupation. This feature is prevalent at an aggregate level and for young and prime age workers. These numbers suggest that workers gain with occupational mobility. Further these patterns survive when we consider the average median re-employment wage changes based on the entire panel rather than in each month. [12]

Major Occupational Groups (2 digits)

Age Group	Aggregate	Occupational Change	Non Occupational Change
All	-6.060	-1.378	-7.773
Young	2.223	10.106	0.128
Prime	-9.102	-7.648	-10.061
Old	-14.357	-15.998	-10.456

Broad Occupational Category (4 digits)

Age Group	Aggregate	Occupational Change	Non Occupational Change
All	-6.742	-0.416	-11.024
Young	2.559	9.281	-2.052
Prime	-9.341	-5.774	-11.635
Old	-13.710	-9.856	-9.464

Table 4: Monthly median re-employment wage changes (%)

We will discuss our interpretation of these findings in section **??**.

2.2 Business Cycle Patterns

To measure the extent of occupational mobility through unemployment we compare the reported occupation at re-employment with all those occupations the individual had performed in past jobs. An occupational change then occurs when the individual performs an occupation that has not been observed before. This measure is more robust than just comparing the new occupation with the immediately previous one as it reduces spurious occupational mobility caused by coding error. It

[11]The wage change is computed using the average wage earned in the job immediately prior and the average wage earned in the job that followed immediately after the unemployment spell. These results are based on the 1996 - 2008 panels as they provide more reliable estimates of re-employment wage changes due to their larger sample size. As from 1996 the sample size of each panel increased from 15,000 to 40,000 individuals.

[12]When considering all the panels i.e. 1986-2008, the results also show that the overall average monthly median and panel median are higher for workers that change occupation and for the young group. However, for the prime age workers re-employment wage growth is slightly higher for those that did not change occupation.

Table 5: Job Finding Rates and Occupational Change for all Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.158	0.072	0.081	0.313	0.523	0.379	0.477	0.008	0.056		
Std. Dev	0.093	0.165	0.101	0.101	0.069	0.095	0.076	0.125	0.129	0.009	0.016
Autocorr.	0.810	0.775	0.775	0.763	0.763	0.785	0.765	0.824	0.916	0.691	0.871
Correlation Matrix											
frate	1.000	0.420	0.715	0.585	0.188	0.630	-0.196	-0.271	-0.725	0.297	0.658
focc		1.000	-0.017	0.641	0.735	0.637	-0.727	-0.733	-0.649	0.468	0.488
fnocc			1.000	0.121	-0.310	0.249	0.313	0.102	-0.409	0.037	0.310
Pocc				1.000	0.426	0.875	-0.415	-0.724	-0.721	0.566	0.727
Cocc					1.000	0.482	-0.989	-0.564	-0.371	0.320	0.285
Pnocc						1.000	-0.471	-0.662	-0.763	0.471	0.653
Cnocc							1.000	0.538	0.360	-0.314	-0.279
Srate								1.000	0.708	-0.647	-0.648
Urate									1.000	-0.521	-0.816
Outpw										1.000	0.828
Output											1.000

also allows us to capture, to some degree, the acquisition of new occupational human capital.[13]

Using this notion of occupational mobility we decompose the number of unemployed workers that reported employment the following month (UE_{t+1}) into those that reported employment in a different occupation ($UEocc_{t+1}$) and those that stayed in the same occupation ($UEnocc_{t+1}$).[14] Noting that the aggregate job finding rate in any given month t is $f_t = UE_{t+1}/Unemp_t$, where $Unemp_t$ denotes the stock of unemployed workers in month t, we obtain that

$$f_t = focc_t + fnocc_t + \epsilon_t,$$

where $focc_t = UEocc_{t+1}/Unemp_t$ and $fnocc_t = UEnocc_{t+1}/Unemp_t$ describe the job finding rates with and without occupational change and ϵ_t captures measurement error due to missing information on occupations.

The first row of Table 1 shows the average monthly values of these measures for the entire period considering occupational changes at 3-digit level along with the separation rate from employment to unemployment and the unemployment rate. These numbers suggest that the occupational mobility of unemployed workers is high and important in accounting for the aggregate job finding rate. The job finding rate involving an occupational change represents 45.4 percent of the total job finding rate, while the job finding rate without an occupational change represent 51.3 percent. The remainder 3.3 percent is due to measurement error.[15] Further, these proportions are relatively stable over the

[13]See Xiong (2008) for a similar definition of occupational change. The main results presented below do not change if one measures occupational mobility by comparing the new occupation with the preceding one.

[14]This decomposition is almost exact as we are able to impute an occupation to most of the unemployed workers that had previous or posterior employment spells. An important advantage of the SIPP is that the proportion of missing occupational information for employed workers is lower than one percent. See Appendix D for details of this imputation.

[15]Even considering occupational change at 2-digits and 1-digit levels we find that $focc$ represents 38.1 and 31.5

period of study. Tables A1-A3, in Appendix E, show that a similar picture arises across gender and different age and educational groups.

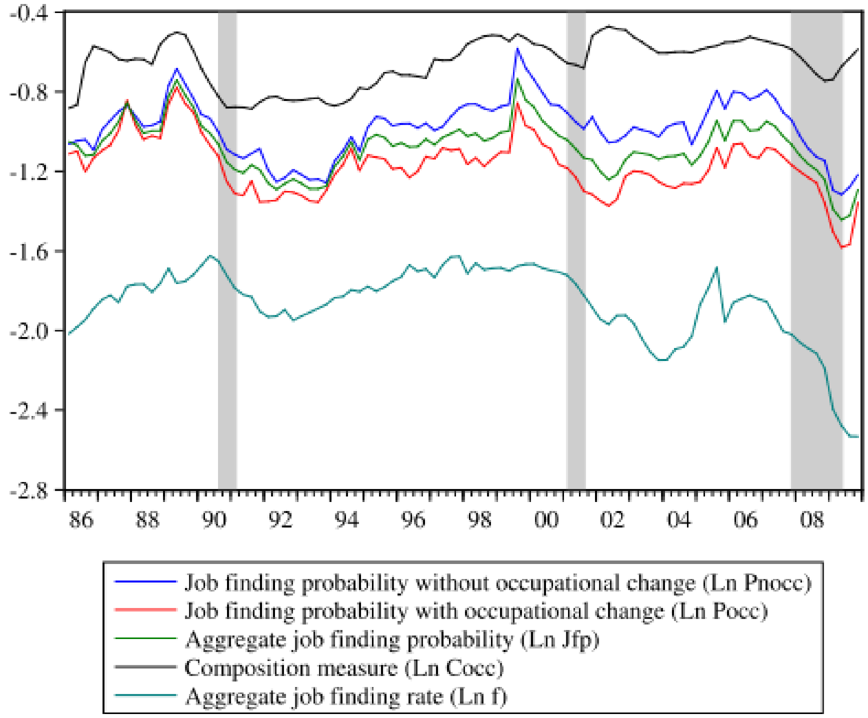


Figure 1: Log of the seasonally adjusted series of f, Jfp, Pocc, Pnocc and Cocc.

We further decompose the job finding rates $focc$ and $fnocc$ such that

$$focc_t = \frac{UEocc_{t+1}}{Uocc_t}\frac{Uocc_t}{Ujob_t}\frac{Ujob_t}{Unemp_t},$$

$$fnocc_t = \frac{UEnocc_{t+1}}{Unocc_t}\frac{Unocc_t}{Ujob_t}\frac{Ujob_t}{Unemp_t},$$

where $Uocc_t$ ($Unocc_t$) denotes the number of unemployed workers at month t that found a job sometime in the future in a different (the same) occupation; and $Ujob_t$ denotes the number of unemployed that found a job sometime in the future.[16] We focus on the first two components of each decomposition. Let $Pocc_t = UEocc_{t+1}/Uocc_t$ and $Pnocc_t = UEnocc_{t+1}/Unocc_t$ and note they reflect the monthly probabilities of changing (not changing) occupation for the pool of unemployed workers that found a job and changed (not changed) occupation at some month $t' > t$. Further, let $Cocc_t = Uocc_t/Ujob_t$ and $Cnocc_t = Unocc_t/Ujob_t$. These proportions reflect the relative importance of those unemployed workers that changed occupations sometime in the future

percent of the total job finding rate.

[16]For both these measures we only consider those workers that reported an uninterrupted spell of unemployment that ended in employment.

on the pool of workers that eventually found a job. They give a sense of how many unemployed workers are looking for a job in a different occupation at any given month.[17] The first row of Table 1 shows the average values of these measures for all workers and Tables A1-A3 shows them for different demographic groups. Once again, these numbers reflect the importance of occupation mobility across unemployed workers.

Figure 1 shows the log series for the aggregate job finding rate together with the log series of $Jfp = PCocc + PCnocc$, the aggregate job finding probability, the job finding probabilities with and without occupational change (Pocc and Pnocc) and our composition measure (Cocc) for all workers. The series at the top of the graph depict the composition effect, the three series immediately below it depict the job finding probabilities and the last series the aggregate job finding rate. This figure shows that these measures move closely together and suggests their procyclicality.[18]

Table 1 also considers the behaviour of the cyclical component of the above series for all workers jointly with that of aggregate output and output per worker as measures of aggregate conditions.[19] We highlight three new findings.[20]

- The job finding rate involving an occupational change is procyclical. This feature is robust to measures of aggregate conditions (output, output per worker and the unemployment rate) and different demographic groups (gender, age and education). The job finding rate without an occupational change is less procyclical than the job finding rate with occupational change. Indeed in some cases $fnocc$ seems to be very close to acyclical.

- The probabilities of finding a job with and without an occupational change (i.e. $Pocc$ and $Pnocc$) are procyclical. This finding is robust across the different measures of aggregate conditions and demographic groups. Once again the procyclicality is stronger for the job finding probabilities involving an occupational change.

- The composition effects described in $Cocc$ and $Cnocc$ are procyclical and countercyclical, respectively. These cyclical patterns are once again robust to the three measures of aggregate

[17] The product of the first two components, $PCocc_t = Pocc_t Cocc_t$ and $PCnocc_t = Pnocc_t Cnocc_t$, then gives the monthly probability of changing (not changing) occupation for the pool of unemployed that became employed at some month $t' > t$. The last term, $CU_t = Ujob_t/Unemp_t$, captures the composition of unemployed workers that found a job at some month $t' > t$ over all those unemployed workers at month t, where the latter includes also those workers that entered the pool of non participants and those who had censored spells at some point in the future.

[18] The correlations of the cycle components of (log) $Pocc$, $Pnocc$ and $Cocc$ with that of (log) Jfp are 0.96, 0.97 and 0.41, respectively; the correlation between the cyclical component of (log) Jfp and (log) f (the aggregate job finding rate) is in turn 0.62. Note that Jfp normalises the monthly UE flow by only those unemployed workers that exited into employment at some point in the future, while f also considers all those unemployed workers that exit into non-participation or had interrupted unemployment spells due censoring or attrition.

[19] See also Tables A1-A3 in Appendix E. Output refers to the seasonally adjusted series of non-farm business output provided by the BLS. Output per worker (Outpw) is constructed using this output measure and the seasonally adjusted employment series from the CPS obtained from the BLS website, http://www.bls.gov. All other variables are based on the SIPP and are seasonally adjusted using the Census Bureau X-12 program. Values are reported in logs as deviation from HP trend with smoothing parameter 1600.

[20] Although the values of the job finding and job separation (to unemployment) rates are lower than those obtained from the CPS, they are consistent with the ones obtained by Mazumder (2007), Fujita, Nekarda and Ramey (2007) and Nagypal (2008) using the SIPP and based on a similar samples albeit using different periods.

conditions and across demographic groups. They suggest that in booms more unemployed workers are successfully looking for a job in a different occupation.

2.3 Discussion

These findings suggest that to study the behaviour of unemployment over the business cycle it is important to take into account workers' decisions to search for jobs in different labor markets. The procyclicality of the aggregate job finding rate, which is the main driving force of unemployment volatility over the period of study,[21] reflects both the cyclical change in the proportion of workers searching for jobs in a new occupation versus in a previous occupation (as captured by the cyclicality of Cocc and Cnocc) and in the probability of finding a job in either kind of occupation (as captured by the cyclicality of Pocc and Pnocc). Our findings suggest that unemployed workers find it more profitable to change occupations when jobs are plentiful and the probability of finding a job in a different occupation is higher.

In the next section we construct a business cycle model of the labor market that is consistent with these features. Our theory shows that workers are more likely to change occupations in booms because the increased probability of finding a job provides extra benefits to undertake this reallocation vis $\acute{a}$ vis a model that considers competitive labor markets within each island as in Lucas and Prescott (1974). In the quantitative section we show that the model is also consistent with other features presented in Table 1 such as countercyclical unemployment and job separation rates, procyclical job finding rate and a high volatility of the cyclical component of the unemployment rate with respect to that of output per worker.

3 Model

Time is discrete, and goes on forever; it is denoted by t. There is a continuum of infinitely lived risk-neutral workers of measure one, located over a continuum of islands, each island indexed by i, such that (almost) all islands are home to a continuum of workers of various measure. Workers can be either employed or unemployed in an island. An unemployed worker receives b each period. The wages of employed workers are determined below.

There is also a continuum of risk-neutral firms that live forever. Each firm has one position, and can decide to enter the labor market in an island of choice. The firm needs a worker to produce a good, with a production function $y(p_t, z_{it})$ that is continuous differentiable, strictly increasing in all arguments, where p_t is the aggregate productivity shock (which impacts all islands in the economy)

[21] Using the Survey of Income Program Participation for the period 1986-2009 we find that 60 percent of the variation of unemployment can be explained by the job finding rate, while the remaining 40 percent is explained by the rate at which employed workers enter unemployment. We decompose the variation of unemployment assuming a two state process (unemployment and employment) and follow the methods for decomposing unemployment and adjusting for time aggregation error on the job finding and separation rates series proposed by Fujita and Ramey (2009). Our figures are very similar to the ones they obtain using the Current Population Survey for a similar period (see Fujita and Ramey's Table 1). However, our decomposition attributes a greater importance to the job separation rate than Shimer (2012) and Hall (2006) suggest.

and z_{it} is the island specific productivity component at a given time t. We assume that the cross-derivatives of productivities are (weakly) positive. Both types of productivities are drawn from bounded intervals and follow stationary Markov-processes. The initial realisations and any future innovations of z's are iid across islands. All agents discount the future using the same discount factor β.

A firm can find a worker by posting a vacancy in a particular island, paying a cost k. There is no on-the-job search, therefore only unemployed workers can decide to search for vacant jobs. A posted job specifies a wage contract contingent on the sequence of realisations of p_t, z_{it} and the duration of the relationship. Let w_{ift} denote the wage paid at firm f in island i at time t. We further specify the matching process within each island below.

Once a matched is formed, firms pay workers according to the posted contract, until the match is broken up. The latter can happen with an exogenous (and constant) probability δ, but in addition also occurs if the worker and the firm decide to do so. Once the match is broken, the worker becomes unemployed in the current island and the firm has to decide to reopen the vacancy or not. A worker that separates from his current employer (voluntarily or not) stays unemployed in his island until the end of the period.

Unemployed workers' can decide to stay in the current island or reallocate. Reallocation, however, involves paying a moving cost c. Further, a worker who decides to reallocate cannot immediately apply for a job and must sit out unemployed in the new island for the rest of the period. Once the worker decides to move, he draws an island from the set of islands with idiosyncratic productivity no lower than z^n, where n is exogenous to the model and represents the n percentile of the distribution of active islands at the moment of reallocation. As this parameter does not play any meaningful role on our theoretical analysis, without loss of generality we will normalise it to zero in the next sections.[22]

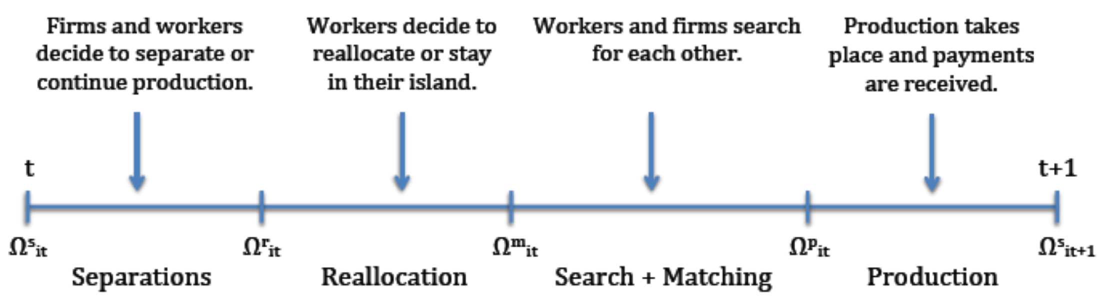


Figure 2: Timing of events within a period

[22]Truncating the distribution of active islands from below allows workers to avoid wasting time visiting those islands with the worse conditions. It is useful purely from a quantitative point of view. It tries to capture the idea that workers gather some information about the occupations they want to avoid but still take time to arrive to their preferred occupation. Note that when $n = 0$ the model encompasses the case of pure random search as in Alvarez and Veracierto (1999) and Veracierto (2008). When $n = 100$ our model encompasses the case of perfectly directed search similar to Menzio and Shi (2011). Indeed, if we allowed for perfectly directed search, the free entry conditions of vacancies implies that all workers chose to visit the islands with the highest idiosyncratic productivity.

Given the above considerations, Figure **??** summarises the timing of the events within a period conditional on the state in island i at time t. A period is subdivided into four stages: separation, reallocation, search and matching and production. Let u_{it}^x and e_{it}^x denote the measure of unemployed and employed workers at the *beginning* of stage x in island i and period t. Also let $\mathcal{E}_t^x$ denote the distribution of unemployed and employed workers over the different islands at the beginning of stage x in period t. The state of island i at the beginning of stage x is then described by the vector $\Omega_{it}^x = \{p_t, z_{it}, \mathcal{E}_t^x\}$. Although we will focus on equilibria in which the relevant state space is described by $\{p_t, z_{it}\}$, for completeness we present the set up of the model using the general state space described by Ω.

3.1 Posting and Matching

In each island firms post contracts to which they are committed. Unemployed workers and advertising firms then match with frictions as in Moen (1997).[23] In particular, in each island there is a continuum of sub-markets, one for each expected lifetime value $\tilde{W}$ that could potentially be offered by a vacant firm. After firms have posted a contract in the sub-market of their choice, workers u can choose which sub-market to visit. Once in their preferred sub-market, workers and firms meet according to a constant returns to scale matching function $m(u, v)$, where u is the measure of workers searching in the sub-market, and v the measure of firms which have posted a contract in this sub-market.

From the above matching function one can easily derived the workers' job finding rate

$$\lambda(\theta) = m(1, v/u), \text{ with } \theta = v/u,$$

and the vacancy filling rate

$$q(\theta) = m(u/v, 1)$$

in the sub-market. The matching function and the job finding and vacancy filling rates are assumed to have the following properties: (i) they are twice-differentiable functions, (ii) nonnegative on the relevant domain, (iii) $m(0, 0) = 0$, (iv) $q(\theta)$ is strictly decreasing, and (v) $\lambda(\theta)$ is strictly increasing and concave.[24]

3.2 Worker's problem

Conditional on the state of the island at the beginning of the production stage, Ω_{it}^p, consider the value function of an unemployed worker

$$W^U(\Omega_{it}^p) = b + \beta \mathbb{E}[W^R(\Omega_{it+1}^r)]. \tag{1}$$

[23]As it will be come apparent, in our framework using Pissarides (2001) with the Hosios condition is equivalent to using Moen's model to describe matching within islands.

[24]We impose two restrictions on beliefs off-the-equilibrium path. Workers believe that, if they go to a sub-market that is inactive on the equilibrium path, firms will show up in such measure to have zero profit in expectation. Firms believe that, if they post in an inactive sub-market, a measure of workers will show up, to make *the measure of deviating firms* indifferent between entering or not. We assume, for convenience, that the zero-profit condition also holds for deviations of a single agent: loosely, the number of vacancies or unemployed, and therefore the tightness will be believed to adjust to make the zero-profit equation hold.

The value of unemployment consists of the flow benefit of unemployment b this period, plus the discounted expected value of being unemployed at the beginning of next period's reallocation stage,

$$W^R(\Omega^r_{it+1}) = \max_{\rho(\Omega^r_{it+1})} \{\rho(\Omega^r_{it+1})R(\Omega^r_{jt+1}) + (1-\rho(\Omega^r_{it+1}))\mathbb{E}[S(\Omega^m_{it+1}) + W^U(\Omega^m_{it+1})]\},$$

where $\rho(\Omega^r_{it+1})$ takes the value of one when the worker decides to reallocate and zero otherwise.

Equation (**??**) includes continuation values for each possible realisation in the matching and reallocation stages. In particular, $R(.)$ denotes the expected benefit of reallocation. Given that workers who reallocate have to sit out one period of unemployment in the new island, this benefit is given by

$$R(\Omega^r_{jt+1}) = -c + \mathbb{E}_{\Omega^p_{jt+1}}[W^U(\Omega^p_{jt+1})].$$

The expected value of staying and searching on the island is given by $\mathbb{E}[S(\Omega^m_{it+1}) + W^U(\Omega^m_{it+1})]$. In this case, $W^U(\Omega^m_{it+1}) = \mathbb{E}[W^U(\Omega^p_{it+1})]$ describes the expected value of not finding a job on the same island, while $S(\Omega^m_{it+1})$ summarizes the expected value *added* of finding a new job on the island. The reallocation decision is captured by the choice between $R(\Omega^r_{jt+1})$ and the expected payoff of search on the current island.[25]

To derive $S(.)$ recall that $\lambda(\theta(\Omega^m_{it}, W_f))$ denotes the probability with which the worker meets a firm f in the sub-market associated with the promised value W_f and tightness $\theta(\Omega^m_{it})$. Further, let $\alpha(W_f)$ denote the probability of visiting such a sub-market. From the set $\mathcal{W}$ of promised values which are offered in equilibrium by firms in this island, the worker only visits with positive probability those sub-markets for which the associated W_f satisfies

$$W_f \in \arg\max_{\mathcal{W}} \lambda(\theta(\Omega^m_{it+1}, W_f))(W_f - W^U(\Omega^m_{it+1})) \equiv S(\Omega^m_{it+1}). \quad (2)$$

Hence equation (**??**) incorporates the worker's optimal visiting decisions to those active sub-markets in island j after he has reallocated, or in island i if he did not move. When the set $\mathcal{W}$ is empty, the expected value added of finding a job in the island is zero and the worker is indifferent between visiting any sub-market.

Now consider the value function at the beginning of the production stage of an employed worker in a contract that currently has a value $\tilde{W}_f(\Omega^p_{it})$. Similar arguments as before imply that

$$\tilde{W}_f(\Omega^p_{it}) = w_{ift} + \beta\mathbb{E}\left[\max_{d(\Omega^s_{it+1})} \{(1-d(\Omega^s_{it+1}))\tilde{W}_f(\Omega^s_{it+1}) + d(\Omega^s_{it+1})W^U(\Omega^s_{it+1})\}\right], \quad (3)$$

where $d(\Omega^s_{it+1})$ take the value of δ when $\tilde{W}_f(\Omega^s_{it+1}) \geq W^U(\Omega^s_{it+1})$ and the worker decides not to quit into unemployment and the value of one otherwise. In equation (**??**), the wage payment w_{ift} at firm f is contingent on state Ω^p_{it}, while the second term describes the worker's option to quit in the separation stage the next period. Note that $W^U(\Omega^s_{it+1}) = \mathbb{E}[W^U(\Omega^p_{it+1})]$ as a worker who separates must stay unemployed in his current island for the rest of the period and $\tilde{W}_f(\Omega^s_{it+1}) = \mathbb{E}[\tilde{W}_f(\Omega^p_{it+1})]$ as the match will be preserved after the separation stage.

[25] Notice that, after paying the reallocation cost c, the worker randomly draws a new island with state vector Ω^p_{jt+1} and, from the next period onwards, any subsequent decisions in the chosen island are the same as the ones described above.

3.3 Firm's problem

Given state vector Ω^p_{it}, consider a firm f in island i, currently employing a worker who has been promised a value $\tilde{W}_f(\Omega^p_{it}) \geq W^U(\Omega^p_{it})$. The expected lifetime discounted profit of this firm can be described recursively as

$$\begin{aligned} J(\Omega^p_{it};\tilde{W}_f(\Omega^p_{it})) \quad &= \quad \max\Big\{y(p_t,z_{it}) - w_{ift} + \beta\mathbb{E}\Big[\max_{\sigma(\Omega^s_{it+1})}\Big\{(1-\sigma(\Omega^s_{it+1}))J(\Omega^s_{it+1};\tilde{W}_f(\Omega^s_{it+1})) \\ &\quad +\sigma(\Omega^s_{it+1})\tilde{V}(\Omega^s_{jt+1})\Big\}\Big]\Big\}, \end{aligned} \tag{4}$$

where $\sigma(\Omega^s_{it+1})$ takes the value of δ when $J(\Omega^s_{it+1};\tilde{W}_f(\Omega^s_{it+1})) \geq \tilde{V}(\Omega^s_{jt+1})$ and the value of one otherwise, $\tilde{V}(\Omega^s_{jt+1}) = \max\{V(\Omega^s_{jt+1}),0\}$ and $V(\Omega^s_{jt+1})$ refers to the value of an unfilled vacancy in market j at time $t+1$ with island-specific state vector Ω^s_{jt+1}. Here the first maximisation is over the wage payment w_{ift} and the promised lifetime utility to the worker $\tilde{W}_f(\Omega^p_{it+1})$. The second maximisation refers to the firm's layoff decision.[26]

Equation (**??**) is subject to the restriction that the wage paid today and tomorrow's promised values have to add up to today's promised value $\tilde{W}_f(\Omega^p_{it})$, according to equation (**??**). Moreover, the workers' option to quit into unemployment, and the firm's option to lay off the worker imply the following participation constraint

$$(J(\Omega^s_{it+1};\tilde{W}_f(\Omega^s_{it+1})) - \tilde{V}(\Omega^s_{jt+1}))\cdot\Big(\tilde{W}_f(\Omega^s_{it+1}) - W^U(\Omega^s_{it+1})\Big) \geq 0, \tag{5}$$

with complementary slackness.

Now consider a firm posting a vacancy. Given cost k, a firm can choose an island where to locate its vacancy, knowing Ω^m_{it}. Further, for each island the firm has to decide which $\tilde{W}_f$ to post given $q(\theta(\Omega^m_{it},\tilde{W}_f))$, the associated job filling probability. Note that this probability summarises the pricing behaviour of other firms and the visiting strategies of workers. Along the same line as above, the expected value of a vacancy in island i solves the Bellman equation

$$V(\Omega^m_{it}) = -k + \max_{\tilde{W}_f}\Big\{q(\theta(\Omega^m_{it},\tilde{W}_f))J(\Omega^m_{it},\tilde{W}_f) + (1-q(\theta(\Omega^m_{it},\tilde{W}_f)))\mathbb{E}_{\Omega^p_{jt}}[V(\Omega^p_{jt})]\Big\}. \tag{6}$$

We assume that in each island there is free entry of firms posting vacancies, which implies that $V(\Omega^x_{it}) = 0,\ \forall\,\Omega^x_{it}, i, t$ at any stage x. The free entry condition then simplifies the vacancy creation condition to

$$k = \max_{\tilde{W}_f} q(\theta(\Omega^m_{it},\tilde{W}_f))J(\Omega^m_{it},\tilde{W}_f).$$

3.4 Worker flows

Until now, we have taken as given the state vectors $\Omega^s_{it},\Omega^r_{it},\Omega^m_{it},\Omega^p_{it}$ and their evolution to discuss agents' optimal decisions. As mentioned earlier p_t, z_{it} follow exogenous processes. However, the

[26] Note that the solution to (**??**) gives the wage payments during the match (for each realisation of Ω^p_{it} for all t). In turn these wages pin down the expected lifetime profits at any moment during the relation, and importantly also at the start of the relationship, where the promised value to the worker is $\tilde{W}_f$.

evolution of the number of unemployed and employed workers is a result of optimal vacancy posting, visiting strategies, separation and reallocation decisions. In Appendix B we provide a derivation of how these measures evolve.

4 Equilibrium

We look for an equilibrium in which the value functions and decisions of workers and firms only depend on the productivity in the aggregate and on the island. Moreover, we are also looking for equilibria where the values offered to all employed workers at a given moment on a given island are equal. Under these considerations the following describe the candidate equilibrium value functions

$$
\begin{aligned}
W^U(p,z) &= b + \beta \mathbb{E}_{p',z'} \Bigg[\max_{\rho(p',z')} \Bigg\{ \rho(p',z') \left[-c + \int W^U(p',z'_i) dF(i) \right] + \\
&\qquad (1-\rho(p',z')) \left[\max_{W^{E\prime}} \left\{ \lambda(\theta(p',z',W^{E\prime}))W^{E\prime} + (1-\lambda(\theta(p,z,W^{E\prime})))W^U(p',z') \right\} \right] \Bigg\} \Bigg] \qquad (7) \\
W^E(p,z) &= w(p,z) + \beta \mathbb{E}_{p',z'} \max_{d(p',z')} \left\{ (1-d(p',z'))W^E(p',z') + d(p',z')W^U(p',z') \right\} \qquad (8) \\
J(p,z,\tilde{W}^E) &= \max_{\{w,\tilde{W}^{E\prime}(p',z')\}} \left\{ y(p,z) - w + \beta \mathbb{E}_{p',z'} \max_{\sigma(p',z')} \{ (1-\sigma(p',z'))J(p',z',\tilde{W}^{E\prime}(p',z')) \} \right\} \qquad (9) \\
V(p,z,\tilde{W}) &= -k + q(\theta(p,z,\tilde{W}))J(p,z,\tilde{W}) = 0, \qquad (10)
\end{aligned}
$$

where $\tilde{W}^E$, w and $\tilde{W}^{E\prime}$ must satisfy (**??**) and the maximisation in (**??**) is subject to the participation constraint (**??**).

The main simplification we achieve by focusing attention in this type of equilibria is that we do not need to keep track of the measures of unemployed and employed workers on each island or their flows between islands to derive agents' decision rules. In turn, this implies that equilibrium outcomes can now be derived in two steps. In the first step, decision rules are solved independently of the heterogeneity distribution that exists across agents and islands using the above four value functions. Once those decision rules are determined, we fully describe the dynamics of these distributions using the workers' flow equations.[27]

Definition 1. *A Block Recursive Equilibrium (BRE) in our island economy is a set of value functions $W^U(p,z)$, $W^E(p,z)$, $J(p,z,W^E)$, workers' policy functions $d(p,z)$, $\rho(p,z)$, $\alpha(p,z)$ (resp. separation, reallocation and visiting strategies), firms' policy functions $\tilde{W}_f(p,z)$, $\sigma(p,z,W^E)$, $w(p,z,W^E)$, $\tilde{W}^{E\prime}(p,z,W^E)$ (resp. contract posted, layoff decision, wages paid, and continuation values promised), tightness function $\theta(\tilde{W},p,z)$, matching probabilities $\lambda(\theta), q(\theta)$, laws of*

[27]This recursive property is common in many search models and in particular in those based on Pissarides (2001). In these models the free entry condition determines the labor market tightness (the key variable of the model) without taking into account the number of unemployed or employed workers in the labor market. These measures are derived using the flow equations that describe workers' transition between employment and unemployment once labor market tightness is obtained. Shimer (2005) and Mortensen and Nagypal (2007) provide recent examples of how this property is preserved when analysing the canonical search and matching model in a business cycle context.

motion of z_{it}, p_t, $F_z(.), F_p(.)$, and a law of motion on the distribution of unemployed and employed workers over islands $\tilde{u}(.) : \mathcal{F}^{[0,1]} \rightarrow \mathcal{F}^{[0,1]}$ and $\tilde{e}(.) : \mathcal{F}^{[0,1]} \rightarrow \mathcal{F}^{[0,1]}$, such that

1. *$\theta(p, z, \tilde{W})$ results from free entry condition $V(p, z, \tilde{W}) = 0$, if $\theta(p, z, \tilde{W}) > 0$ and $V(p, z, \tilde{W}) \leq 0$ if $\theta(p, z, \tilde{W}) = 0$, defined in (**??**), and given value function $J(p, z, \tilde{W})$.*

2. *Matching probabilities $\lambda(.)$ and $q(.)$ are only functions of labor market tightness $\theta(.)$, according to the definitions in section **??**.*

3. *Given firms' policy functions, laws of motion F_z, F_p, and implied matching probabilities from $\lambda(.)$, the value functions W^E and W^U satisfy (**??**) and (**??**), while $d(.), \rho(.), \alpha(.)$ are the associated policy functions.*

4. *Given workers optimal separation, reallocation and application strategies, implied by $W^E(.)$ and $W^U(.)$, and the laws of motions on p_t, z_{it}, firms' maximisation problem is solved by $J(.)$, with associated policy functions $\{\sigma(.), w(.), \tilde{W}^{E\prime}(.)\}$.*

5. *$\tilde{W}^{E\prime}(p, z) = W^E(p, z)$.*

6. *$\tilde{u}$ and $\tilde{e}$ map initial distributions of unemployed and employed workers (respectively) over islands into next period's distribution of unemployed and employed workers over islands, according to policy functions and exogenous separation, and then according to equations in Appendix B.*

4.1 Characterization

We start the characterisation of equilibria by showing that in each matching stage, firms offer a unique $\tilde{W}_f$ with associate tightness $\tilde{\theta}(p, z)$. To do so, consider an island i that is characterised by state vector (p, z). For any promised value W^E, the joint value of the match is defined as $W^E + J(p, z, W^E) \equiv \tilde{M}(p, z, W^E)$. Lemma 1 now shows that under risk neutrality the value of a match is constant in W^E and J decreases one-to-one with W^E.

Lemma 1. *The joint value $\tilde{M}(p, z, W^E)$ is constant in $W^E \geq W^U(p, z)$ and hence we can uniquely define $M(p, z) \overset{def}{=} \tilde{M}(p, z, W^E),\ \forall\ M(p, z) \geq W^E \geq W^U(p, z)$ on this domain. Further, $J_W(p, z, W^E) = -1,\ \forall\ M(p, z) > W^E > W^U(p, z)$, and endogenous match breakup decisions are efficient from the perspective of the match.*

The proof of Lemma 1 crucially relies on the firms' ability to offer workers intertemporal wage transfers such that the value of the match is not affected by the (initial) promised value. Lemma 2 now shows that firms offer a unique $\tilde{W}_f$ in the matching stage and there is a unique θ associated with it.

Lemma 2. *Assume free entry of firms, $J_W(p, z, W^E) = -1$ for each p, z, and a matching function that exhibits constant returns to scale, with a vacancy filling function $q(\theta)$ that is nonnegative and strictly decreasing, while the job finding function $\lambda(\theta)$ is nonnegative, strictly increasing and concave. If the elasticity of the vacancy filling rate is weakly negative in θ, there exists a unique $\theta^*(p, z)$ and $W^*(p, z)$ that solve (**??**), subject to (**??**).*

The requirement that the elasticity of the job filling rate with respect to θ is non-positive is automatically satisfied when $q(\theta)$ is log concave, as is the case with the urn ball matching function.[28] Alternatively, one can use the Cobb-Douglas matching function as it implies a constant $\varepsilon_{q,\theta}(\theta)$. Both matching functions imply that the job finding and vacancy filling rates have the properties described in Lemma 2 and hence guarantee a unique pair $\tilde{W}_f, \theta$. To simplify the analysis that follows, we assume a Cobb-Douglas matching function. Using η to denote the (constant) elasticity of the job finding rate with respect to θ, we find the well-known division of the surplus according to the Hosios' rule

$$\eta(W^E - W^U(p,z)) - (1-\eta)J(p,z,W^E) = 0. \tag{11}$$

Finally, since in every period there is at most one $\tilde{W}_f$ offered in the matching stage, a worker's visiting strategy, α, is to visit the sub-market associated with $\tilde{W}_f$ with probability one when $S(p,z) > 0$ and to randomly visit any sub-market when $S(p,z) = 0$.

The last step in our characterisation is to derive the reallocation and separation policy functions, $d(p,z)$, $\sigma(p,z)$ and $\rho(p,z)$. Lemmas **??** and **??**, below, show that for every p, there exists a (potentially trivial) reservation productivity $z^s(p)$ below which any match, if it exists, is broken up such that $d(p,z) = \sigma(p,z) = 1$ for all $z < z^s(p)$ and $d(p,z) = \sigma(p,z) = \delta$ otherwise. Further, for every p, there exists a reservation productivity $z^r(p)$ such that $\rho(p,z) = 1$ (a worker reallocates) for all $z < z^r(p)$ and $\rho(p,z) = 0$ (a worker does not reallocate) otherwise.[29]

4.2 Existence

To show existence of equilibrium it is useful to consider the operator T mapping a value function $\tilde{M}(p,z,n)$ for $n = 0,1$ into the same function space such that $\tilde{M}(p,z,0) = M(p,z)$, $\tilde{M}(p,z,1) = W^U(p,z)$ and

$$T(\tilde{M}(p,z,0)) = y(p,z) + \beta\mathbb{E}_{p',z'}\Big[\max_{d^T}\{(1-d^T)M(p',z') + d^T W^U(p',z')\}\Big]$$

$$T(\tilde{M}(p,z,1)) = b + \beta\mathbb{E}_{p',z'}\Big[\max_{\rho^T}\{(\rho^T\Big(\int W^U(p',\tilde{z})dF(\tilde{z}) - c\Big) + (1-\rho^T)(S^T(p',z') + W^U(p',z'))\}\Big]$$

where by virtue of the free entry condition

$$S^T(p',z') \overset{def}{=} \max_{\theta(p',z')}\Big\{\lambda(\theta(p',z'))\Big(M(p',z') - W^U(p',z')\Big) - \theta(p',z')k\Big\}.$$

A fixed point $\tilde{M}(p,z,n)$, $n = 0,1$ describes the problem faced by unemployed workers and firm-worker matches in the decentralised economy. In the proof of Proposition **??** we show that all equilibrium functions and the evolution of the economy can be derived completely from the fixed point of the mapping T. For that purpose, we assume that the probability distribution of tomorrow's z conditional on today's z first-order stochastically dominates the corresponding distribution conditional on a z' that is lower today.

[28] The urn-ball matching function is the one that arises endogenously within a directed search model a la Burdett, Shi and Wright (2001). In this case q exhibits a negative elasticity, $\frac{-\frac{1}{\theta}}{e^{1/\theta}-1} - \frac{\frac{1}{\theta^2}e^{1/\theta}}{(e^{1/\theta}-1)^2} < 0$.

[29] Note that the reservation productivities depend on n, the parameter that determines the set of islands to which a worker could reallocate. To ease notation we leave this dependency implicit.

Assumption 1. *$F_z(z_{it+1}|p_t, z_{it}) < F_z(z_{it+1}|p_t, z'_{it})$, for all i, z_{it+1}, p_t if $z_{it} > z'_{it}$.*

Thus, a higher island-specific productivity today leads, on average, to a higher productivity tomorrow and hence the ranking of island-specific z productivity is -in this sense- persistent. The next result derives the essential properties of T.

Lemma 3. *T is (i) a well-defined operator mapping functions from the closed space of bounded continuous functions $\tilde{M}$ into itself, (ii) a contraction and (iii) maps functions $M(p, z)$ and $W^U(p, z)$ that are increasing in z into itself.*

A direct implication of the above Lemma is that the optimal reallocation policy is a reservation-z policy as described above, as both $S(p, z)$ and $W^U(p, z)$ are increasing in z, but $R(p, z_j)$ is constant. The next result implies that the optimal quit policy is also a reservation-z policy as described above.

Lemma 4. *If $\delta + \lambda(\theta(p, z)) < 1$, $M(p, z) - W^U(p, z)$ in the fixed point of T is increasing in z.*

Lemma **??** and equations (**??**) and (**??**) together imply that in each island labor market tightness $\theta(p, z)$ and the job finding rate $\lambda(\theta(p, z))$ are also increasing functions of z if $\delta + \lambda(\theta(p, z)) < 1$.[30]

Note that the above policy functions describe the decision rules in our candidate equilibrium. Since $\tilde{W}(p, z) = M(p, z) - J(p, z, \tilde{W})$ and $J(p, z, \tilde{W}) = (1 - \eta)(M(p, z) - W^U(p, z)) = k/q(\theta(p, z, \tilde{W}))$, $\tilde{W}(p, z)$, $J(p, z, \tilde{W})$ and $\theta(p, z, \tilde{W})$ can be constructed from $M(p, z)$ and $W^U(p, z)$. This is done in the proof of Proposition **??**, where the existence and uniqueness of equilibrium are 'inherited' from the existence and uniqueness of the fixed point of the mapping T.

Proposition 1. *(i) A Block Recursive Equilibrium exists and is unique.*
(ii) Moreover, under assumption 1 and the condition in Lemma 4, the behaviour of agents can be summarised in two functions of the aggregate state p, the reallocation cutoff of island-specific productivity $z^r(p)$ and the cutoff level of island-specific productivity for separations $z^s(p)$.

4.3 Planner's Problem and Efficiency

The social planner, currently in the production stage, in this economy solves the problem of maximising total discounted output. Namely,

$$\max_{\{d_{it}(\Omega_t^s), \rho_{it}(\Omega_t^r), v_{it}(\Omega_t^m), \alpha_i(\Omega_t^m)\}} \mathbb{E}\left[\sum_t \beta^t \int_I [u_{it}b + e_{it}y(p_t, z_{it}) - (c\rho_{it}u_{it} + kv_{it})]\, di\right]$$

subject to the laws of motion and initial conditions

$$u_{it+1} = (1 - \rho_{it})u_{it} + (e_{it} - e_{it+1}) + \int_I \rho_{jt}u_{jt}dj$$
$$e_{it+1} = (1 - d_{it})e_{it} + (1 - \rho_{it})u_{it}\lambda\left(\frac{v_{it}}{(1 - \rho_{it})u_{it}}\right)$$
$$\mathcal{E}_0 \text{ given}, \; v_{i0} = 0, \text{ for all } i,$$

where I denotes the set of islands the worker can potential visit after reallocation and $\theta_{it} = v_{it}/(1-\rho_{it})u_{it}$.

[30] Tables 1 and 2 suggest that this parametric restriction is easily satisfied in the data.

Proposition 2. *The equilibrium identified in Proposition* **??** *is constrained efficient.*

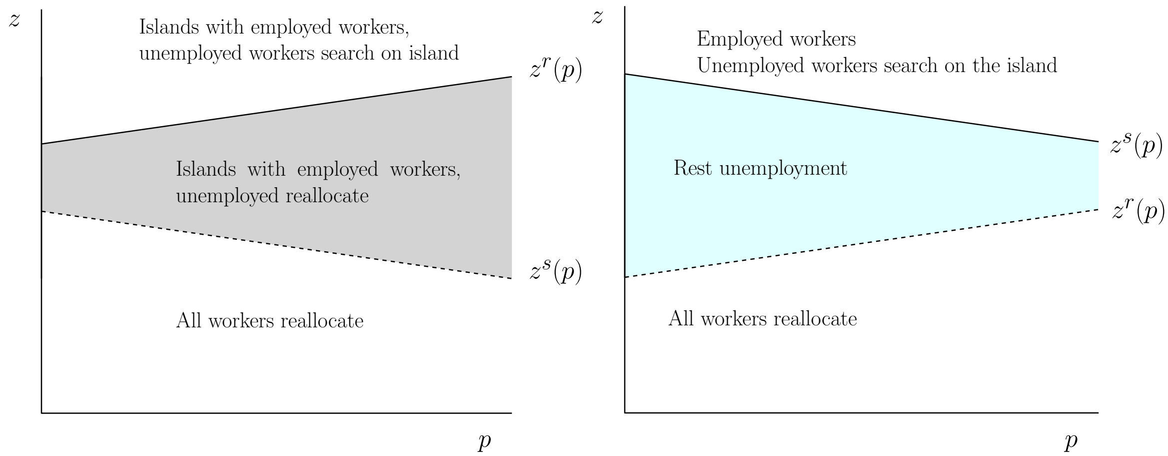
The crucial insight behind Proposition 2 is that the social planner's value functions are linear in the number of unemployed and employed on each island. The remaining dependence on p and z_i is equivalent to the one derived from the fixed point of T. Given the value functions of unemployed workers and worker-firm matches, the outcome at the matching stage is efficient and the Hosios' condition is thus satisfied. Proposition 2 also implies that workers' reallocation decisions are efficient. This is intuitive as the value of an unemployed worker who always remains on the island equals the shadow value of this worker in the social planner's problem, and reallocation decisions are made by comparing the expectation over the value of unemployment at other islands with the value of unemployment on the current island.

5 Implications

The decision to separate from an existing match, or to reallocate to a new island is characterised by a cutoff property, as it would be in the simple McCall search model, but now these cutoffs are varying with aggregate productivity p. The aggregate outcomes in the economy depend (i) on the characteristics of the cutoff functions $z^r(p), z^s(p)$ and their relative position; (ii) on the dynamic processes of z and p, which change the conditions of a given island over time; and (iii) on the resulting dynamics of the distribution of workers over islands.

Hence we can gain insight about the behaviour of the labor market with respect to the aggregate state of the economy, by analysing the response of workers' reallocation, search and separation decisions to aggregate productivity. Our model allows us to study these features analytically when aggregate productivity is perceived to be permanently fixed. The comparative statics of this situation coincide with the response to a one-time unexpected permanent change in productivity p, which is a standard device to gain intuition about the responses to more general persistent productivity shock processes (see also Shimer, 2005, Mortensen and Nagypal, 2007, and Hagedorn and Manovskii, 2008).

The relative position of the two cutoff functions dictates whether an island has employed workers, unemployed workers, both, or, possibly, neither. To illustrate these features, Figure **??** depicts them when $z^r(p)$ is an increasing function and $z^s(p)$ is a decreasing function of aggregate productivity, such that reallocations are procyclical and separations countercyclical. In islands with productivity z such that $z^r(p) > z > z^s(p)$, employed workers prefer to stay in their jobs and therefore also stay on these islands, but those who are unemployed prefer to reallocate to different markets. In these islands there are no new matches being created. Conversely, if $z^s(p) > z > z^r(p)$, these islands have rest unemployment. Firms and workers prefer to dissolve existing job matches. Unemployed workers prefer not to have a job and firms do not find it profitable to post vacancies. Unemployed workers, however, prefer to remain on these islands than to reallocate elsewhere. Finally, if z is above both cutoffs, workers want to remain on the island, firms create new vacancies and unemployed workers move into new jobs over time; while if z is below both cutoffs, all workers (employed and unemployed) prefer to reallocate and no firm creates new vacancies. Below we show the conditions under which z^r is increasing and z^s is decreasing with p.



(a) $z^r > z^s$

(b) $z^s > z^r$

Figure 3: Relative positions of cutoff productivities

In what follows it will be useful to note that in our model wages are described by a standard Pissarides style wage equation

$$w(p,z) = (1-\eta)y(p,z) + \eta b + \beta(1-\eta)\theta(p,z)k.$$

A formal derivation of this equation can be found in Appendix B. Using the free-entry condition and the Cobb-Douglas specification for the matching function we have that θ then solves

$$\theta(p,z)^{\eta-1}\frac{\eta(y(p,z)-b)-\beta(1-\eta)\theta(p,z)k}{1-\beta(1-\delta)} - k \equiv E(\theta;p,z) = 0,$$

where differentiation implies that θ is increasing in both p and z,

$$\theta_x(p,z) = \frac{\theta(p,z)y_x(p,z)}{w(p,z)-b}, \tag{12}$$

and the subscript x denotes differentiation with respect to $x = p, z$.

5.1 Cyclicality of Worker Reallocation Flows

We first turn to analyse whether a higher aggregate productivity leads to more or less reallocation, given the same initial distribution of employed and unemployed workers over islands. Note that at islands where the island-specific productivity equals z^r (the reservation productivity for the reallocation decision) it holds that the value of reallocation equals the value of staying and searching in the local labor market,

$$\int_{\underline{z}}^{\bar{z}} W^U(p,z)dF(z) - c = W^U(p,z^r) + \lambda(\theta(p,z^r))(W^E(p,z^r) - W^U(p,z^r)). \tag{13}$$

In a stationary environment, described by p, z, the value of unemployment at islands with $z < z^r$

is given by $W^U(p,z) = W^U(p,z^r)$.[31] On the other hand, the value of unemployment at islands with $z \geq z^r$ is given by

$$W^U(p,z) = \frac{b + \beta\lambda(\theta(p,z))(W^E(p,z) - W^U(p,z))}{1-\beta}. \tag{14}$$

Equation (**??**) can then be expressed as

$$\frac{(1-\eta)k}{\eta}\left(\beta\int_{\underline{z}}^{\bar{z}} \max\{\theta(p,z),\theta(p,z^r)\}dF(z)\right) - c(1-\beta) = \frac{(1-\eta)k}{\eta}\theta(p,z^r), \tag{15}$$

where the LHS describes the net benefit of reallocating to a different island and the RHS the benefit of staying in the same island.[32] Hence the response to a positive (and permanent) productivity shock is more reallocation (a higher reservation productivity) if $dz^r/dp > 0$.

Proposition 3, below, derives the conditions under which procyclical reallocation arises, taking into account on the one hand that the value of switching to become unemployed on an island with a higher z than the current island is increasing in p; and on the other that this gain realises only one period after arriving to the new island (as workers cannot search during the same period they arrived to the island), and that the cost of missing out on one period of higher productivity also goes up with p.[33] Proposition 3 also compares the cyclicality of reallocation in our setting (where there is search frictions on islands) with a setting where markets on the islands are competitive *á* la Alvarez and Veracierto (1999).

To make precise the comparison, consider the same environment as above, with the exception that workers can match instantly with firms.[34] This implies that every worker will earn his marginal product $y(p,z)$. Importantly, we keep the reallocation frictions the same: workers who reallocate have to forgo production for a period, and arrive at a random island at the end of the period. In the simple case of permanent productivity (p,z), the value of being in island z, conditional on $y(p,z) > b$ is $W^c(p,z) = y(p,z)/(1-\beta)$, where to simplify we have not consider job destruction shocks.[35]

Block recursiveness, given the free entry condition, is preserved, so again, decisions are only functions of (p,z). Unemployed workers optimally choose to reallocate, and the optimal policy is

[31] This follows since over this range of z's, $\int_{\underline{z}}^{\bar{z}} W^U(p,z)dF(z) - c \geq W^U(p,z) + S(p,z)$ and unemployed workers prefer to reallocate the period after arrival. The stationary version of (**??**) then implies $W^U(p,z) = W^U(p,z^r)$ for all $z < z^r$.

[32] This equation is obtain by noting that (**??**) can be expressed as

$$\beta\int_{\underline{z}}^{\bar{z}}\Big(\max\{\lambda(\theta(p,z))(W^E(p,z)-W^U(p,z)),\lambda(\theta(p,z^r))(W^E(p,z^r)-W^U(p,z^r))\}\Big)dF(z)$$
$$= \lambda(\theta(p,z^r))(W^E(p,z^r)-W^U(p,z^r)) + c(1-\beta).$$

Using $\eta\lambda(\theta)(W^E(p,z)-W^U(p,z)) = (1-\eta)\lambda(\theta)J(p,z) = (1-\eta)\theta(p,z)k$, we have (**??**).

[33] The absence of the qualification $\delta + \lambda(\theta) < 1$ is because in Lemma 2 we put very little restrictions on the stochastic process for z. Here, with a one-time unexpected increase, we do not need this restriction.

[34] As before, we assume free entry (without vacancy costs), and constant returns to scale production.

[35] Note that if island productivity was stochastic, rest unemployment can occur on these competitive islands as in Jovanovic (1987), Hamilton (1988), Gouge and King (1997) and Alvarez and Shimer (2011).

a reservation quality, z_c^r, characterised by the following equation

$$\beta \int \max\{y(p,z), y(p,z_c^r)\} dF(z) + (b-c)(1-\beta) = y(p,z_c^r). \tag{16}$$

The LHS describes the net benefit of switching islands, while the RHS the value of of staying employed earning y in the (reservation) island.

Proposition 3. *Given an increase in aggregate productivity:*

1. *Search frictions on the island make reallocation more procyclical relative to the competitive benchmark case with the same $F(z)$ and the same initial reservation productivity $z^r = z_c^r$.*
2. *With search frictions, if the production function is modular or supermodular (i.e. $y_{pz} \geq 0$), there exists a $c \geq 0$ under which reallocation is procyclical. With competitive markets on islands, if the production function is modular, reallocation is countercyclical, for any $\beta < 1$ and $c \geq 0$.*

Note that the first part of the Proposition does not say anything about the sign of dz^r/dp or dz_c^r/dp and hence if reallocation is procyclical our countercyclical in either the frictional or the competitive case. It does imply, however, that $dz^r/dp > dz_c^r/dp$ at $z^r = z_c^r$ and, hence, that search frictions within islands make reallocation more attractive to worker. The crucial difference between the two cases arises since the benefits of reallocation increase proportionally more when labor markets are frictional than when they are competitive. In particular, with competitive markets a higher aggregate productivity increases the expected gain of reallocation only through an increase in wages relative to the reservation island, $\mathbb{E}[y_p(p,z)/y_z(p,z^r) \mid z \geq z^r]$. With search frictions a higher aggregate productivity increases the expected gain of reallocation through both wages and the probability of finding employment, leading to $\mathbb{E}[(\theta(p,z)/\theta(p,z^r))(y_p(p,z)/(w(p,z)-b))((w(p,z^r) - b)/y_z(p,z^r)) \mid z \geq z^r]$. The term $(\theta(p,z)/\theta(p,z^r))$ shows the increase in the job finding rate relative to the reservation island, while the other terms describe the proportional increase in wages relative to the reservation island. Since workers are paid less than their marginal product, this proportional increase is higher in the frictional case, generating an extra benefit for reallocation.

The second part of the Proposition presents restrictions on the production technology that guarantee countercyclical reallocation with competitive labor markets, but is able to generate procyclical reallocation in the frictional case. It is useful to note, however, that in both cases dz^r/dp is more likely to be positive when the production function exhibits a higher degree of supermodularity. For example, when p and z are complements in total output (i.e. $y = pz$), the benefits of reallocation are $z/z^r > 1$ times higher than in the case in which p and z are perfect substitutes (i.e. $y = p + z$).

The next result shows that reallocations become more procyclical when workers are able to restrict their search, as implied by an increase in z^n. To show this, we once again consider a one time increase in aggregate productivity using the same stationary environment as before. Let z_{rs}^r describe the reservation reallocation productivity when workers sample from $z \geq z^n$ and let z^r describes the reservation reallocation productivity in the baseline case where $n = 0$.

Lemma 5. *Let $z_{rs}^r = z^r$. Then $dz_{rs}^r/dp > dz^r/dp$.*

This result follows as z^n does not change with aggregate productivity and workers who reallocate always draw islands from the set $z \in [z^n, \overline{z}]$. The associated payoffs in each of the new islands, $W^U(p, z)$, only increase through p. It follows that the higher z^n, the higher the expected gain of reallocation. However, as p increases and z^r also increases, the worker looses the benefit of moving to islands slightly above the reservation island as now the worker also prefers to reallocate away from those islands. The proof of Lemma 5 shows that the former always dominates the latter.

5.2 Countercyclicality of Job Separations Flows

Although the results presented in Section 2 show that job separations are countercyclical, in our model procyclical reallocation flows can add a force that pushes separation flows in a procyclical direction as the increased attractiveness of reallocation can feed back into separation decisions. The strength of this force and hence the implications of our model with respect to the cyclicality of job separations depends on whether there is rest unemployment.

As mentioned earlier, rest unemployment occurs when $z^s(p) > z^r(p)$. In this case, job separations are always countercyclical. This follows as the value of being unemployed does not depend directly on the value of reallocation. It depends on the island-specific value of unemployment, which rises less with p than the value of the match $M(p, z)$. Therefore for a higher p, the value of a match $M(p, z)$ will equal the value of unemployment $W^U(p, z)$ at a lower island z.

Formally, consider a one-time aggregate productivity shock, with permanent island components of productivity z. Note that all islands with rest unemployment have the same value of unemployment: $\underline{W}^U = b/(1-\beta)$. Then one can derive the slope of $z^s(p)$ from $M(p, z^s(p)) = W^U(p, z^s(p)) = \underline{W}^U$. Namely,

$$M(p, z) = y(p, z) + \beta[(1-\delta)M(p, z) + \delta W^U(p, z)] \Rightarrow (1-\beta)\underline{W}^U = y(p, z^s(p)) \tag{17}$$

$$\Rightarrow \frac{dz^s(p)}{dp} = -\frac{y_p(p, z^s(p))}{y_z(p, z^s(p))} \tag{18}$$

When $z^r(p) > z^s(p)$ and any worker that becomes unemployed in islands with $z \in [z^s(p), z^r(p)]$ prefers to reallocate (rather than rest), countercyclical job separations cannot be guaranteed. As long as the island is on or below the reallocation cutoff, the value of unemployment now is the value of reallocating (after sitting out one period of unemployment before being able to reallocate), $R(p) \stackrel{def}{=} W^U(p, z^r(p))$. The next result shows that the slope of $z^s(p)$ is an affine combination of (**??**) and the slope of $z^r(p)$.

Lemma 6. *With permanent island-specific productivity, and $z^s(p) < z^r(p)$ for p, it holds that*

$$-\frac{y_p(p, z^s(p))}{y_p(p, z^r(p))} + \frac{\beta\lambda(\theta(p, z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p, z^r(p))}\left(1 + \frac{y_z(p, z^r(p))}{y_p(p, z^r(p))}\frac{dz^r(p)}{dp}\right) = \frac{y_z(p, z^s(p))}{y_p(p, z^r(p))}\frac{dz^s(p)}{dp}. \tag{19}$$

The first term, $\frac{y_p(p,z^s(p))}{y_p(p,z^r(p))}$, is less than one when the production function is (super)modular and $z^r(p) > z^s(p)$. The second term, $\frac{\beta\lambda(\theta(p,z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z^r(p))}$, is positive and its magnitude depends on $dz^r(p)/dp$ (normalised by $\frac{y_z(p,z^r(p))}{y_p(p,z^r(p))}$). Lemma **??** then implies that the procyclicality of reallocation can feed back into the separation decisions. Alternatively, we can derive the slope of z^s explicitly as

$$\frac{dz^s}{dp} = -\frac{\frac{y_p(p,z^s)}{1-\delta} - \beta\frac{1-\eta}{\eta}\frac{\theta(p,z^r)y_p(p,z^r)}{w(p,z^r)-b}k}{\frac{y_z(p,z^s)}{1-\delta} - \beta\frac{1-\eta}{\eta}\frac{\theta(p,z^r)y_z(p,z^r)}{w(p,z^r)-b}k}.$$

It is not difficult to verify that dz^s/dp becomes negative when the production technology has a sufficiently low degree of supermodularity.

These results show that when no rest unemployment occurs, the degree of supermodularity of the production function plays a crucial role in determining the size of the feedback effect reallocation decisions have on separation decision. Namely, a higher degree of supermodularity makes procyclical reallocations more likely, while making countercyclical separations less likely. A lower degree of supermodularity does the opposite.

5.3 The Occurrence of Rest Unemployment

The previous section highlighted the importance of the relative position of the reservation functions with aggregate productivity for reallocation, $z^r(p)$, and separation, $z^s(p)$, for the extent of rest, search and reallocation unemployment. Importantly, their relative position is important: $z^s(p) > z^r(p)$ implies rest unemployment. In this section we want to study forces that affect the relative position of these two cutoffs. For simplicity, we do this for an aggregate productivity p held constant, while now incorporating that island specific productivity varies over time. Allowing for the latter is crucial to understand the occurrence of rest unemployment because a worker decides to stay unemployed in his island, even though there are no jobs currently around, when there is a high enough probability that the island's productivity will become sufficiently high in the future. The arguments below hold for any p, and hence tell us also how the entire reservation lines $z^r(p), z^s(p)$ move relative to each other.

An analytically tractable way to allow for time-varying island productivities, is to introduce a shock that triggers a new island productivity, randomly redrawn from the unconditional distribution of island productivities.[36] Using this setup we study how the expected lifetime values of remaining on an island, or reallocating to a different island, are affected by permanent and unexpected changes in the reallocation cost c, unemployment benefit flow b, and the degree of persistence of the island productivity shocks. Given that we hold p constant, we re-label $z^s(p)$ and $z^r(p)$ and use $\hat{z}^s$ as the reservation island productivity below which workers separate and $\hat{z}^r$ as the reservation island productivity below which workers reallocate, and drop the explicit reference to p elsewhere.

In this environment, the value of reallocation is given by

$$R = -c + \int W^U(z')dF(z'), \tag{20}$$

[36]This is a shock process similar to the one in Mortensen and Pissarides (1994), but now it shocks islands instead of firms. In the calibration below we will use an autoregressive process for island shocks; for this calibration, the same properties as derived below apply.

while the value of being unemployed on island with productivity z (measured at the production stage),

$$W^U(z) = \gamma\Big(b + \beta \max\{R, W^U + \max\{\lambda(\theta)(1-\eta)(M(z) - W^U(z)), 0\}\}\Big) + (1-\gamma)\mathbb{E}[W^u(z)], \quad (21)$$

where $(1-\gamma)$ is the probability that the island productivity is drawn anew. We have also used the fact that the value of unemployment does not change if the z-shock arrives at the beginning of period $t+1$ or at the beginning of the production stage in period t. The two ways for an unemployed worker on an unproductive island to return to production can be seen clearly in (**??**). Passively, he can wait till the current island's conditions improve, when $\max\{R, W^U + \max\{\lambda(\theta)(1-\eta)(M(z) - W^U(z)), 0\}\} = W^s$. Or, actively, he can pay the fixed cost and sample the productivity from a different island, which makes this term equal R.[37]

The term $W^s - R$ captures the difference in expected value between waiting unemployed on the same island for one period, and reallocating immediately. If $R > W^s$, then $z^r > z^s$; when $R < W^s$, then $z^s > z^r$; when $R = W^s$, $z^r = z^s$. From (**??**) and (**??**) it follows that

$$W^s - R = \frac{1}{1-\beta\gamma A(z^r)}\left(-\beta\gamma \int_{z^r} \lambda(\theta(z))(1-\eta)(M(z) - W^u(z))dF(z) + (1-\beta\gamma)c\right); \quad (22)$$

where $A(z^r) = F(z^r)$ if $z^r > z^s \geq \underline{z}$, and $A(z^r) = 1$ if $z^r \leq z^s$. The expected utility difference between waiting (or having to wait) at least period and reallocating instantly involves paying the reallocation cost c (taking into account that waiting possibly means paying the reallocation cost the next period), and on the other side, being able to apply for jobs on the new island tomorrow– whereas a stayer, if he does not get an island shock, does not apply for jobs tomorrow. As the cost of reallocation, the unemployment benefit flows, or the persistent of the island productivity changes, the value of waiting one period (with the option of reallocating the next period) and immediate reallocation can move closer or further apart.

The difference $W^s - R$ directly affects the distance between z^s and z^r: an increase in the former often directly leads to an increase in the latter. Denote the parameter of interest generically by ω; below we will make explicit the dependence on this parameter where necessary for clarity. The reservation quality for separation and reallocation then satisfy implicitly, respectively

$$M(\omega, z^s(\omega)) - W^s(\omega) = 0 \quad (23)$$

$$\lambda(\theta(\omega, z^r(\omega)))(1-\eta)(M(\omega, z^r(\omega)) - W^s) + (W^s(\omega) - R(\omega)) = 0 \text{ if } R(\omega) > W^s(\omega) \quad (24)$$

while, with this stochastic process for island-specific shocks, if $W^s(\omega) > R(\omega)$, then $z^r(\omega) = \underline{z}$. Consider the case of $R(\omega) > W^s(\omega)$ first. We can see that (**??**)-(**??**) defines $z^r(\omega), z^s(\omega)$ as implicit functions of $M(\omega, z) - W^s(\omega)$ and $W^s(\omega) - R(\omega)$. The first term is given by

$$\begin{aligned} M(\omega, z^s(\omega)) - W^s(\omega) = {} & xz^s(\omega) - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega, z) - W(\omega, z), W^s(\omega) - R(\omega)\}] \\ & + \beta\gamma(W^s(\omega) - R(\omega)) \quad (25) \\ M(\omega, z^r(\omega)) - W^s(\omega) = {} & xz^r(\omega) - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega, z) - W(\omega, z), W^s(\omega) - R(\omega)\}] \\ & + \beta\gamma(1 - \lambda(\theta(\omega, z^r(\omega))(1-\eta)(M(\omega, z^r(\omega)) - W^s(\omega)). \quad (26) \end{aligned}$$

[37] The fixed cost of island reallocation is important. If there was only the time cost of sampling alternative islands, an unemployed worker on an inactive island has no opportunity cost of sampling new islands.

For the response of $z^s(\omega), z^r(\omega)$, we can take the derivative of (**??**)-(**??**), which leads to the equationswhere we make explicit the dependence on ω if and only if the derivative has to be taken with respect to it.

$$x\frac{dz^s(\omega)}{d\omega} = -\frac{d}{d\omega}\bigg(xz^s - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega,z) - W(\omega,z), W^s(\omega) - R(\omega)\}] \\ + \beta\gamma(W^s - R)\bigg) - \beta\gamma\frac{d(W^s(\omega) - R(\omega))}{d\omega} \tag{27}$$

$$x\frac{dz^r(\omega)}{d\omega} = -\frac{d}{d\omega}\bigg(xz^r - b + \beta(1-\gamma)\mathbb{E}_z[\max\{M(\omega,z) - W(\omega,z), W^s(\omega) - R(\omega)\}] \\ + \beta\gamma(1-\lambda(1-\eta))(M(z^r) - W^s)\bigg) - \frac{1-\beta\gamma(1-\lambda)}{\lambda}\frac{d(W^s(\omega) - R(\omega))}{d\omega}. \tag{28}$$

It follows that the sign of the derivative of $z^r - z^s$ with respect to b or c is the opposite of the sign of the corresponding derivative $W^s - R$, because $\beta\gamma < \frac{1-\beta\gamma(1-\lambda)}{\lambda}$, and when taking derivatives the differential terms within the large brackets in (**??**) and (**??**) are identical.

The next result shows that higher reallocation cost work in favor rest unemployment, as does a higher value of leisure b. The idiosyncratic shock process for islands is also an important force for the extent of rest unemployment: less persistence in island-specific productivity pushes workers towards rest unemployment, everything else equal.

Lemma 7. *We can derive the following about the response in values of reallocation, waiting, surplus and unenployment, and the cutoffs of reallocation and separation, to changes in parameters*

- *(i)* $\frac{d(R-W^s)}{dc} < 0$*, (ii)* $\frac{d(M(z)-W^u(z))}{dc} > 0$ *for all active islands; (iii) and* $z^r - z^s$ *is decreasing in c, strictly if* $z^r > z^s$*.*
- *(i)* $\frac{d(R-W^s)}{db} < 0$*, (ii)* $\frac{d(M(z)-W^u(z))}{db} < 0$ *for all active islands; (iii) and* $z^r - z^s$ *is decreasing in* b *(while both* z^r *and* z^s *are increasing in* b*)*
- *(i)* $\frac{d(R-W^s)}{d\gamma} > 0$*, (ii) there exists a cutoff* $z^\gamma > \max\{z^r, z^s\}$ *such that for all* $z > z^\gamma$ $\frac{d(M(z)-W^u(z))}{d\gamma} > 0$ *and* $\frac{dW^u(z)}{d\gamma} > 0$*; while* $\frac{d(M(z)-W^u(z))}{d\gamma} < 0$ *and* $\frac{dW^u(z)}{d\gamma} < 0$ *for* $z^\gamma > z > z^\gamma > \max\{z^r, z^s\}$*. In expectation,* $\frac{d(\mathbb{E}_z[M(z)-W^u(z)])}{d\gamma} > 0$ *and* $\frac{d\mathbb{E}_z[W^u(z)]}{d\gamma} > 0$*. And (iii), if* $z^r > z^s$ *and* $R - W^s$ *is not too large, or if* $z^s > z^r$*, then* $z^r - z^s$ *is increasing in* γ*.*

The first result is intuitive: an increase in the direct cost of reallocation should make the worker more conservative about changing islands. One should however, incorporate that if $z^r > z^s$, the value of waiting incorporates that, if nothing changes in island productivity, the worker will incur the same additional reallocation cost one period later; but island productivity might instead change. In effect, the worker has two ways of improving his productivity: moving to a different island, at fixed cost c; and waiting for productivity to improve, at a time cost. If the fixed cost c goes up, incurring the time cost has become relatively more attractive. The increase in the explicit reallocation cost also leads to large surpluses of employment for islands of given productivity. Behind this lies that if a bad island-productivity shock hits, the unemployed worker can reallocate immediately, while the employed worker has to wait one period. Since the value of waiting has increased, the utility loss of being employed instead of unemployed when a bad shock hits is smaller; this raises the value of employed relative to unemployment. However, on the RHS of (**??**), the surplus of unemployment does not go up enough to offset the rise in c , thus leading to a decline in $R - W^s$.

From the perspective that improving ones productivity takes either an explicit reallocation cost, or time (or both), it is intuitive that a rise in b lowers the effective cost of waiting, and thus drives down $R - W^s$. Perhaps it is worth noting here is that b *unambiguously* leads to a smaller difference $R - W^s$, and a smaller difference $z^r - z^s$, while the behavior in response to aggregate productivity depended also on the complementarity between aggregate and island productivity.

The persistence of island-specific productivity is also important for reallocation behavior, because it affects the expected time until the island productivity of a bad island improves, and simultaneously, the expected time that currently high island productivity can be enjoyed. These forces are very clear at the extreme values of γ. Consider first $\gamma = 0$ and $c > 0$. In this case, a worker would never reallocate for any $F(z)$, as (**??**) implies that $W^U(z) = W^s$ for every z, and therefore $R = W^s - c$. Intuitively, a worker would not pay the reallocation cost if tomorrow's productivity on his own island is as uncertain as the productivity of any other island.[38] At the other extreme, $\gamma = 1$ and $c > 0$ is sufficiently low such that $(R - c)(1 - \beta) > b$. In this case, there will be no rest unemployment as islands never change productivity, and in addition the value of consuming b forever is lower than reallocating and becoming unemployed on a randomly drawn new island. Consider now the intermediate case, $\gamma \in (0, 1)$, and increase γ. This implies a force that pushes the value of waiting downwards, since the probability that the island is subject to a shock improves the workers productivity without reallocation has dropped. Simultaneously, the value of reallocation is pushed upwards, because finding a good island now means that its productivity can be enjoyed for considerably longer time; this positive effect on the better islands dominates the negative effect on the worse islands, in expectation, yielding a higher average value of unemployment across islands.

5.4 Decomposition of the Unemployment Rate

We now turn to analyse the aggregate unemployment rate in our economy. We first consider a dynamic decomposition of the unemployment rate based on the flow equations described in Appendix B and the reservation productivities, z^r and z^s. We then consider a decomposition based on the cross-sectional distribution of unemployed workers in the economy, to disentangle the degree of mismatch across islands.

5.4.1 Dynamic Decomposition

Consider the case in which $z^s > z^r$. For all islands with idiosyncratic productivities $z > z^s$, the sources of unemployment are (i) search frictions, workers not being lucky enough to get a job, (ii) reallocation frictions, workers transiting from one island to another and (iii) exogenous

[38] No reallocation, however, does not necessarily mean workers decide to become rest unemployed. For rest unemployment to arise we require $F(z)$ to be sufficiently dispersed, such that some islands have productivities so low that the current loss incurred in production more than offsets the expected gain of being matched and potentially productive in the future. Rest unemployment occurs in the islands in which there is no gain of forming, or continuing existing matches, and thus $\hat{z}^s > \hat{z}^r = \underline{z}$.

job separations. The flow equations in section 2.4 then imply that, given the unemployment and employment rates at the start of the period, u_t and e_t, respectively, the next period unemployment rate in any island with $z > z^s$ is given by

$$u_{t+1}(z) = (1 - \lambda(\theta(p,z)))u_t(z) + \int_{\underline{z}}^{z^r} u_t(z')dF(z') + \delta e_t(z). \tag{29}$$

For islands that exhibit productivities $z \in [z^r, z^s]$ we have that the sources of unemployment are (i) rest unemployment, (ii) reallocation frictions and (iii) endogenous separations. Hence, the next period unemployment rate in any of these islands is given by

$$u_{t+1}(z) = u_t(z) + \int_{\underline{z}}^{z^r} u_t(z')dF(z') + e_t(z). \tag{30}$$

Note that in this case, all employed workers decide to separate. For islands with $z < z^r$ the only source of unemployment is due to reallocation frictions since workers that reallocate arrive randomly to this islands every period and those that started unemployed reallocated some where else. The next period unemployment rate in any of these islands is given by

$$u_{t+1}(z) = \int_{\underline{z}}^{z^r} u_t(z')dF(z'). \tag{31}$$

Integrating across islands then gives the dynamics of the unemployment rate in the economy.

When $z^r > z^s$ we have that $u_{t+1}(z)$ is given by equation (**??**) for those islands with $z > z^r$. For islands with $z \in [z^s, z^r]$ we have that the unemployment pool is made up of those employed workers that exogenously displaced and those workers that arrive due to reallocation. Namely,

$$u_{t+1}(z) = \int_{\underline{z}}^{z^r} u_t(z')dF(z') + \delta e_t(z). \tag{32}$$

For islands with $z < z^s$ we have that $u_{t+1}(z)$ is given by equation (**??**). As before, integrating across islands gives the dynamics of the aggregate unemployment rate for this case.

We use these decompositions to show how the proportions of workers that are unemployed due to search frictions, reallocation frictions, job destruction or are rest unemployed change over the business cycle.

5.4.2 Mismatch

In our economy reallocation frictions prevent workers from going to the island with the best conditions. If the planner could eliminate these frictions, then the allocation that maximises output would be to move all workers to that island. Free entry would guarantee that enough firms post vacancies in this island. The aggregate unemployment rate would then be determined by the degree of search frictions present in such an island. Our model then implies that to measure the degree of mismatch one should compare such an unemployment rate with the unemployment rates across islands obtained when reallocation frictions are present. This measure, however, penalises quite heavily islands with low realisations of z.[39]

As an alternative we compare the unemployment rates across islands with the aggregate unemployment rate, $u(\widetilde{z})$, that arises if all islands had productivity $\widetilde{z}$, the average productivity of the ergodic distribution of z. Following Jackman and Roper (1987), the number of mismatched unemployed workers as a proportion of the aggregate unemployment rate u_t (the one that prevails in the economy with search and reallocation frictions) can be measured by

$$M_t^u = \frac{1}{2}\sum_{i=1}^{I} \mid \frac{u_{it} - u(\widetilde{z})}{u_t} \mid . \tag{33}$$

Using $u(\widetilde{z})$ in the above index is attractive as it provides a mismatch measure based on the long run expected rate of unemployment that arises in our economy. Indeed, since all islands face the same distribution of z, as $t \to \infty$ our island economy converges to a representative agent one where the average unemployment rate face by each island is $u(\widetilde{z})$. In this limit, reallocation frictions do not bind and search frictions become the only source of unemployment (as in Pissarides, 2001).[40]

6 Occupational Human Capital

Kambourov and Manovskii (2009a) argue that there are substantial returns to occupational tenure, in the order of 20% for a tenure of ten years. To capture this feature, we now consider a simple extension of the model that allows employed workers to accumulate occupational human capital. We assume three levels of occupational human capital, such that x_j denotes the productivity of a worker with human capital $j = 1, 2, 3$ and $x_3 \geq x_2 \geq x_1$. The total output of a firm in island i, at time t and employing a worker with human capital level j is then $y(p_t, z_{it}, x_j)$, where y increases in x_j. Human capital accumulation follows a Markov chain with transition matrix

$$\begin{bmatrix} \gamma_{11} & \gamma_{12} & 0 \\ 0 & \gamma_{22} & \gamma_{23} \\ 0 & 0 & \gamma_{33} \end{bmatrix},$$

where γ_{jm} denotes the per period probability that an employed worker with productivity x_j changes his productivity to x_m.

We assume that productivities are drawn at the start of the period and human capital increases step-wise. We also assume that a worker's human capital remains constant throughout any unemployment spell in his current island. Hence, the only way for a worker to loose his human capital

[39] In a recent paper Sahin, Song, Topa and Violante (2010) consider a similar allocation problem. In their case, however, the social planner takes as given the distribution of employed workers and vacancies and chooses the unemployment and next period's vacancy rates in each island to maximise total output. This implies that, for a given distribution of vacancies, unemployed workers are allocated in such way that their marginal contribution to the matching process is equalises across islands.

[40] Jackman, Layard and Savouri (1991) also consider a mismatch indicator based on a comparison between the current unemployment rates across island and with a measure of long run unemployment. In their case, however, the latter is the unemployment rate consistent with price stability, the NAIRU.

is to reallocate. In such a case the worker arrives to the new island with the lowest human capital level.

To keep the analysis as parsimonious as possible, we divide an island into three distinct sub-markets, one for each x_j. A new arrival to the island enters the sub-market x_1. Once employed this worker can increase his human capital to x_2 with probability γ_{12} each period, in which case he transits to sub-market x_2. Once in this sub-market an employed worker can move to sub-market x_3 with probability γ_{23} every period. We also assume that each firm observes the conditions in each island and corresponding sub- market when deciding to post vacancies and that the free entry condition holds at the level of each sub-market.

Note that under this extension the structure of the basic model remains intact. Our assumptions imply that we have effectively tripled the mass of labor markets, allowing employed workers to randomly transit between the labor markets that exist within each island according to the above transition matrix. In Appendix C we show that we can use the arguments of Lemmas 3 and 4 to show that Propositions 1 and 2 also hold in this case and guarantee existence, uniqueness and efficiency of equilibrium.

The importance of occupation-specific human capital is that it creates ex-post heterogeneity among workers. For each human capital level j there are now two reservation productivities: (i) $z^r(p, x_j)$ that characterise the reallocation decisions of workers with human capital x_j; and (ii) $z^s(p, x_j)$ that characterises these workers separation decisions.

A higher level of human capital implies that a given island becomes more attractive to remain in as an unemployed worker, but also that matches become more productive, and workers would be more willing to stay employed. In other words, both z^r and z^s drop as human capital x rises. Still, an increase in human capital tends to make a worker more attached to the island than to employment. In the setting of the previous section, we can show that the cutoff for z^r will drop more than the cutoff in z^s, when we increase x only for the island in which the worker currently is. (For simplicity, this increase is taken to be permanent – this would correspond to a one-time, permanent, unexpected increase in island-specific human capital).

Lemma 8. *Consider the setting in section* **??** *without aggregate productivity changes, and an island-specific production function xz. Consider a one-time, unexpected, permanent increase in x in the current island, wlog from starting situation $x = 1$. This productivity gain lost forever when reallocating to a different island (x will be reset to 1 forever). Then*

$$\frac{d(z^r(x) - z^s(x))}{dx} < 0.$$

These reservation productivities imply, for example, that less experienced workers are more likely to reallocate, while the more experience workers are more likely to become rest unemployed. This is intuitive as a worker's opportunity cost of reallocating is increasing with occupational human capital.

7 Quantitative Analysis

In this section we analyse the quantitative properties of the model. Here we evaluate the version with occupational human capital accumulation. The block recursive structure allow us to solve the equilibrium computationally from a fixed point of a mapping by simply iterating on the value functions with state variables p, z and x only.[41] This feature makes it relatively easy to calibrate our model using Simulated Minimum Distance (SMD).

7.1 Calibration

We implement the SMD procedure by first setting a few parameters values and functional forms. In particular, we set the time period to a week and the average working life to 40 years, with a constant probability of death. The discount rate is set such that the implied yearly interest rate is 4 percent and hence $\beta = 0.9992$. We assume that p and z satisfy an AR(1) processes where ρ_i and σ_i describe the persistence parameter and variance of the process, respectively, for $i = p, z$, these we will estimate below. We normalise the lowest occupational human capital level $x_1 = 1$. We set γ_{11} and γ_{22} to obtain an average occupational tenure of 5 and 10 years and set $\gamma_{33} = 1$. We consider a multiplicative aggregate productivity function such that $y = pzx$ and assume a Cobb-Douglas matching function within each island $m(\theta) = \theta^{\eta}$. We let unemployed workers who reallocate draw randomly from the top half of the island distribution. The vector of parameters that are left to be estimated is $(\delta, k, c, b, \eta, \rho_p, \rho_z, \sigma_p, \sigma_z, \underline{z}_{corr}, x_2, x_3)$, $\underline{z}_{corr}$ is a simply rescaling of the island-distributions, such that average island-productivity equals 1 in the absence of business cycle shocks.

We calibrate the remaining 11 parameters by targeting the following sets of moments. We target the time-series average of labor market flows and stocks: the mean of the aggregate unemployment, job finding and separation rates and the mean of aggregate job finding rate involving an occupational change. Linked to η, we target the elasticity of the aggregate job finding rate with respect to aggregate labor market tightness ($\widehat{\beta}_1$), which is obtained from the OLS estimation of the reduce form matching function $ln(\lambda_t) = \beta_0 + \beta_1 ln(\theta_t) + \varepsilon_t$, where ε_t is assumed to be gaussian noise.[42] Intuitively, these moments contain information about the first five parameters in the vector above. We target the autocorrelation and volatility of aggregate output per worker, which relate closely to σ_p, ρ_p, but not perfectly, as aggregate output per worker is affected by mobility responses of agents, e.g. when unproductive matches are broken up endogenously, and workers move to better islands. The average returns to tenure at 5 and 10 years as they give information about x_2 and x_3. To calibrate the island specific productivity shocks, ρ_z and σ_z we target the the aggregate unemployment duration distribution, and the probability of repeat mobility after changing island once (from Kambourov and Manovskii 2009). This yields the parameter values in table **??**.[43]

Two moments that we will take from the literature are the 5 and 10 year returns to occupational

[41] This stands in contrast, e.g. to Lkhagvasuren (2010), who is only able to solve a model with reallocation flows in the absence of aggregate shocks - citing computational difficulties.

[42] The estimates of β_0 and β_1 are significant to a 1% level and the regression's $R^2 = 0.81$.

[43] In the near future, we will also use the SIPP to estimate most of the moments described above.

Table 6: Calibrated Parameters

δ	k	c	b	η	ρ_p	σ_p	ρ_z	σ_z	$\underline{z}_{corr}$	x_2	x_3
0.007	70.5	32.5	0.74	0.25	0.985	0.004	0.998	0.01	0.3	1.25	1.5

Table 7: Model (and data) moments targeted

	u	θ	y	wage	sep	reall	jf
	0.06	0.008	1.062	0.909	0.022	0.002	0.46
	ten5yr	ten10yr	dur<5wk	dur<15wk	dur<27	dur>27	
model	0.09	0.14	0.49	0.24	0.09	0.18	
data	0.12	0.18	0.38	0.30	0.14	0.18	

tenure, given that the SIPP has a panel structure that is relatively short, making it difficult to estimate these returns accurately. For this reason we use the IV-GLS estimates for 3-digit occupations reported in Kambourov and Manovskii (2009a) - Table 4 (column 15) based on the Panel Survey of Income Dynamics (PSID).

In the current version, we target the average job finding rate and separation rate from CPS. We take an agnostic elasticity of the matching function, 0.5, in the range of Petrongolo and Pissarides (2001). In Table **??**, we report the model generated moments, in combination with the corresponding data moments, for returns to occupational tenure and the unemployment duration distribution. The unemployment duration distribution comes from FRED2.

Two things deserve perhaps some additional highlighting: first, notice that the measured returns to occupational tenure are lower than the returns to tenure keeping island-quality constant. This occurs because of selection: workers stay long in occupations that are initially good, and become significantly more productive over time relative to workers without occupational tenure on an island. However, good islands tend to regress towards the mean, implying that relative to the rest of the labor market they are losing ground. What remains is a measured 9% return to 5 years of occupational tenure, and 14% return to 10 years of occupational tenure. Secondly, notice that the model is quite successful at matching the unemployment duration distribution, creating a large enough percentage of long-term unemployed. In the next section, we will see what is behind this.

7.2 Results

Table **??** shows the comparison of the business cycle data moments generated by the model. A number of things are noteworthy: this model produces significant amplification of business cycle shocks for unemployment. The unemployment fluctuations in the data (logged HP filtered series, filtered with factor 1600), at about 90% of those in the data (0.949 vs 0.125). Separations are countercyclical (though not extremely so, correlated with output per worker at -0.15), while reallocation through unemployment is procyclical (a correlation of 0.81). Meanwhile, the Beveridge curve is preserved, and in fact going strong at a correlation between output and vacancies of -0.9044. Other correlations are also solid: job finding is highly correlated with labor market tightness (0.99), while both

Table 8: Business Cycle Statistics of the calibrated model

statistc	u	v	tghtness	sep	jf	prod	wage	reall
stdev	0.0949	0.0410	0.1331	0.0118	0.0717	0.0158	0.0180	0.0365
autocorr	0.8100	0.6821	0.7941	-0.0766	0.7665	0.7401	0.7401	0.4338
u	1	-0.9044	-0.9913	-0.0427	-0.9717	-0.9485	-0.9484	-0.6266
v		1	0.9526	-0.2186	0.9781	0.9920	0.9921	0.8595
theta			1	-0.0369	0.9938	0.9816	0.9816	0.7114
sep				1	-0.0936	-0.1563	-0.1561	-0.3344
jf					1	0.9937	0.9938	0.7617
prod						1	1.0000	0.8101
wage							1	0.8102
reall								1

are highly correlated with output per worker.

To see what is behind this, it is worthwhile to look at the decision rules for reallocation and separation. In Figure **??**, we can see three sets of $z^s(p), z^r(p)$ functions and each set corresponds to a occupational human capital level. In the graph aggregate productivity is on the x-axis and island-productivity rank on the y-axis. First, the highest two (dark-blue and dark-green) lines are for workers without any occupational human capital. For a small set of islands , these workers will rest in a downtown. However, a good many of these unemployed will be either looking for a job in their occupation or, if their island worsens below the dark blue line, reallocate to better islands. Thus, although some countercyclical separations are generated, these workers are not terribly attached to their occupation, and will often move to other occupations. In good times, inexperienced workers will directly reallocate upon an endogenous separation; there is no rest unemployed for these workers.

We then see that workers get progressively more attached to their island as their occupational human capital increases. Those with the highest level of occupational human capital will only leave in case the island-productivity is almost at the lowest island productivity (in the calibration). When the economy improves, resting workers become more eager to reallocate if their island has not improved in the mean time, as the clear upward slope on the yellow line (for the highest human capital level) and the red line (for the intermediate level) demonstrate. Workers with occupational human capital tend to quit at a significantly higher level of island productivity than reallocate, and these quits are clearly negatively correlated with the business cycle. This calibration is suggestive that the presence of human capital creates significant rest unemployment.

In Figure **??**, we plot the mass of unemployed workers as a function of aggregate productivity on the x-axis. One can see that most of the unemployment fluctuations are indeed caused by workers waiting in their island for both island and aggregate conditions to improve.

It is now intuitive that amplification of unemployment fluctuations can occur. In bad times, workers separate endogenously into unemployment; but these workers do not separate to reallocate as reallocation is less attractive in bad times. Instead, they wait in their market for the aggregate

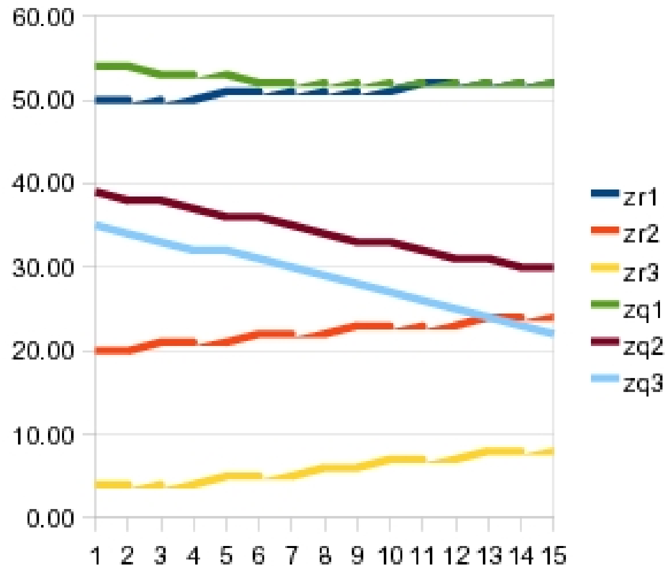


Figure 4: Reservation functions for three occupational human capital levels (3=highest)

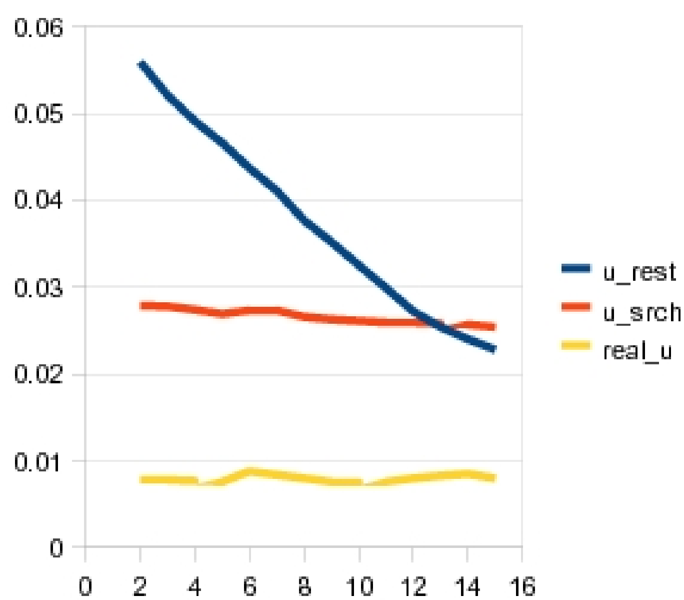


Figure 5: Mass of unemployed as a function of aggregate productivity state (blue=rest unemployment; red=search unemployment; yellow=reallocation unemployment)

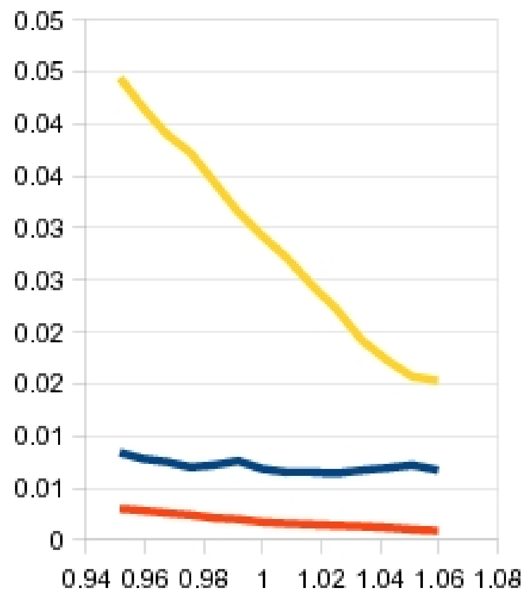


Figure 6: Mass of rest unemployed for different human capital levels; yellow: highest human capital level, blue: intermediate, and red: lowest level.

situation and the island-specific situation to improve. The inflow into unemployment is not met by an increase in vacancies, because these workers are in unattractive markets.

If the island-specific situation stays bad enough, but the aggregate situation becomes more favorable, rest unemployment workers might consider reallocating. Upon arrival at islands with enough island-specific productivity, there will be vacancies for these workers, lifting them out of unemployment, and causing the overall unemployment rate to decline. It is important that this only happens when aggregate productivity has improved enough to make reallocation profitable for unemployed workers. If on the other hand, the island-specific situation improves, vacancies can also start to be posted in the original market again, and the workers are able to move back into employment. However, as long as the situation in the market and in the aggregate is bad enough, no vacancies are created for these workers.

There is a second force that amplifies volatility. Islands in which the surplus of employment is very small (workers are close to separating into unemployment) have low job finding rates, but those are sensitive to aggregate shocks, along the lines of Hagedorn and Manovskii (2008). Search unemployed workers are however not randomly distributed over islands, but tend to be relatively more prevalent in the small-surplus islands at a moment in time (in better islands, they would be hired out of the unemployment pool already). Thus, the job finding response of active islands is weighted towards islands with more unemployed, which tend to be islands with small match surpluses, which have stronger responses to aggregate fluctuations.

Finally, we can study explicitly how workers with different human capital levels contribute to the mass of rest unemployed, as a function of the aggregate productivity state. We see in figure **??** that the largest proportion of rest unemployed are those with the highest level of human capital. This, however, should be seen from the perspective that with the three-type distribution in the calibration, this is also the largest group of workers. What is noteworthy from Figure **??** is that the mass of rest unemployed of those with high human capital is more sensitive to the business cycle.

8 Conclusions

In this paper we have presented a tractable general equilibrium framework to study the evolution of aggregate unemployment over the business cycle by considering different sources of unemployment. We focused on workers' decisions to search, rest, reallocate and separate as causes of unemployment. The model provides a tractable analysis of the interaction between search and reallocation frictions. We show that when search frictions are present in local labor markets, worker reallocation is more procyclical than if labor markets are competitive. This is consistent with the observed procyclicality of workers across occupations and regions. Further, we present a decomposition of the unemployment rate into its constituent parts and provide a measure of mismatch for our economy. We then calibrate our model and provide quantitative evaluation of its implications.

An important restriction of our analysis is to not consider the implications of job to job transitions as a source of reallocation across (or within) islands. Recent evidence has shown that job transitions without an intervening spell of unemployment account for a sizable part of workers flows over the business cycle. A simple way of introducing such flows in our framework is by allowing employed workers to quit, reallocate and apply for jobs somewhere else during the same period. The drawback of this approach is that the worker looses his current job as an outside option. Unfortunately, allowing for the later complicates matters dramatically in our setup. Menzio and Shi (2011) suggest using directed search across islands to gain tractability. Although we believe that this will be indeed the case, we leave this extension for future research.

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Appendix

A Proofs

Proof of Lemma 1 Consider a firm that promised $W \geq W^U(p,z)$ to the worker such that the expected payoff to the firm is given by $J(p,z,W)$ solving (**??**). Now consider an alternative offer $\hat{W} \neq W$ which is also acceptable to the unemployed worker, provides the same contingent continuation values $\tilde{W}^{E'}(p',z'|p,z,W)$ to the worker as W and implies $J(p,z,\hat{W})$ solves equation (**??**). Risk neutrality then implies that

$$J(p,z,W) \geq J(p,z,\hat{W}) + (\hat{W} - W),$$

if the firm provides the worker with $\hat{W}$ using the optimal policy associated with providing W. Note that the last term in the RHS of the inequality makes up for the difference in value by offering the worker a payment (reduction) today. Similarly, if the firm provides the worker with W using the optimal policy associated with $\hat{W}$, we have that

$$J(p,z,\hat{W}) \geq J(p,z,W) - (\hat{W} - W).$$

Hence it must be that $J(p,z,\hat{W}) = J(p,z,W) + W - \hat{W}$ for *all* $M(p,z) \geq W, \hat{W} \geq W^U$. Differentiability of J with slope -1 follows immediately. Moreover, $M(p,z,W) = W + J(p,z,\hat{W}) + \hat{W} - W = M(p,z,\hat{W}) \equiv M(p,z)$. Finally, if $W'(p',z') < W^U(p',z')$ is offered tomorrow while $M(p',z') > W^u(p',z')$, it is a profitable deviation to offer $W^U(p',z')$, since $M(p',z') - W^U(p',z') = J(p',z',W^U(p',z')) > 0$ is feasible. This completes the proof of Lemma 1.

Proof of Lemma 2 Since we confine ourselves to one island, with known continuation values $J(w,p,z)$ and $W^U(p,z)$ in the production stage, we drop the dependence on p,z for ease of notation. Free entry implies $k = q(\theta)J(W) \Rightarrow \frac{dW}{d\theta} < 0$. Notice that it follows that the maximand of workers in (**??**), subject to (**??**) is continuous in W, and provided $M > W^U$, has a zero at $W = M$ and at $W = W^U$, and a strictly positive value for intermediate W: hence the problem has an interior maximum on $[W^U, M]$. What remains to be shown is that the first order conditions are sufficient for the maximum, and the set of maximizers is singular.

Solving the worker's problem of posting an optimal value subject to tightness implied by the free entry condition yields the following first order conditions (with multiplier μ):

$$\begin{aligned} \lambda'(\theta)[W - W^U] - \mu q'(\theta)J(W) &= 0 \\ \lambda(\theta) - \mu q(\theta)J'(W) &= 0 \\ k - q(\theta)J(W) &= 0 \end{aligned}$$

Using the constant returns to scale property of the matching function, one has $q(\theta) = \lambda(\theta)/\theta$. This implies, combining the three equations above, to solve out μ and $J(W)$,

$$0 = \lambda'(\theta)[W(\theta) - W^U] + \frac{\theta q'(\theta)}{q(\theta)}k \equiv G(\theta),$$

where we have written W as a function of θ, as implied by the free entry condition. Then, one can derive $G'(\theta)$ as

$$G'(\theta) = \lambda''(\theta)[W(\theta) - W^U] + \lambda'(\theta)W'(\theta) + \frac{d\varepsilon_{q,\theta}(\theta)}{d\theta},$$

where $\varepsilon_{q,\theta}(\theta)$ denotes the elasticity of the vacancy filling rate with respect to θ and

$$\frac{d\varepsilon_{q,\theta}(\theta)}{d\theta} = \frac{q'(\theta)k}{q(\theta)} + \frac{\theta[q''(\theta)q(\theta) - q'(\theta)^2]k}{q(\theta)^2}.$$

Since the first two terms in the RHS are strictly negative, G' is strictly negative when $\varepsilon_{q,\theta}(\theta) \leq 0$. The latter then guarantees there is a unique $\tilde{W}_f$ and corresponding θ that maximizes the worker's problem. This completes the proof of Lemma 2.

Proof of Lemma ?? First we show that the operator T maps continuous functions into continuous functions. Note that $\theta \in [0, 1]$, for all p, z and $W^U(p, z)$, $M(p, z)$ and $\lambda(\theta)$ are continuous functions. The Theorem of the Maximum then implies that $S(p, z)$ is also a continuous function. That T maps continuous functions into continuous functions then follows as the $\max\{M(p', z'), W^U(p', z')\}$ is also a continuous function. Moreover, since the domain of p, z is bounded, the resulting continuous functions are also bounded.

To show that T defines a contraction, consider two functions $\tilde{M}, \tilde{M}'$, such that $\|\tilde{M} - \tilde{M}'\|_{\text{sup}} < \varepsilon$. Then it follows that $\|W^U(p, z) - W^{U\prime}(p, z)\|_{\text{sup}} < \varepsilon$ and $\|M(p, z) - M'(p, z)\|_{\text{sup}} < \varepsilon$, where W^U, M are part of $\tilde{M}$ as defined in the text. Since $\|\max\{a, b\} - \max\{a', b'\}\| < \max\{\|a - a'\|, \|b - b'\|\}$, as long as the terms over which to maximize do not change by more than ε in absolute value, the maximized value does not change by more ε. The only maximization for which it is nontrivial to establish this is $\max\{\int W^U(p, z)dF(z) - c, S(p, z) + W^U(p, z)\}$. The first part can be established readily: $\| \int \Big(W^U(p, z) - W^{U\prime}(p, z)\Big)dF(z)\| < \varepsilon$. We now show that this property holds for $\|S(p, z) + W^U(p, z) - S'(p, z) - W^{U\prime}(p, z)\|$.

Consider first the case that $M - W > M' - W'$. Then, we must have $\varepsilon > W' - W \geq M' - M > -\varepsilon$. Construct $M'' = W' + (M - W) > M'$ and $W'' = M' - (M - W) < W'$. Call $S(M - W)$ the maximized surplus $\max_\theta\{\lambda(\theta)(M - W) - \theta k\}$ and θ the maximizer; likewise $S(M' - W')$ and θ'. Then

$$\begin{aligned} -\varepsilon < S(M' - W'') + W'' - S(M - W) - W &\leq S(M' - W') + W' - S(M - W) - W \\ &\leq S(M'' - W') + W' - S(M - W) - W < \varepsilon \end{aligned}$$

where $S(M' - W'') = S(M - W) = S(M'' - W')$ by construction. Note that the outer inequalities follow because $M - M' > -\varepsilon, W' - W < \varepsilon$.

Likewise, consider the case where $M' - W' > M - W \geq 0$. Then

$$\begin{aligned} \varepsilon > S(M' - W'') + W'' - S(M - W) - W &> S(M' - W') + W' - S(M - W) - W \\ &> S(M'' - W') + W' - S(M - W) - W > -\varepsilon \end{aligned}$$

Hence $\|S(p, z) + W^U(p, z) - S'(p, z) - W^{U\prime}(p, z)\| < \varepsilon$. It then follows that $\|T(\tilde{M}(p, z, 1)) - T(\tilde{M}'(p, z, 1)\| < \beta\varepsilon$ for all p, z, and $\|\tilde{M} - \tilde{M}'\| < \varepsilon$. Hence, the operator is a contraction.

It is now trivial to show that if M and W^U are increasing in z, T maps them into increasing functions. This follows since the $\max\{M(p',z'), W^U(p',z')\}$ is also an increasing function. Assumption 1 is needed so higher z today implies (on average) higher z tomorrow. Since the value of reallocation is constant in z, the reservation policy for reallocation follows immediately. This completes the proof of Lemma 3.

Proof of Lemma ?? T maps the subspace of functions $\tilde{M}$ into itself with $M(p,z)$ increasing weakly faster in z than $W^U(p,z)$. To show this let $M(p,z) - W^U(p,z)$ be weakly increasing in z and z^s denote the reservation productivity such that for and $z < z^s$ a firm-worker match decide to terminate the match. Using $\max_\theta\{\lambda(\theta)(M - W^U) - \theta k\} = \lambda(\theta^*)(M - W^U) - \lambda'(\theta^*)(M - W^U)\theta^* = \lambda(\theta^*)(1-\eta)(M - W^U)$, we construct the following difference

$$T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1) = y(p,z) - b + \beta\mathbb{E}_{p',z'}\Big[(1-\delta)\max\{M(p',z') - W^U(p',z'), 0\} - \max\Big\{\int W^U(p',\tilde{z})dF(\tilde{z}) - c - W^U(p',z'), \lambda(\theta^*)(1-\eta)\big(M(p',z') - W^U(p',z')\big)\Big\}\Big].$$

The first part of the proof shows the conditions under which $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in z. Consider the range of $z \in [\underline{z}, z^r)$, where $z^r < z^s$. In this case, the term under the expectation sign in the above equation reduces to $-\int W^U(p',\tilde{z})dF(\tilde{z}) + c + W^U(p',z')$. It is then immediate that when W^U increases in z', this term also increases in z. Since $y(p,z)$ is increasing in z, then $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is also increasing in z. Now suppose $z \in [z^r, z^s)$. In this case, the term under the expectation sign becomes zero (as $M(p',z') - W^U(p',z') = 0$) and $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in z in this range. Next suppose that $z \in [z^s, z^r)$. In this case, the term under the expectation sign reduces to $(1-\delta)(M(p',z') - W^U(p',z'))$. Since $M(p,z) - W^U(p,z)$ is weakly increasing in z, $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is weakly increasing in z in this case. Finally consider the range of $z \geq z^r > z^s$ or $z \geq z^s > z^r$, such that there employed workers do not quit nor reallocate. In this case the term under the expectation sign equals $(1-\delta)[M(p',z') - W^U(p',z')] - \lambda(\theta^*)(1-\eta)[M(p',z') - W^U(p',z')]$. When $M - W^U$ is increasing in z, a sufficient condition for the latter term to also increase in z is that $1 - \delta - \lambda(\theta^*) > 0$. In this case $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is increasing in z when such a condition holds.

The set of functions with increasing differences between the first and second coordinate is closed in the space of bounded and continuous functions. In particular, consider the set of functions $F \stackrel{def}{=} \{f \in \mathcal{C} | f : X \times Y \to \mathbb{R}^2, |f(x,y,1) - f(x,y,2)| \text{ increasing in } y\}$, where $f(.,.,1), f(.,.,2)$ denote the first and second coordinate, respectively, and $\mathcal{C}$ the metric space of bounded and continuous functions endowed with the sup-norm.

The next step in the proof is to show that fixed point of $T\tilde{M}(p,z,0) - T\tilde{M}(p,z,1)$ is also weakly increasing in z. To show we first establish the following result.

Lemma A.1: *F is a closed set in $\mathcal{C}$*

Proof. Consider an $f' \notin F$ that is the limit of a sequence $\{f_n\}, f_n \in F, \forall n \in \mathbb{N}$. Then there exists an $y_1 < y$ such that $f'(x,y_1,1) - f'(x,y_1,2) > f'(x,y,1) - f'(x,y,2)$, while $f_n(x,y_1,1) - f_n(x,y_1,2) \leq f_n(x,y,1) - f_n(x,y,2)$, for every n. Define a sequence $\{s_n\}$ with $s_n = f_n(x,y_1,1) -$

$f_n(x, y_1, 2) - f_n(x, y, 1) - f_n(x, y, 2)$. Then $s_n \geq 0$, $\forall n \in \mathbb{N}$. A standard result in real analysis guarantees that for any limit s of this sequence, $s_n \to s$, it holds that $s \geq 0$. Hence $f'(x, y_1, 1) - f'(x, y_1, 2) \leq f'(x, y, 1) - f'(x, y, 2)$, contradicting the premise. □

Thus, the fixed point exhibits this property as well and the optimal quit policy is a reservation-z policy given $1 - \delta - \lambda(\theta^*) > 0$. Furthermore, since $\lambda(\theta)$ is concave and positively valued, $\lambda'(\theta)(M - W^U) = k$ implies that job finding rate is also (weakly) increasing in z. This completes the proof of Lemma 4.

Proof of Proposition 1 The proof is basically an exercise to construct candidate equilibrium functions from the fixed point value and policy functions of T, and then verify these satisfy all equilibrium conditions. From the fixed point functions $M(p, z)$ and $W^U(p, z)$ with policy functions $\gamma_\theta^T(p, z)$ and $\gamma_W^T(p, z)$ define the function $J(p, z, W) = \max\{M(p, z) - W, 0\}$, and $\theta(p, z, W)$ and $V(p, z, W)$ from $0 = V(p, z, W) = -k + q(\theta(p, z, W))J(p, z, W)$. Also define $W^E(p, z) = M(p, z) - k/q(\gamma_\theta^T(p, z)) = \gamma_W^T(p, z)$ if $M(p, z) > W^U(p, z)$, and $W^E(p, z) = M(p, z)$ if $M(p, z) \leq W^U(p, z)$, using $W^U(p, z)$ from the fixed point. Finally, define $\delta(p, z) = \delta^T(p, z)$, $\sigma(p, z) = \delta^T(p, z)$, $\rho(p, z) = \rho^T(p, z)$, $\tilde{W}^{E\prime}(p', z') = \gamma_W^T(p', z')$, $\tilde{W}^f = \gamma_W^T(p, z)$ and $w(p, z)$ derived from (**??**) given all other functions.

Now (**??**) is satisfied by construction. Given the construction of $J(p, z, W)$, $\theta(p, z, W)$ indeed satisfies the free entry condition. $J(p, z, W)$ is satisfied if we ignore the maximization problem. However, $w(p, z, W^E)$, $\tilde{W}^{E\prime}(p', z'|p, z, W^E)$ satisfying (**??**) all yield the same $J(p, z, W^E)$ as long as $M(p, z) \geq W^E > W^U(p, z)$, $M(p', z') \geq \tilde{W}^{E\prime}(p', z'|p, z, W^E) \geq W^U(p, z)$, which is indeed the case. Hence, $J(p, z, W)$ is also satisfies (**??**), provided the separation decisions coincide, which is the case as the matches are broken up if and only if it is efficient to do so according to $M(p, z)$ and $W^U(p, z)$.

Given the constructed $W^U(p, z)$, the constructed $\rho(p, z)$ also solves the maximization decision in the decentralized setting. Finally, we have to verify $W^U(p, z)$. It is easy to see that this occurs if $S(p, z) = S^T(p, z)$. Consider the unemployed worker's application maximization problem that gives $S(p, z)$,

$$\max_{\{\tilde{\theta}(p,z), \tilde{W}(p,z)\}} \lambda(\tilde{\theta}(p, z))(\tilde{W}(p, z) - \tilde{W}^U(p, z)),$$

subject to

$$J(p, z, \tilde{W}(p, z))q(\tilde{\theta}(p, z)) - k = 0.$$

From Lemma 1, we know that $\tilde{W}(p, z) = M(p, z) - J(p, z, W(p, z))$. Substitute in the latter equation to get rid of J, and we see that the maximization problem for $S^T(p, z)$ is equivalent to the problem for the worker in the competitive equilibrium. Finally, $\tilde{W}^f(p, z)$ is consistent with profit maximization and thus here with the free entry condition, since any $W \in [W^U(p, z), M(p, z)]$ by construction of $\theta(p, z, W)$ is made consistent with free entry.

Hence, the constructed value functions and decision rules satisfy all conditions of the equilibrium, and the implied evolution of the distribution of employed and unemployed workers will also be the same.

Uniqueness follows from the same procedure in the opposite direction, by contradiction. Suppose the equilibrium is not unique. Then a second set of functions exists that satisfy the equilibrium conditions. Construct $\hat{M}$ from these. Since in any equilibrium the breakup decisions have to be efficient, the reallocation decision and application is captured in T, $\hat{M}$ and $\hat{W}^U$ must be fixed point of T, contradicting the uniqueness of the fixed point established by Banach's Fixed Point Theorem. This completes the proof of Proposition 1.

Proof of Proposition ?? Consider the mapping T^{SP}, with 'aggregate' states at the moments of decision making abbreviated to $(p', \{z_i', e_i', u_i'\}_I) \overset{def}{=} \mathcal{S}'$. The values are 'measured' at the beginning of the period, and tomorrow is denoted by a prime.

$$
\begin{aligned}
T^{SP}W^{SP}(p, \{z_i, e_i, u_i\}_I) = \max_{\{d_i(\mathcal{S}'), \rho_i(\mathcal{S}'), v_i(\mathcal{S}')\}} &\int_I (u_i b + e_i y(p, z_i)) di \\
&+ \beta \mathbb{E}_{\mathcal{S}'} \left[-\left(c \int_I \rho_i(S') u_i di + k \int_I v_i(S') di \right) + W^{SP}(p', \{z_i', e_i', u_i'\}_I) \right]
\end{aligned}
$$

subject to

$$
\begin{aligned}
u_i' &= (1 - \rho_i) u_i + (e_i - e_i') + \int_I \rho_j u_j dj \\
e_i' &= (1 - d_i) e_i + (1 - \rho_i) u_i \lambda \left(\frac{v_i}{(1 - \rho_i) u_i} \right) \\
&\mathcal{S}_0 \text{ given}, \ v_{i0} = 0, \ \forall i.
\end{aligned}
$$

Note that the decisions of the social planner here are: (i) reallocate people on an island (ρ_i), (ii) break up matches (d_i), (iii) set the number of vacancies for the unemployed (v_i). With $v_i = \theta_i(1 - \rho_i)u_i$, we can change the last decision variable to the tightness, by substitution.

The next step is to show that as W^{SP} is linear in u_i and e_i, then T^{SP} maps this function into a function that is likewise linear in these variables. Linearity of W^{SP} implies that it can be written as

$$
W^{SP}(\mathcal{S}) = \int_I \left(W^U(p, z_i) u_i + M(p, z_i) e_i \right) di.
$$

Moreover, under linearity the value of reallocation for u workers leaving their island is $\int_I W^U(p, z_j) u dj - uc$,, and hence we can write

$$
\begin{aligned}
T^{SP}W^{SP}(p, \{z_i, e_i, u_i\}_I) = \max_{\substack{d_i(\mathcal{S}'), \rho_i(\mathcal{S}') \\ v_i(\mathcal{S}')}} \int_I \Bigg(& u_i b + \beta \mathbb{E}_{p', z_i'} \Bigg[\left(\left(\int_I W^U(p', z_j') dj - c \right) \rho_i(\mathcal{S}') u_i \right. \\
& + (1 - \rho_i(\mathcal{S}')) u_i \left[\lambda\left(\theta_i(\mathcal{S}')\right) M(p', z_i') - \theta_i(\mathcal{S}') k + (1 - \lambda\left(\theta_i(\mathcal{S}')\right)) W^U(p', z_i') \right] \Bigg] \\
& + e_i(p, z_i) y(p, z_i) \\
& + \beta \mathbb{E}_{p', z_i'} \left[e_i(p', z_i') \left[(1 - d_i(\mathcal{S}')) M(p', z_i') + d_i(\mathcal{S}') W^U(p', z_i') \right] \right] \Bigg) di
\end{aligned}
$$

Further we can completely isolate the terms with u_i and e_i and within these terms we can isolate u_i and e_i and take the maximization over the remaining terms such that

$$T^{SP}W^{SP}(p,\{z_i,e_i,u_i\}_I) = \int_I \left[W^U_{max}(p,z_i)u_i + M_{max}(p,z_i)e_i\right] di$$

where

$$\begin{aligned}
W^U_{max}(p,z_i) = \max_{\substack{\rho_i(\mathcal{S}')\\ v_i(\mathcal{S}')}} \Bigg\{ b + \beta \mathbb{E}_{p',z_i'} \Bigg[\left(\int_I W^U(p',z_j')dj - c \right) \rho_i(\mathcal{S}') \\
+ (1-\rho_i(\mathcal{S}')) \left[\lambda(\theta_i(\mathcal{S}') \left[M(p',z_i') - W^M(p',z_i') \right] - \theta_i(\mathcal{S}')k + W^U(p',z_i') \right] \\
M_{max}(p,z_i) = \max_{d_i(\mathcal{S}')} \Bigg\{ y(p,z_i) \\
+ \beta \mathbb{E}_{p',z_i'} \left[\left(d_i(\mathcal{S}') W^U(p',z_i') + (1-d_i(\mathcal{S}'))M(p',z_i') \right) \right] \Bigg\}
\end{aligned}$$

The maximized value depends only on p and z_i, and hence T^{SP} maps a value function that is linear in u_i and e_i into a value function with the same properties. Moreover, using the definitions of W^U_{max} and M_{max} it follows that from the fixed point of the mapping T^{SP} we can derive a W^{U*}_{max} and M^*_{max} that constitutes a fixed point to T, and vice versa. Hence, the allocations of the fixed point of T are allocations of the fixed point of T^{SP}, and hence the equilibrium allocation is the efficient allocation. This completes the proof of Proposition 2.

Proof of Proposition 3 For simplicity assume that $n = 0$ such that workers that decide to reallocate randomly visit an island from the set of all active islands. The reservation island productivity for the competitive and search case, satisfies, respectively,

$$b + \beta \int_{\underline{z}}^{\bar{z}} \frac{\max\{y(p,z), y(p,z_c^r)\}}{1-\beta} dF(z) - \frac{y(p,z_c^r)}{1-\beta} - c_c = 0 \tag{34}$$

$$\frac{(1-\eta)k}{\eta} \left(\beta \int_{\underline{z}}^{\bar{z}} \frac{\max\{\theta(p,z), \theta(p,z^r)\}}{1-\beta} dF(z) - \frac{\theta(p,z^r)}{1-\beta} \right) - c_s = 0 \tag{35}$$

Using (**??**), the response of the reservation island productivity, for the competitive, and the frictional case, is then given by

$$\frac{dz_c^r}{dp} = \frac{\beta F(z_c^r) \frac{y_p(p,z_c^r)}{y_z(p,z_c^r)} + \beta \int_{z_c^r}^{\bar{z}} \frac{y_p(p,z)}{y_z(p,z_c^r)} dF(z) - \frac{y_p(p,z_c^r)}{y_z(p,z_c^r)}}{1-\beta F(z_c^r)} \tag{36}$$

$$\frac{dz^r}{dp} = \frac{\beta F(z^r) \frac{y_p(p,z^r)}{y_z(p,z^r)} + \beta \int_{z^r}^{\bar{z}} \frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)} \frac{y_p(p,z)}{y_z(p,z^r)} dF(z) - \frac{y_p(p,z^r)}{y_z(p,z^r)}}{1-\beta F(z^r)} \tag{37}$$

Choosing c_c, c_s appropriately such that $z_c^r = z^r$, the above expressions imply that $\frac{dz^r}{dp} > \frac{dz_c^r}{dp}$ if $\frac{\theta(p,z)}{w(p,z)-b} > \frac{\theta(p,z^r)}{w(p,z^r)-b}, \ \forall\, z > z^r$. Hence we now need to show that $\frac{\theta(p,z)}{w(p,z)-b}$ is increasing in z.

$$\frac{d\left(\frac{\theta(p,z)}{w(p,z)-b}\right)}{dz} = \frac{\theta y_z(p,z)}{(w-b)^2} - \theta \left(\frac{(1-\eta) + (1-\eta)\beta \frac{\theta}{w-b}k}{(w-b)^2} \right) y_z(p,z),$$

which has the same sign as $\eta - (1-\eta)\beta k \frac{\theta}{w-b}$ and the same sign as

$$\eta(1-\eta)(y(p,z)-b) + \eta(1-\eta)\beta\theta k - (1-\eta)\beta\theta k$$
$$= (1-\eta)(\eta(y(p,z)-b) - (1-\eta)\beta\theta k).$$

But $\eta(y(p,z)-b) - (1-\eta)\beta\theta k = y(p,z) - w > 0$ and thus we have established Part *1* of the Proposition.

For Part *2*, note that modularity implies that $y_p(p,z) = y_p(p,\tilde{z}),\ \forall z > \tilde{z}$; while supermodularity implies $y_p(p,z) \geq y_p(p,\tilde{z}),\ \forall z > \tilde{z}$. Hence modularity implies

$$\frac{dz_c^r}{dp} = \frac{1}{1-\beta F(z_c^r)} \frac{y_p(p,z_c^r)}{y_z(p,z_c^r)} \left(\beta F(z_c^r) + \beta \int_{z_c^r}^{\bar{z}} \frac{y_p(p,z)}{y_p(p,z_c^r)} dF(z) - 1\right) < 0,\ \forall\, \beta < 1.$$

In the case with frictions,

$$\frac{dz^r}{dp} = \frac{1}{1-\beta F(z^r)} \frac{y_p(p,z^r)}{y_z(p,z^r)} \left(\beta F(z^r) + \beta \int_{z^r}^{\bar{z}} \frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)} \frac{y_p(p,z)}{y_p(p,z^r)} dF(z) - 1\right).$$

If we can show that the integral becomes large enough, for c large enough, to dominate the other terms, we have established the claim. First note that $\frac{y_p(p,z)}{y_p(p,z^r)}$ is weakly larger than 1, for $z > z^r$ by the (super)modularity of the production function. Next consider the term $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)}$. Note that

$$\lim_{z \downarrow y^{-1}(b;p)} \frac{\theta(p,z)}{w(p,z)-b} = \frac{\lambda(\theta(p,z))}{1-\beta+\beta\lambda(\theta(p,z))} = 0,$$

because $\theta(p,z) \downarrow 0$, as $y(p,z^r) \downarrow b$. Hence, fixing a z such that $y(p,z) > b$, $\frac{\theta(p,z)(w(p,z^r)-b)}{\theta(p,z^r)(w(p,z)-b)} \to \infty$, as $y(p,z^r) \downarrow b$. Since this holds for any z over which is integrated, the integral term becomes unboundedly large, making dz^r/dp strictly positive if reservation z^r is low enough. Since the integral rises continuously but slower in z^r than the also continuous term $\frac{\theta(p,z^r)}{1-\beta}$, it can be readily be established that z^r depends continuously on c, and strictly negatively so as long as $y(p,z^r) > b$ and $F(z)$ has full support. Moreover, for some $\bar{c}$ large enough, $y(p,\underline{z}^r) = b$. Hence, as $c \uparrow \underline{z}^r$, $\frac{dz^r}{dp} > 0$. This completes the proof of Proposition 3.

Proof of Lemma 5 First note that when workers sample new islands with $z \geq z^n$ in the event of reallocation, (**??**) is now described by

$$\frac{(1-\eta)k}{\eta}\left(\beta \int_{z^n}^{\overline{z}} \frac{\theta(p,z)}{1-F(z^n)} dF(z)\right) - c_{rs}(1-\beta) = \frac{(1-\eta)k}{\eta}\theta(p,z_{rs}^r),$$

where c_{rs} describes the explicit cost of reallocation in this case. Implicit differentiation then yields

$$\frac{dz_{rs}^r}{dp} = \beta \int_{z^n}^{\overline{z}} \frac{\theta_p(p,z)}{\theta_z(p,z_{rs}^r)} \frac{dF(z)}{1-F(z^n)} - \frac{\theta_p(p,z)}{\theta_z(p,z_{rs}^r)}.$$

Using (**??**) and choosing appropriately c_{rs} and c such that $z^r = z_{rs}^r$ we have that

$$sign\left[\frac{dz_{rs}^r}{dp} - \frac{dz^r}{dp}\right] = sign\left[\int_{z^n}^{\overline{z}} \frac{\theta_p(p,z)}{1-F(z^n)} dF(z) - \left(\frac{1-F(z^r)}{1-\beta F(z^r)}\right) \int_{z^r}^{\overline{z}} \frac{\theta_p(p,z)}{1-F(z^r)} dF(z)\right].$$

Given (**??**) implies that $\theta_p(p,z)$ is increasing in z and $\beta \leq 1$, it then follows that $dz_{rs}^r/dp > dz^r/dp$. This completes the proof of Lemma 5.

Proof of Lemma 6 Note that $R(p) = \frac{b+\beta\theta(p,z^r(p))k(1-\eta)/\eta}{1-\beta}$. The derivative of this function with respect to p equals

$$\frac{\beta k(1-\eta)}{(1-\beta)\eta}\frac{\theta}{w(p,z^r(p))-b}\left(y_p(p,z^r(p))+y_z(p,z^r(p))\frac{dz^r(p)}{dp}\right). \tag{38}$$

Since $w(p,z^r(p))-b = (W^E(p,z^r(p))-W^U(p,z^r(p)))(1-\beta(1-\delta)+\beta\lambda(\theta(p,z^r(p))))$ and $\frac{\theta\beta k(1-\eta)}{(1-\beta)\eta} = \beta\lambda(\theta(p,z^r(p)))(W^E(p,z^r(p))-W^U(p,z^r(p))$, we find that (**??**) reduces to

$$\frac{\beta\lambda(\theta(p,z^r(p)))}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z^r(p))}\left(y_p(p,z^r(p))+y_z(p,z^r(p))\frac{dz^r(p)}{dp}\right). \tag{39}$$

From the cutoff condition for separation, we find $(1-\beta)R(p) = y(p,z^s(p))$. Taking the derivative with respect to p implies the left side equals (**??**) and the right side equals $y_p(p,z^s(p)) + y_z(p,z^s(p))\frac{dz^s(p)}{dp}$. Rearranging yields (**??**). This completes the proof of Lemma 6.

Proof of Lemma 7 We divide the proof into three sections. To simplify notation we consider the transformation $y = y(z)$, where $y(.)$ is the common island production function, and let F denote the cdf of y. Accordingly, let $y^r = y(\hat{z}^r)$ and $y^s = y(\hat{z}^s)$.

Comparative statics wrt c Consider the difference $W^s - R$ and values of c such that $R \geq W^s$. In this case we have that

$$W^s = (1-\gamma)(R+c)+\gamma(b+\beta R),$$
$$W^s - R = -\gamma(1-\beta)R+(1-\gamma)c+\gamma b.$$

Suppose towards a contradiction that $d(W^s-R)/dc < 0$. The above equations imply that $\frac{dR}{dc} > \frac{(1-\gamma)}{\gamma(1-\beta)} > 0$. We will proceed by showing that under $d(W^s-R)/dc < 0$ both the expected (post-island-shock) surplus and the surplus on active islands decrease, which implies that the value of unemployment decreases, which in turn implies $\frac{dR}{dc} < 0$, which is our contradiction.

Consider an active island with $W^U(y) > R$, the surplus on this island is given by

$$\begin{aligned} M(y)-W^U(y) = \gamma(y-b+\beta(1-\lambda(\theta(y))(1-\eta))(M(y)-W^U(y))) \\ + (1-\gamma)(\mathbb{E}_y[M(y)-W^U(y)]+(y-\mathbb{E}[y])), \end{aligned} \tag{40}$$

where $\mathbb{E}_y[M(y)-W^U(y)]$ describes the expected surplus after an island shock (after the search stage). Note that $\frac{d}{d(M(y)-W^U(y))}(\lambda(\theta(y))(1-\eta)(M(y)-W^U(y))) = \lambda(\theta(y))$, since (dropping the y argument for brevity) $(1-\eta)(M-W^U) = \frac{(1-\eta)}{\eta}J = \frac{1-\eta}{\eta}\frac{k}{q(\theta)}$, and hence $\lambda(\theta)(1-\eta)(M-W^U) = \frac{1-\eta}{\eta}k\theta$. Moreover, $\frac{d\theta}{d(M-W^U)} = \frac{\eta}{1-\eta}\frac{\lambda(\theta)}{k}$. Putting the last two expressions together, we find that the above derivative equals $\lambda(\theta)$. From (**??**), it follows that

$$0 < \frac{d(M-W^U)}{d(\mathbb{E}_y[M(y)-W^U(y)])} = \frac{1-\gamma}{1-\gamma\beta(1-\lambda(\theta))} < 1. \tag{41}$$

Expected match surplus measured after the search stage is

$$\mathbb{E}_y[M(y) - W^U(y)] = \int_{y^r} y - b + \beta(1 - \lambda(\theta(y))(1-\eta))(M(y) - W^U(y))dF(y) + \int_{y^s}^{y^r} y - b + \beta(M(y) - R)dF(y) + \int^{y^s} y - b + \beta(W^s - R)dF(y), \tag{42}$$

note that the $(1-\gamma)$ shock integrates out. The third term of the expression above is decreasing in c, by our contradiction supposition. The second term, $\int_{y^s}^{y^r}[M(y) - W^U(y)]dF(y)$, can be rewritten as

$$M - W^s = \gamma(y - b + \beta(M - W^s + W^s - R)) + (1-\gamma)(\mathbb{E}_y[M(y) - W^U(y)] + y - \mathbb{E}[y]),$$

and rearranging yields

$$M - W^s = \frac{\gamma}{1-\gamma\beta}(y - b + \beta(W^s - R)) + \frac{1-\gamma}{1-\gamma\beta}\mathbb{E}_y[M(y) - W^U(y)],$$

where $\frac{\gamma}{1-\gamma\beta}(y-b+\beta(W^s-R))$ is decreasing. For the first term, note that $M(y)-W^U(y)$ responds to c through $\mathbb{E}_y[M(y) - W^U(y)]$, from (**??**). Combining all the elements (**??**), (**??**) and the last two equations, we find that

$$\begin{aligned}\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} &= \int_{y^r} \frac{(1-\gamma)\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} \\ &\quad + (F(y^r) - F(y^s))\left(\frac{\gamma\beta}{1-\gamma\beta}\frac{d(W^s-R)}{dc} + \frac{1-\gamma}{1-\gamma\beta}\frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc}\right) \\ &\quad + F(y^s)\beta\frac{d(W^s-R)}{dc} \\ \iff \frac{d\mathbb{E}_y[M(y) - W^U(y)]}{dc} &= C \cdot \frac{d(W^s-R)}{dc} < 0,\end{aligned} \tag{43}$$

where C is a positive constant. From this it follows that $\frac{d[M(y)-W^U(y)]}{dc} < 0$, by (**??**).

Next consider $\frac{dW^U}{dc}$, and $\frac{d\mathbb{E}[W^U]}{dc}$. For $y \le y^r$, $W^U(y) = W^s = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta\mathbb{E}[W^U] - \beta c)$. For $y > y^r$, $W^U(y) = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(\lambda(\theta(y))(1-\eta)(M(y) - W^U(y)) + \beta W^U(y)))$. It follows that $\mathbb{E}[W^U] = F(y^r)(b + \beta\mathbb{E}[W^U] - \beta c) + \int_{y^r}(b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y)) + \beta W^U(y))dF(y)$. Combining the latter equation with

$$W^U = \frac{1-\gamma}{1-\beta\gamma}\mathbb{E}[W^U] + \frac{\gamma}{1-\beta\gamma}(b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y))),$$

we have that

$$\begin{aligned}&\left(1 - \beta F(y^r) - \beta\frac{1-\gamma}{1-\beta\gamma}(1 - F(y^r))\right)\mathbb{E}[W^U] \\ &\quad = F(y^r)(b - \beta c) + \int_{y^r}\frac{b + \beta\lambda(\theta(y))(1-\eta)(M(y) - W^U(y))}{1-\beta\gamma}dF(y)\end{aligned}$$

Taking the derivative with respect to c, we find that both the first and second terms on the RHS are negative, the latter because we have established that $\frac{d(M(y)-W^U(y))}{dc} < 0$. It then follows that $\frac{d\mathbb{E}[W^U]}{dc} < 0$, which implies that $\frac{dR}{dc} = \frac{d\mathbb{E}[W^U]}{dc} - 1 < 0$, which contradicts our premise.

Now consider values of c such that $R < W^s$. Here there is rest unemployment. In this case, $W^s = \gamma(b + \beta W^s) + (1-\gamma)\mathbb{E}[W^U]$ and $\frac{dW^s}{dc} = 0$, since workers in islands with productivities $y \leq y^s$ will never reallocate. Doing so implies paying a cost $c > 0$ and randomly drawing a new island from the productivity distribution, while staying in the current island also implies (with probability $1-\gamma$) obtained a free draw from the productivity distribution. Hence, $d(R-W^s)/dc = dR/dc$. Noting that workers in islands $y > y^s$ prefer employment in their current island, the above arguments imply W^U is independent of the value of reallocation for any y. It then follows that $\frac{dR}{dc} = \frac{d\mathbb{E}[W^U]}{dc} - 1 = -1 < 0$, which contradicts our premise.

Comparative Statics with respect to b. Once again consider the difference $W^s - R$ such that $R \geq W^s$. Writing W^s and W^U, for islands above the separation cutoff, as

$$W^s = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(R - W^s)) + \gamma\beta W^s \tag{44}$$

$$W^U(y) = (1-\gamma)\mathbb{E}[W^U] + \gamma(b + \beta(\lambda(\theta)(1-\eta)(M(y) - W^U(y)))) + \gamma\beta W^U(y), \tag{45}$$

we find that $W^s - \mathbb{E}[W^U] = \int_{y^r}(W^s - W^U(y))dF(y)$, which in turn implies

$$W^s - R = \frac{1}{1-\gamma\beta F(y^r)}\left(-\beta\gamma\int_{y^r}\lambda(\theta)(1-\eta)(M(y) - W^U(y))dF(y) + (1-\gamma\beta)c\right). \tag{46}$$

That is, the difference between waiting one period to reallocate and reallocating now is the forgone possibility of searching for a job in the new island next period, but on the other hand, the reallocation cost only has to be incurred next period with probability γ, and discounted at rate β.

Next consider the relationship between $M(y) - W^U(y)$ and $\mathbb{E}[M(y) - W^U(y)]$. From (**??**) and (**??**) , we find that

$$\frac{d(M(y) - W^U(y))}{db} = \frac{1-\gamma}{1-\gamma\beta(1-\lambda(\theta))}\frac{d\mathbb{E}[M(y) - W^U(y)]}{db} - \frac{\gamma}{1-\gamma\beta(1-\lambda(\theta))}. \tag{47}$$

Note that $\frac{d(M(y)-W^U(y))}{db}$ must have the same sign for all y, which is positive if and only if

$$\frac{d\mathbb{E}[M(y) - W^U(y)]}{db} > \frac{\gamma}{1-\gamma}.$$

Towards a contradiction, suppose $\frac{d(W^s-R)}{db} < 0$. Then, we have $\frac{d(W^s-R)}{db} = \frac{d(W^s-\mathbb{E}[W^U])}{db}$, which equals $\frac{d}{db}\left(-\int_{y^r}\max\{W^U(y) - W^s, 0\}dF(y)\right)$. By the envelope condition, the effect $\frac{dy^r}{db}$ disappears. By the previous argument and (**??**) subtracted by (**??**), it follows that $\frac{d(M(y)-W^U(y))}{db} > 0$ and by (**??**), $\frac{d\mathbb{E}[M(y)-W^U(y)]}{db} > 0$.

Along the lines of (**??**), we find

$$\begin{aligned}\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} &= -1+\int_{y^r}\frac{\beta(1-\lambda(\theta(y)))-\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db}\\ &\quad -\int_{y^r}\frac{\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db}\\ &\quad +(F(y^r)-F(y^s))\left(\frac{\gamma\beta^2}{1-\gamma\beta}\frac{d(W^s-R)}{db}+\frac{\beta(1-\gamma)}{1-\gamma\beta}\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db}\right)\\ &\quad -(F(y^r)-F(y^s))\frac{\gamma\beta}{1-\gamma\beta}+F(y^s)\beta\frac{d(W^s-R)}{db} \qquad (48)\end{aligned}$$

$$\implies \frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} = C_2\cdot\frac{d(W^s-R)}{db}-C_3<0,$$

with C_2, C_3 are positive-valued terms. This is the desired contradiction.

Next consider the case that $W^s > R$. Then, equation (**??**) becomes instead

$$W^s-\mathbb{E}[W^U] = -\frac{\beta\gamma}{1-\beta\gamma}\int_{y^s}\lambda(\theta)(1-\eta)(M(y)-W^U(y))dF(y) \qquad (49)$$

Similarly, if we start from the premise that $\frac{d(W^s-R)}{db} < 0$, this will imply again by (**??**) that $\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} > 0$. Note that in this case, in equation (**??**) reduces to

$$\mathbb{E}_y[M(y)-W^U(y)] = \int_{y^s} y-b+\beta\lambda(\theta(y))(1-\eta)(M(y)-W^U(y))dF(y), \qquad (50)$$

and (**??**) reduces to

$$\begin{aligned}\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db} &= -1+\int_{y^s}\frac{\beta(1-\lambda(\theta(y)))-\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db}\\ &\quad -\int_{y^s}\frac{\gamma\beta(1-\lambda(\theta(y)))}{1-\gamma\beta(1-\lambda(\theta))}dF(y)\frac{d\mathbb{E}_y[M(y)-W^U(y)]}{db}, \qquad (51)\end{aligned}$$

which again implies that $\frac{d\mathbb{E}[M-W^U]}{db} < 0$, a contradiction.

Comparative statics with respect to γ We start with the case where $R > W^s$. Towards a contradiction, assume that $\frac{d(W^s-R)}{d\gamma} > 0$. From equation (**??**), we find that

$$\begin{aligned}\frac{d(W^s-R)}{d\gamma} =& \frac{\beta F(y^r)}{1-\beta\gamma}(W^s-R)+\frac{1}{1-\beta\gamma F(y^r)}\left(-\int_{y^r}\lambda(\theta)(1-\eta)(M(y)-W^u(y)dF(y)-\beta c\right)\\ &-\int_{y^r}\beta\gamma\lambda(\theta)\frac{d(M(y)-W^u(y))}{d\gamma}dF(y) \qquad (52)\end{aligned}$$

From our premise it follows that

$$\begin{aligned}-\int_{y^r}\beta\gamma\lambda(\theta)\frac{d(M(y)-W^u(y))}{d\gamma}dF(y) \geq& \frac{\beta F(y^r)}{1-\beta\gamma}(R-W^s)\\ &+\frac{1}{1-\beta\gamma F(y^r)}\left(\int_{y^r}\lambda(\theta)(1-\eta)(M(y)-W^u(y))dF(y)+\beta c\right)>0 \qquad (53)\end{aligned}$$

Now, let us look at the implications for $\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}$. We can rewrite (**??**), bringing tomorrow's continuation values to the LHS as

$$\begin{aligned}(1-\beta)\mathbb{E}_y[M(y)-W^u(y)] = &\int_{y^r} y-b-\beta\lambda(\theta(y))(1-\eta)(M(y)-W^u(y))dF(y)\\ &+\int_{y^s}^{y^r} y-b+\beta(W^s-R))dF(y)\\ &+\int^{y^s} y-b+\beta(W^s-R)-\beta(M(y)-W^s)dF(y), \end{aligned}\tag{54}$$

Taking derivatives with respect to γ, we find

$$\begin{aligned}(1-\beta)\frac{d\mathbb{E}_y[M(y)-W^u(y)]}{d\gamma} =&-\beta\int_{y^r}\lambda(\theta(y))\frac{d(M(y)-W^u(y))}{d\gamma}dF(y)\\ &+\int_{y^s}^{y^r}\beta\frac{d(W^s-R)}{d\gamma})dF(y)\\ &+\int^{y^s}\beta\frac{d(W^s-R)}{d\gamma}-\beta\frac{d(M(y)-W^s)}{d\gamma}dF(y)\\ &>0\end{aligned}\tag{55}$$

For $y<y^s$ it holds that

$$\begin{aligned}\frac{d(M(y)-W^s)}{d\gamma} =&(1-\gamma)\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}+\gamma\beta\frac{d(W^s-R)}{d\gamma}\\ &+(y-b+\beta(W^s-R)-\mathbb{E}[M(y)-W^u(y)]).\end{aligned}\tag{56}$$

The first two terms on the RHS are positive, the last term on the RHS negative. In the RHS of (**??**) all terms are positive, except for $F(y^s)(1-\gamma)\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}$ and $F(y^s)\gamma\beta\frac{d(W^s-R)}{d\gamma}$ associated with $\frac{d(M(y)-W^s)}{d\gamma}$. However, one can see that $-F(y^s)\gamma\beta\frac{d(W^s-R)}{d\gamma}$ is more than offset by $\beta\frac{d(W^s-R)}{d\gamma}$ on the same line, while we can bring $F(y^s)(1-\gamma)\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}$ to the LHS, to find that $\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}$ is premultiplied by $(1-F(y^s)\beta\gamma)>0$. Hence, it follows that $\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}>0$.

From $M(y)-W^u(y)=(1-\gamma)\mathbb{E}[M(y)-W^u(y)]+\gamma(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^u(y))$, it follows that for $y>y^r$

$$\begin{aligned}\beta\gamma\lambda(\theta)\frac{dM(y)-W^u(y)}{d\gamma} =&\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\bigg(\Big(y-b+\beta(1-\lambda(\theta)(1-\eta))(M(y)-W^u(y))\\ &-\mathbb{E}[M(y)-W^u(y)]\Big)+(1-\gamma)\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}\end{aligned}\tag{57}$$

Integrating this term over all $y>y^r$, we have

$$\begin{aligned}\beta\gamma\int_{y^r}\lambda(\theta(y))\frac{d(M(y)-W^u(y))}{d\gamma}dF(y)\geq &\frac{\beta\gamma\lambda(\theta(y^r))}{1-\beta\gamma+\beta\gamma\lambda(\theta(y^r))}\bigg(\int_{y^r}(1-\gamma)\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}dF(y)\\ &+\frac{1}{\gamma}\int_{y^r}M(y)-W^u(y)-\mathbb{E}[M(y)-W^u(y)]dF(y)\bigg)>0,\end{aligned}\tag{58}$$

where the last inequality follows from the fact that $M(y) - W^u(y) - \mathbb{E}[M(y) - W^u(y)]$ is increasing in y, and $\frac{\beta\lambda(\theta(y))}{1-\beta\gamma+\beta\gamma\lambda(\theta(y))}$ similarly is increasing in y. Then $\int_{y^r} M(y) - W^u(y) - \mathbb{E}[M(y) - W^u(y)]dF(y)$ is larger than zero. The LHS of (**??**) is positive, but this contradicts our premise in (**??**).

For the case that $W^s > R$, we can derive directly that

$$(1-\beta)\frac{d\mathbb{E}_y[M(y) - W^u(y)]}{d\gamma} = -\beta\int_{y^r}\lambda(\theta(y))\frac{d(M(y) - W^u(y))}{d\gamma}dF(y); \tag{59}$$

with this in hand, we can derive from (**??**) that

$$-(1-\beta)\frac{d\mathbb{E}_y[M(y) - W^u(y)]}{d\gamma} = \int_{y^s}\left(\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\left(\left(y - b + \beta(1-\lambda(\theta)(1-\eta))(M(y) - W^u(y))\right.\right.\right.$$
$$\left.\left.\left. - \mathbb{E}[M(y) - W^u(y)]\right) + (1-\gamma)\frac{d\mathbb{E}[M(y) - W^u(y)]}{d\gamma}\right)dF(y). \tag{60}$$

Isolating $\frac{d\mathbb{E}[M(y)-W^u(y)]}{d\gamma}$ on the LHS, we find

$$\frac{d\mathbb{E}_y[M(y) - W^u(y)]}{d\gamma}\left((1-\beta) + \frac{F(y^s)\beta\gamma\lambda(\theta)(1-\gamma)}{1-\beta\gamma+\beta\gamma\lambda(\theta)}\right) =$$
$$-\int_{y^s}\left(\frac{\beta\gamma\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\left(y - b + \beta(1-\lambda(\theta)(1-\eta))(M(y) - W^u(y))\right.\right.$$
$$\left.\left. - \mathbb{E}[M(y) - W^u(y)]\right)dF(y) < 0. \tag{61}$$

From (**??**), it follows that $\frac{d\mathbb{E}_y[M(y)-W^u(y)]}{d\gamma} < 0$, and therefore that, from (**??**) for the relevant case that $W^s > R$, the above equation, and equation **??**, $W^s - R$ is increasing in γ:

$$\frac{d(W^s - R)}{d\gamma} = \frac{d(W^s - \mathbb{E}[W^u]}{d\gamma} = -\frac{1}{1-\beta}\int_{y^s}\gamma\lambda\frac{d(M(y) - W^u)}{d\gamma} > 0. \tag{62}$$

Proof of lemma 8 (Occupation-specific Human Capital) In the above setting, we introduce human capital x premultiplying island productivity y. Normalize (without loss of generality, for the results we are deriving here) $x = 1$. If have an incremental improvement in x that is island-specific, the value of reallocating will stay constant, at $R = \mathbb{E}_y[W^u(1,y)] - c$, where now we denote $W^u(x,y)$ by the productivity component that is enjoyed by every worker on the island, and x is the worker's island-specific human capital. The value of W^s will increase, as $W^s = (1-\gamma)\mathbb{E}[W^u(x,y)] + \gamma(b + \beta\max\{R, W^s\}$ implies that W^s is increasing in $\mathbb{E}[W^u(x,y)]$.

Suppose that $R > W^s$. The value of unemployment on islands where (if the

$$W^s(x) = (1-\gamma)\mathbb{E}_y[W^u(x,y)] + \gamma(b + \beta R) \tag{63}$$

$$W^u(x,y) = (1-\gamma)\mathbb{E}_y[W^u(x,y)] + \gamma(b + \beta\lambda(\theta)(1-\eta)(M(x,y) - W^u(x,y)) + \beta W^u(x,y)). \tag{64}$$

When comparing the value of reallocating (and hence, a reset to $x = 1$), we see that in the difference $W^s(x) - \mathbb{E}_y[1, y]$, equals a number of terms that do not depends on x, denoted by C, and the value of unemployment after a $(1-\gamma)$ redraw of island productivity:

$$(W^s(x) - \mathbb{E}_y[W^u(1, y)]) = (1-\gamma)(\mathbb{E}_y[W^u(x, y)] + C. \tag{65}$$

As a result, $\frac{d(W^s(x) - \mathbb{E}_y[W^u(1,y)])}{dx} = (1-\gamma)\frac{d\mathbb{E}_y[W^u(x,y)]}{dx}$. Rewriting $\mathbb{E}[W^u(x, y)]$, using $W^u(x, y) = (1-\gamma)\mathbb{E}[W^u(x, y)] + \gamma(b + \beta\big(\lambda(\theta)(1-\eta)(M(x, y) - W^u(x, y))\big) + \beta W^u(x, y)$ we find

$$\mathbb{E}[W^u(x, y)] = (1 - F(y^r(x))(b + \beta R) + \int_{y^r} b + \beta\lambda(\theta)(M(x, y) - W^u(x, y)) + \beta W^u(x, y) dF(y)$$

from which it follows that

$$\begin{aligned} &\mathbb{E}[W^u(x, y)]\Big(1 - \frac{\beta(1-\gamma)}{1-\beta\gamma}(1-F(y^r(x)))\Big) = \\ &\quad F(y^r(x)(b+\beta R) + \frac{\beta\gamma}{1-\beta\gamma}b(1-F(y^r(x)) + \frac{\beta}{1-\beta\gamma}\int_{y^r(x)}\lambda(\theta(x, y))(1-\eta)(M(x, y) - W^u(x, y))dF(y). \end{aligned} \tag{66}$$

In turn, (using the envelope condition, which implies that the term premultiplying $dy^r(x)/dx$ again equals zero), this means

$$\begin{aligned} &\frac{d\mathbb{E}_y[W^u(x, y)]}{dx} = \\ &\quad \frac{\beta}{(1-\beta\gamma) - \beta(1-\gamma)(1-F(y^r(x)))}\frac{d}{dx}\Big(\int_{y^r}\lambda(\theta(x, y))(1-\eta)(M(x, y) - W^u(x, y))dF(y)\Big) \end{aligned} \tag{67}$$

Let us now look at the behavior of the expected surplus, from

$$\begin{aligned} \mathbb{E}_y[M(x, y) - W^u(x, y)] = &\int_{\underline{y}}^{\bar{y}}(yx - b)dF(y) + \int_{y^r(x)}\beta\lambda(\theta(x, y))(1-\eta)(M(x, y) - W^u(x, y))dF(y) \\ &+ \int_{y^s(x)}^{y^r(x)}\beta(M(x, y) - W^s)dF(y) + \beta\int^{y^r(x)}(W^s(x) - R)dF(y). \end{aligned} \tag{68}$$

The surplus at a given island with $y \geq y^r(x)$, respectively $y^r > y \geq y^s$, behaves as

$$\begin{aligned} \frac{d(M(x, y) - W^u(x, y))}{dx} &= (1-\gamma)\frac{d\mathbb{E}_y[M(x, y) - W^u(x, y)]}{dx} + (1-\gamma)(y - \mathbb{E}[y]) + \gamma y \\ &\quad + \beta\gamma(1 - \lambda(\theta(x, y)))\frac{d(M(x, y) - W^u(x, y))}{dx} \end{aligned} \tag{69}$$

$$\begin{aligned} \frac{d(M(x, y) - W^s(x))}{dx} &= (1-\gamma)\frac{d\mathbb{E}_y[M(x, y) - W^u(x, y)]}{dx} + (1-\gamma)(y - \mathbb{E}[y]) + \gamma y \\ &\quad + \beta\gamma\frac{d(M(x, y) - W^s(x))}{dx} + \beta\gamma\frac{d(W^s(x) - R)}{dx} \end{aligned} \tag{70}$$

The derivative of $\int_{y^r}\beta\lambda(\theta(x, y))(1-\eta)(M(x, y) - W^u(x, y))dF(y)$ wrt to x equals

$$\int_{y^r}\left(\frac{\beta\lambda(\theta(x, y))(1-\gamma)}{1-\beta\gamma(1-\lambda)}\left(\frac{d\mathbb{E}_y[M(x, y) - W^u(x, y)]}{dx} + y - \mathbb{E}[y]\right) + \frac{\beta\lambda(\theta(x, y))\gamma}{1-\beta\gamma(1-\lambda(\theta))}y\right)\, dF(y), \tag{71}$$

where we note that $\int_{y^r} \frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda)} dF(y) \leq \frac{\beta\lambda(\theta(x,\bar{y}))(1-\gamma)}{1-\beta\gamma(1-\lambda(x,\bar{y}))}(1-F(y^r)) < 1-F(y^r)$. Let us now look at the behavior of the third term in (**??**): the derivative of $\beta(W^s(x)-R)$ is given by

$$\frac{\beta(1-\gamma)}{1-\beta+\beta(1-\gamma)F(y^r(x))}\cdot \tag{72}$$
$$\int_{y^r}\left(\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda)}\quad\left(\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\lambda(\theta(x,y))\gamma}{1-\beta\gamma(1-\lambda(\theta))}y\right)dF(y)$$

Likewise, we can look at the derivative of $\beta(M(x,y)-W^u(x,y))+\beta(W^s(x)-R)$ with respect to x is

$$\frac{\beta(1-\gamma)}{1-\beta\gamma}\left(\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\gamma}{1-\beta\gamma}y+\left(\frac{1}{1-\beta\gamma}\frac{\beta(1-\gamma)}{1-\beta+\beta(1-\gamma)F(y^r)}\cdot\right.$$
$$\left.\int_{y^r}\left(\frac{\beta\lambda(\theta(x,y))(1-\gamma)}{1-\beta\gamma(1-\lambda)}\left(\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx}+y-\mathbb{E}[y]\right)+\frac{\beta\lambda(\theta(x,y))\gamma}{1-\beta\gamma(1-\lambda(\theta))}y\right)dF(y)\right) \tag{73}$$

We want to make sure that all terms premultiplying $\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx}+y-\mathbb{E}[y]$ on the RHS do not add up to a number larger than 1. First, note that the terms premultiplying the derivative of the expected surplus in (**??**) are larger than in (**??**); hence if we can show if we replace the premuplication term in (**??**) with the corresponding term in (**??**), and still are able to show that the entire term premultiplying the derivative of the expected surplus on the RHS is less than one, we have established this step of the proof. The contribution of these premultiplication terms in the second term on the RHS of (**??**) is smaller than $\beta(1-\gamma)(1-F(y^r(x)))$. Hence, if

$$\left(\beta\frac{d(M(x,y)-W^u(x,y))}{dx}+\beta\frac{d(M(x,y)-W^u(x,y))}{dx}\right)F(y^r(x))<1-\beta(1-\gamma)(1-F(y^r(x))) \tag{74}$$

we have established the desired property. Starting from collecting the terms premultiplying the derivative of the expected surplus, $\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx}+y-\mathbb{E}[y]$, and substituting these into the LHS of (**??**), we can develop

$$F(y^r)\left(\frac{\beta(1-\gamma)(1-\beta+\beta(1-\gamma)F(y^r(x)))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}+\frac{\beta(1-\gamma)(\beta(1-\gamma)(1-F(y^r(x))))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}\right)$$
$$=\frac{\beta(1-\gamma)(1-\beta+\beta(1-\gamma))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))}F(y^r(x)) \tag{75}$$

The RHS of (**??**) can be rewritten as $1-\beta+\beta\gamma)+\beta(1-\gamma)F(y^r(x))$. We will show that $\beta\gamma+\beta(1-\gamma)F(y^r(x))$ are larger than (**??**), from which the desired result follows (as the remaining term, $1-\beta$, is larger than zero, and therefore means that the desired inequality is additionally slack).

$$\beta\gamma>\frac{\beta\gamma(\beta(1-\gamma))\Big(1-\beta+\beta(1-\gamma)\Big)F(y^r(x))}{(1-\gamma\beta)(1-\beta+\beta(1-\gamma)F(y^r(x)))} \tag{76}$$

$$\beta(1-\gamma)F(y^r(x))>\frac{\beta(1-\gamma)(1-\gamma\beta)\big(1-\beta+\beta(1-\gamma)\big)F(y^r(x))}{(1-\gamma\beta)(1-\beta+\beta F(y^r(x)))} \tag{77}$$

Adding up (**??**) and (**??**), we find that the RHS equals precisely the term in (**??**).

Bringing all terms involving $\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx} + y - \mathbb{E}[y]$, it is now straightward to see that the remaining terms on the RHS premultiplying y, are positive. (Integrating terms $y - \mathbb{E}[y]$, will also yield a positive term.) Hence, $\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx} > 0$. It follows from (**??**) that $\frac{d(M(x,y)-W^u(x,y))}{dx} > 0$, and therefore, by (**??**), $\frac{d\mathbb{E}_y[W^u(x,y)]}{dx} > 0$, and subsequently, $\frac{d(W^s(x)-R)}{dx} > 0$.

Consider next the case that $W^s > R$. In this case again $\frac{d(R-W^s(x))}{dx} = (1-\gamma)\frac{d\mathbb{E}_y[W^u(x,y)]}{dx}$. In this case

$$(1-\beta)\frac{d\mathbb{E}_y[W^u(x,y)]}{dx} = \int_{y^s(x)} \beta\lambda(\theta(x,y))\frac{d(M(x,y)-W^u(x,y))}{dx}dF(y) \tag{78}$$

The surplus $M(x,y) - W^u(x,y)$ responds to changes in x is given by

$$\begin{aligned}\frac{d(M(x,y)-W^u(x,y))}{dx} &= (1-\gamma)\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx} + (1-\gamma)(\mathbb{E}_y[y]-y)\\ &\quad + \gamma(y+\beta(1-\lambda(x,y))\left(\frac{d(M(x,y)-W^u(x,y))}{dx}\right), \end{aligned}\tag{79}$$

while the expected surplus evolves according to

$$\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx} = \int_{y^s(x)} y + \beta(1-\lambda(x,y))\frac{d(M(x,y)-W^u(x,y))}{dx}, \tag{80}$$

Substituting (**??**) into (**??**), it follows that $\frac{d\mathbb{E}_y[M(x,y)-W^u(x,y)]}{dx} > 0$, from which in turn it follows that (**??**) is also positive.

Finally, let us look at the implications for the cutoff in terms of island productivities $y^s(x), y^r(x)$. Consider first the case that $y^r(x) > y^s(x)$. The reservation quality for separation and reallocation satisfy implicitly, respectively

$$M(x, y^s(x)) - W^s = 0 \tag{81}$$

$$\lambda(\theta(x,y^r(x)))(1-\eta)(M(x,y^r(x)) - W^u(x,y^r(x))) + (W^s(x) - R) = 0. \tag{82}$$

We can see this defines $y^r(x), y^s(x)$ as implicit functions of $M(x,y) - W^s(x)$ and $W^s - R$. The first term is given by

$$\begin{aligned}M(x,y^s(x)) - W^s &= xy^s(x) - b + \beta(1-\gamma)\mathbb{E}_y[\max\{M(x,y)-W(x,y), W^s - R\}]\\ &\quad + \beta\gamma(W^s(x)-R) \end{aligned}\tag{83}$$

$$\begin{aligned}M(x,y^r(x)) - W^s &= xy^r(x) - b + \beta(1-\gamma)\mathbb{E}_y[\max\{M(x,y)-W(x,y), W^s - R\}]\\ &\quad + \beta\gamma(1-\lambda(\theta(x,y^r(x))(1-\eta)(M(x,y^r(x)) - W^s(x)). \end{aligned}\tag{84}$$

Taking derivatives with respect to x (taking into account the implicit relationship $y^s(x), y^r(x)$), we find

$$y^s(x) + \beta(1-\gamma)\frac{d}{dx}\Big(\mathbb{E}_y[\max\{M(x,y)-W(x,y), W^s-R\}]\Big) + \beta\gamma\frac{d(W^s-R)}{dx} + x\frac{dy^s(x)}{dx} = 0 \tag{85}$$

$$\begin{aligned}&\frac{\lambda(\theta)}{1-\beta\gamma(1-\lambda(\theta))}\left(y^r(x) + \beta(1-\gamma)\frac{d}{dx}\Big(\mathbb{E}_y[\max\{M(x,y)-W(x,y), W^s-R\}]\Big) + x\frac{dy^r(x)}{dx}\right)\\ &\quad + \frac{d(W^s-R)}{dx} = 0\end{aligned} \tag{86}$$

B Omitted Derivations in the Benchmark Model

Derivation of Workers Flows Changes over time in the unemployment and employment rates in an island i are described by the sum of four types of flows. The within-market flows of unemployment to employment and vice versa. The between-market flows of unemployed and the direct flow of employed workers who separate from their current employment to look for jobs as unemployed workers in other islands (after paying cost c).

Consider an island i at the beginning of period t with state vector Ω^s_{it}. Assume that on such an island all firms during the matching stage offer the same $\tilde{W}^*_f(\Omega^m_{it})$. As shown below, this will be indeed the case in equilibrium. Given u^s_{it} and e^s_{it}, the number of unemployed workers in this island at the beginning of the reallocation state is given by

$$u^r_{it} = \Big(\delta + (1-\delta)\mathbb{I}[\tilde{W}^*_f(\Omega^s_{it}) < W^U(\Omega^s_{it})]\Big)e^s_{it} + u^s_{it},$$

where $\mathbb{I}$ denotes a standard indicator function. The first term takes into account that a measure $\delta e^s_{it}$of employed workers gets displaced, while the rest of employed workers quit to unemployment if is optimal to do so. The number of unemployed u^r_{it} is given by summing this flow to the number of unemployed at the beginning of the period. The number of employed at the beginning of the reallocation stage is simply $e^r_{it} = e^s_{it} - (u^r_{it} - u^s_{it})$.

Now consider the number of unemployed and employed workers at the beginning of the matching stage. To derive these numbers we have to consider the flows between islands. It is important to remember that only those unemployed workers at the beginning of the period in each island, u^s_{it}, are allowed to reallocate. The flow from any island i to another island j is then given by

$$outflow(i,j) = u^s_{it}\mathbb{I}[R(\Omega^r_{jt}) > \mathbb{E}[S(\Omega^m_{it}) + W^U(\Omega^m_{it})]]dF_j.$$

This expression captures the transitions of the unemployed from island i to island j, where dF_j is the probability of drawing island j after deciding to reallocate. Since islands' identities are on the unit interval and are drawn randomly using a uniform distribution, $dF_j = 1$. The inflow into island i is given by

$$inflow(i) = \int_0^1 outflow(j,i)dj.$$

Hence the number of unemployed workers at the beginning of the matching stage is

$$u^m_{it} = u^r_{it} + inflow(i) - outflow(i,j),$$

from which only $u^s_{it} - outflow(i,j)$ are allowed to search for jobs, since workers that reallocated to this island at time t have to wait until the following period to search for jobs. Note that the number of employed workers at the beginning of the matching period is the same as the number of employed workers at the beginning of the reallocation period; that is, $e^m_{it} = e^r_{it}$.

Finally, the number of unemployed workers at the beginning of the production stage is given by

$$u^p_{it} = u^m_{it} - \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f))[u^s_{it} - outflow(i,j)].$$

When there is rest unemployment in the island, however, we have that $u^p_{it} = u^m_{it}$.[44] The number of employed workers is given by

$$e^p_{it} = e^m_{it} + \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f))[u^s_{it} - outflow(i,j)]$$

and $e^p_{it} = e^m_{it}$ in the case of rest unemployment.

Derivation of the 'Pissarides wage equation' Given that an employed worker value in steady state is

$$W^E(p,z) = w(p,z) + \beta(1-\delta)W^E(p,z) + \beta\delta W^U(p,z),$$

then

$$W^E(p,z)-W^U(p,z) = w(p,z)-b-\beta\lambda(\theta(p,z))(W^E(p,z)-W^U(p,z))+\beta(1-\delta)(W^E(p,z)-W^U(p,z)),$$

or

$$W^E(p,z) - W^U(p,z) = \frac{w(p,z)-b}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z))}.$$

From the combination of the free entry condition and the Hosios condition, we have

$$\eta\frac{w(p,z)-b}{1-\beta(1-\delta)+\beta\lambda(\theta(p,z))} = (1-\eta)k/q(\theta(p,z)). \quad (87)$$

Moreover, from the value of the firm, we have

$$\frac{k}{q(\theta(p,z))} = \frac{y(p,z)-w(p,z)}{1-\beta(1-\delta)}$$

Solving the latter equation for $w(z)$ gives

$$w(p,z) = y(p,z) - \frac{k}{q(\theta(p,z))}(1-\beta(1-\delta)).$$

Substituting this in (**??**), we find

$$\eta(y(p,z)-b) - \frac{k}{q(\theta(p,z))}(1-\beta(1-\delta)) - \beta\theta(p,z)(1-\eta)k = 0.$$

If we replace the middle term with $y(p,z) - w(p,z)$, we get the desired wage equation.

C Occupational Human Capital

Consider occupational human capital accumulation as described in the main text. Our assumptions imply that a labor market is now determined by a given occupational human capital within an island, rather than just an island as in the baseline model. As before we focus on Block Recursive Equilibria. Further, we focus attention on equilibria in which the values offered to all employed

[44] The case in which no worker decided to visit the sub-market is capture by the possibility that $\theta(\Omega^m_{it}, \tilde{W}^*_f) = \lambda(\theta(\Omega^m_{it}, \tilde{W}^*_f)) = 0$. As shown later, rest unemployment occurs for sufficiently low values of z. In this case, new firms will not enter these islands and hence setting $\theta(\Omega^m_{it}, \widetilde{W}^*_f) = \lambda(\theta(\Omega^m_{it}, \widetilde{W}^*_f)) = 0$.

workers in island i with productivity x_j at time t are equal. As in the baseline model, let z^n denote the lowest productivity of the set of islands from which a worker randomly draws if he decides to reallocate. The Bellman equations that described the candidate equilibrium are then given by

$$W^U(p,z,x_j) = b + \beta \mathbb{E}_{p',z'} \left[\max_{\rho(p',z',x_j)} \left\{ \rho(p',z',x_j) \left[-c + \int W^U(p',z'_i,x_1) dF(i) \right] + \right.\right.$$
$$\left.\left. (1-\rho(p',z',x_j)) \left[\max_{W_j{}^{E\prime}} \left\{ \lambda(\theta(p',z',x_j,W_j{}^{E\prime})) W_j{}^{E\prime} + (1-\lambda(\theta(p,z,x_j,W_j{}^{E\prime}))) W^U(p',z',x_j) \right\} \right] \right\} \right]$$

$$W^E(p,z,x_j) = w(p,z,x_j)$$
$$+\beta \mathbb{E}_{p',z'} \left\{ \gamma_{jj} \left[\max_{d(p',z',x_j)} (1-d(p',z',x_j)) W^E(p',z',x_j) + d(p',z',x_j) W^U(p',z',x_j) \right] \right.$$
$$\left. +(1-\gamma_{jj}) \left[\max_{d(p',z',x_{j+1})} (1-d(p',z',x_{j+1})) W^E(p',z',x_{j+1}) + d(p',z',x_{j+1}) W^U(p',z',x_{j+1}) \right] \right\} \tag{88}$$

$$\begin{aligned} J(p,z,x_j,\tilde{W}_j^E) \;=\; & \max_{\{w_j,\tilde{W}^{E\prime}(p',z',x_i)\}} \left\{ y(p,z,x_j) - w_j \right. \\ & +\beta \mathbb{E}_{p',z'} \left\{ \gamma_{jj} \left[\max_{\sigma(p',z',x_j)} \{(1-\sigma(p',z',x_j)) J(p',z',x_j,\tilde{W}^{E\prime}(p',z',x_j))\} \right] \right. \\ & \left.\left. +(1-\gamma_{jj}) \left[\max_{\sigma(p',z',x_{j+1})} \{(1-\sigma(p',z',x_{j+1})) J(p',z',x_{j+1},\tilde{W}^{E\prime}(p',z',x_{j+1}))\} \right] \right\} \right\} \end{aligned} \tag{89}$$

$$V(p,z,x_j,\tilde{W}) = -k + q(\theta(p,z,x_j,\tilde{W})) J(p,z,x_j,\tilde{W}) = 0,$$

where $\tilde{W}_j^E$, w_j and $W_j{}^{E\prime}$ must satisfy (**??**) and the first maximization in (**??**) is subject to the participation constraint (**??**) for each of the corresponding sub-markets.

C.0.1 Characterization

To characterize the equilibrium consider a sub-market x_j in island i at time t. Let the aggregate and idiosyncratic productivities be p and z. Given the free entry of firms at a sub-market level, Lemmas 1 and 2 can be directly applied here. All firms in a sub-market offer the same $W^*(p,z,x_j)$ with associated tightness $\theta^*(p,z,x_j)$ and the match surplus is divided according to (**??**). The application strategies of workers in each of these sub-markets are then the same as in the baseline model. That is, $\alpha = 1$ when $S(p,z,x_j) > 0$ and $\alpha = 0$ when $S(p,z,x_j) = 0$.

Similarly, in each sub-market the reallocation and separation policy functions are such that there exists a (potentially trivial) reservation productivity $z^s(p,x_j)$ below which any match in sub-market x_j, if it exists, is broken up with $d(p,z,x_j) = \sigma(p,z,x_j) = 1$ for all $z < z^s(p,x_j)$ and $d(p,z,x_j) = \sigma(p,z,x_j) = \delta$ otherwise. Further, there exists a reservation productivity $z^r(p,x_j)$ below which a worker in sub-market x_j reallocates with $\rho(p,z,x_j) = 1$ for all $z < z^r(p,x_j)$ and $\rho(p,z,x_j) = 0$ otherwise. As in the case of the baseline model, the existence of these reservation productivities is shown within the equilibrium's existence proof, to which we now turn.

C.0.2 Existence and Efficiency

Consider the operator T mapping a value function $\widetilde{M}(p,z,x_j,n)$ for $n=0,1$ and $j=1,2,3$ into the same functional space such that $\widetilde{M}(p,z,x_j,0) = M(p,z,x_j) \equiv J(p,z,x_j,W_j^E) + W^E(p,z,x_j)$, $\widetilde{M}(p,z,x_j,1) = W^U(p,z,x_j)$, and

$$\begin{aligned} T(\widetilde{M}(p,z,x_j,0)) &= y(p,z,x_j) + \beta\mathbb{E}_{p',z'}\Big\{\gamma_{jj}\left[\max_{d_j^T}(1-d_j^T)M(p',z',x_j) + d_j^T W^U(p',z',x_j)\right] \\ &\quad +(1-\gamma_{jj})\left[\max_{d_{j+1}^T}(1-d_{j+1}^T)M(p',z',x_{j+1}) + d_{j+1}^T W^U(p',z',x_{j+1})\right]\Big\} \end{aligned}$$

$$\begin{aligned} T(\tilde{M}(p,z,x_j,1)) = b + \beta\mathbb{E}_{p',z'}\Big[\max_{\rho_j^T}\{(\rho_j^T\left(\int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) - c\right) \\ +(1-\rho_j^T)(S^T(p',z',x_j) + W^U(p',z',x_j))\}\Big] \end{aligned}$$

where by virtue of the free entry condition

$$S^T(p',z',x_j) \stackrel{def}{=} \max_{\theta(p',z',x_j)}\Big\{\lambda(\theta(p',z',x_j))\Big(M(p',z',x_j) - W^U(p',z',x_j)\Big) - \theta(p',z',x_j)k\Big\}.$$

As with the baseline model the aim is to show that (i) the operator T is a contraction, mapping continuous functions, $M(p,z,x_j)$ and $W^U(p,z,x_j)$ for all j, that are increasing in z into itself; and (ii) to show that $M(p,z,x_j) - W^U(p,z,x_j)$ for all j in the fixed point of T is increasing in z. The main difference with the baseline model is that by adding three sub-markets we have increased the dimensionality of the operator T by three. To show (i) we invoke once more Assumption 1 and apply the same arguments in Lemma 3. Note that when showing that T is a contraction, choosing two functions $\widetilde{M}$ and $\widetilde{M}'$ such that $\| \widetilde{M} - \widetilde{M}' \|_{\sup} < \varepsilon$ implies that $\| M(p,z,x_j) - M'(p,z,x_j) \|_{\sup} < \varepsilon$ and $\| W^U(p,z,x_j) - W^{U\prime}(p,z,x_j) \|_{\sup} < \varepsilon$ *for each* $j=1,2,3$. Using this insight it is straightforward to verify Lemma 3 for this case and that there exists a reservation productivity $z^r(p,x_j)$ for every $j=1,2,3$ such that for $z < z^r(p,x_j)$ workers in sub-market x_j prefer to reallocate.

To establish (ii) we follow similar arguments as in Lemma 4. Consider the difference

$$\begin{aligned} & T(\widetilde{M}(p,z,x_j,0)) - T(\widetilde{M}(p,z,x_j,1)) \\ = \; & y(p,z,x_j) - b + \beta\mathbb{E}_{p',z'}\Big\{\gamma_{jj}\Big((1-\delta)\max[M(p',z',x_j) - W^U(p',z',x_j),0] + W^U(p',z',x_j)\Big) \\ & +(1-\gamma_{jj})\Big((1-\delta)\max[M(p',z',x_{j+1}) - W^U(p',z',x_{j+1}),0] + W^U(p',z',x_{j+1})\Big) \\ & -\max\Big\{\int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) - c, \lambda(\theta_j^*)(1-\eta)(M(p',z',x_j) - W^U(p',z',x_j)) + W^U(p',z',x_j)\Big\}\Big\}, \end{aligned}$$

for all $j=1,2,3$. We now need to show the conditions under which $T(\widetilde{M}(p,z,x_j,0)) - T(\widetilde{M}(p,z,x_j,1))$ is weakly increasing in z.

First suppose that an employed worker with x_j did not increase his human capital. In this case, the above expression can be simplified such that

$$\begin{aligned}
& T(\widetilde{M}(p,z,x_j,0)) - T(\widetilde{M}(p,z,x_j,1)) \\
= \; & y(p,z,x_j) - b + \beta \mathbb{E}_{p',z'}\Big\{(1-\delta)\max[M(p',z',x_j) - W^U(p',z',x_j),0] \\
& - \max\Big\{\int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) - c - W^U(p',z',x_j), \lambda(\theta_j^*)(1-\eta)(M(p',z',x_j) - W^U(p',z',x_j))\Big\}\Big\}.
\end{aligned}$$

The arguments of Lemma 4 can be directly applied to show that a sufficient condition for $T(\widetilde{M}(p,z,x_j,0)) - T(\widetilde{M}(p,z,x_j,1))$ to be weakly increasing is given by $1-\delta-\lambda(\theta_j^*) > 1$ for all j.

Next consider the case in which an employed worker with x_j for $j = 1,2$ did increase his human capital. We then have that

$$\begin{aligned}
& T(\widetilde{M}(p,z,x_j,0)) - T(\widetilde{M}(p,z,x_j,1)) \\
= \; & y(p,z,x_j) - b + \beta \mathbb{E}_{p',z'}\Big\{(1-\delta)\max[M(p',z',x_{j+1}) - W^U(p',z',x_{j+1}),0] \\
& + W^U(p',z',x_{j+1}) - W^U(p',z',x_j) \\
& - \max\Big\{\int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) - c - W^U(p',z',x_j), \lambda(\theta_j^*)(1-\eta)(M(p',z',x_j) - W^U(p',z',x_j))\Big\}\Big\}.
\end{aligned}$$

First let $j = 2$. Consider the range of $z \in [\underline{z}, z^s(p,x_3))$, where $z^r(p,x_2) > z^s(p,x_3)$ such that employed workers with x_3 voluntarily quit into unemployment and unemployed workers with x_2 reallocate. Under these conditions the terms under the expectation simplify to $-\int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) + c + W^U(p',z',x_3)$. It then follows that since $W^U(p',z',x_3)$ increases in z, then $T(\widetilde{M}(p,z,x_2,0)) - T(\widetilde{M}(p,z,x_2,1))$ also increases in z. Note that the previous arguments also hold when $z \in [\underline{z}, z^r(p,x_2))$, where $z^r(p,x_2) < z^s(p,x_3)$, as in this case employed workers with x_3 voluntarily quit into unemployment and unemployed workers with x_2 reallocate. Now suppose that $z \in [z^s(p,x_3), z^r(p,x_2))$. The term under the expectation then simplifies to $(1-\delta)[M(p',z',x_3) - W^U(p',z',x_3)] + W^U(p',z',x_3) - \int W^U(p',\widetilde{z},x_1)dF(\widetilde{z}) + c$. Since both $M(p',z',x_3) - W^U(p',z',x_3)$ and $W^U(p',z',x_3)$ are weakly increasing in z, $T(\widetilde{M}(p,z,x_2,0)) - T(\widetilde{M}(p,z,x_2,1))$ also weakly increases in z.

Next suppose that $z^r(p,x_2) < z^s(p,x_3)$ and consider a $z \in [z^r(p,x_2), z^s(p,x_3))$. In this case, instead of establishing that the contraction $T(\widetilde{M}(p,z,x_2,0)) - T(\widetilde{M}(p,z,x_2,1))$ maps increasing difference into increasing difference and hence its fixed point also implies that $M(p,z,x_2) - W^U(p,z,x_2)$ increases in z, we now show that *at* the fixed point $M(p,z,x_2) - W^U(p,z,x_2)$ is increasing in z. To show the latter we follow a contradiction argument.

Suppose that at the fixed point of $T(\widetilde{M}(p,z,x_2,0)) - T(\widetilde{M}(p,z,x_2,1))$, $M(p,z,x_2) - W^U(p,z,x_2)$ is decreasing in z such that $M(p,z,x_2) < W^U(p,z,x_2)$ for all $z > z^s(p,x_2)$. Consider a $z \in [z^r(p,x_2), z^s(p,x_3))$ such that $z > z^s(p,x_2)$. This implies that at the fixed point

$$M(p,z,x_2) - W^U(p,z,x_2) = \; y(p,z,x_2) - b + \beta \mathbb{E}_{p',z'}\Big\{W^U(p',z',x_3) - W^U(p',z',x_2)\Big\}.$$

Since $y(p,z,x_2) > b$ and $W^U(p',z',x_j)$ is increasing in x, the LHS of the above expression is strictly positive, which contradicts that at a $z \in [z^r(p,x_2), z^s(p,x_3))$ with $z > z^s(p,x_2)$, $M(p,z,x_2) < W^U(p,z,x_2)$.

Now consider a $z > z^s(p, x_3) > z^r(p, x_2)$ such that $z > z^s(p, x_2)$ or a $z > z^r(p, x_2) > z^s(p, x_3)$ such that $z > z^s(p, x_2)$. In both of these cases we have that at the fixed point

$$M(p, z, x_2) - W^U(p, z, x_2) = \quad y(p, z, x_2) - b + \beta \mathbb{E}_{p', z'} \Big\{ (1 - \delta)[M(p', z', x_3) - W^U(p', z', x_3)] \\ + W^U(p', z', x_3) - W^U(p', z', x_2) \Big\}.$$

Since LHS is strictly positive and implies $M(p, z, x_2) - W^U(p, z, x_2) > 0$, we once again obtain our require contradiction. Hence, at the fixed point of $T(\widetilde{M}(p, z, x_2, 0)) - T(\widetilde{M}(p, z, x_2, 1))$, $M(p, z, x_2) - W^U(p, z, x_2)$ is weakly increasing in z.

Now let $j = 1$. It easy to verify that the same argument used above imply that at the fixed point of $T(\widetilde{M}(p, z, x_1, 0)) - T(\widetilde{M}(p, z, x_1, 1))$, $M(p, z, x_1) - W^U(p, z, x_1)$ is weakly increasing in z.

Taken together, the above arguments imply that at the fixed point of $T(\widetilde{M}(p, z, x_j, 0)) - T(\widetilde{M}(p, z, x_j, 1))$ for all $j = 1, 2, 3$, $M(p, z, x_j) - W^U(p, z, x_j)$ is weakly increasing in z and hence there exists a reservation productivity $z^s(p, x_j)$ such that for $z < z^s(p, x_j)$ workers and firms will decide to dissolve the match and for $z > z^s(p, x_j)$ workers and firms decide to continue production in the match.

Finally, it is straightforward to verify that Propositions 1 and 2 also hold in this environment and hence a Block Recursive Equilibrium with occupational human capital exists, is unique and efficient.

D Data Construction

The SIPP is a longitudinal data set based on a representative sample of the US civilian noninstitutionalized population. It is is divided into multi-year panels. Each panel comprise a new sample of individuals and is subdivided into four rotation groups. Individuals in a given rotation group are interviewed every four months such that information for each rotation group is collected each month. At each interview individuals are asked, among other things, about their employment status as well as their occupations and industrial sectors during employment in the last four months.[45]

There are several advantages of using the SIPP to other data sets like the Current Population Survey (CPS) or the Panel Study of Income Dynamics (PSID), which also have been used to measure labor market flows and/or occupational and sectoral mobility. The SIPP's longitudinal dimension, high frequency interview schedule and explicit aim to collect information on worker turnover allows us to construct reliable measures of occupational mobility and labor market flows.[46]

We consider the period 1986 - 2009. To cover this period we use the 1986-1988, 1990-1993, 1996, 2001, 2004 and 2008 panels. Although the SIPP started in 1984, our period of study reflects two considerations. The first one is methodological. Since 1986 the US Census Bureau has been using dependent interviewing in the SIPP's survey design, which helps to reduce measurement error problems. The second reason is that such a period allows us to study the behavior of unemployment,

[45] See http://www.census.gov/sipp/ for a detailed description of the data set.

[46] See Mazumder (2007), Fujita, Nekarda and Ramey (2007) and Nagypal (2008) for recent studies that document labor market flows and Xiong (2008) for a study that documents occupational mobility using the SIPP. To our knowledge there is no study that uses the SIPP to jointly study labor market flow and occupational/sectoral mobility.

labor market flows between unemployment and employment and occupational mobility during the "great moderation" and also capture some aspects of the last recession.

For the panels 1986-1988 and 1990-1993 we have used the Full Panel files as the basic data set, but appended the monthly weights obtained from the individual waves. We have used the Full Panel files as the individual waves do not have clear indicators of the job identifier. Since the US Census Bureau does not provide the Full Panel file for the 1989 data set, which was discontinued and only three waves are available, we opted for not using this data set. This is at a minor cost as the 1988 panel covers up to September 1989 and the 1990 panel collects data as from October 1989. For the panels 1996, 2001, 2004, 2008 there is no Full Panel files, but one can easily construct the full panel by appending the individual wave information using the individual identifier "lgtkey". In this case, the job identifier information is clearly specify.

Two important differences between the post and pre-1996 panels are worth noting. The pre-1996 panels have an overlapping structure and a smaller sample size. Starting with the 1996 panel the sample size of each panel doubled in size and the overlapping structure was dropped. To overcome these differences and make the sample sizes somehow comparable, we constructed our pre-1996 indicators by obtaining the average value of the indicators obtained from each of the overlapping panels. On the other hand, the SIPP's sample design implies that in *all* panels the first and last three months have less than 4 rotation groups and hence a smaller sample size. For this reason we only consider months that have information for all 4 rotation groups. The data also shows the presence of seams effects between waves. To reduce the seam bias we average the value of the indicator over the four months that involve the seam. For the panels 1990-2008 the indicators are based on the employment status variable at the second week of each month, "wesr2" for the 1990-1993 panels and "rwkesr2" for the 1996-2008 panels. Given that for the panels 1986-1988 we do not have a weekly employment status variable, our indicators are based on the employment status monthly recode variable "esr". The choice of the second week is to approximate the CPS reference week when possible.[47]

For the 1990-2008 panels, a worker is considered employed if he/she was *attached* to a job. Namely if the individual was (1) with job/business - working, (2) with job/business - not on layoff, absent without pay and (3) with job/business - on layoff, absent without pay. A worker is considered unemployed if he/she was *not attached* to a job and looking for work. Namely if the individual was with (4) no job/business - looking for work or on layoff. A worker is then considered out of the labor force (non-participant) if he/she was with (5) no job/business - not looking for work and not on layoff. For the 1986-1988 panels we follow the same principle. A worker is considered employed if he/she was (1) with a job the entire month, worked all weeks, (2) With a job all month, absent from work w/out pay 1+ weeks, absence not due to layoff, (3) with job all month, absent from work w/out pay 1+ weeks, absence due to layoff, (4) with a job at least 1 but not all weeks, no time on layoff and no time looking for work and (5) with job at least 1 but not all weeks, some weeks on layoff or looking for work. A worker is considered unemployed if he/she was with (6) no job all

[47] See Fujita, Nekarda and Ramey (2007) for a similar approach. We have also performed our analysis by constructing the labor market status of a worker based on the employment status monthly recode variable for all panels and our results do not change.

month, on layoff or looking for work all weeks and (7) no job, at least one but not all weeks on layoff or looking for work. The worker is considered out of the labor force if he/she was with (8) no job, no time on layoff and no time looking for work.

The SIPP collects information on a maximum of two jobs an individual might hold simultaneously. For each of these jobs we have information on, among other things, hours worked, total earnings, three digit occupation and three digit industry codes. If the individual did hold two jobs simultaneously, we consider the main job as the one in which the worker spent more hours. We break a possible tie in hours by using total earnings. The job with the highest total earnings will then be considered the main job. In most cases individuals report to work in one job at any given moment. In the vast majority of cases in which individuals report two jobs, the hours worked are sufficient to identify the main job. Once the main job is identified, the worker is assigned the corresponding three digit occupation.[48]

Using the derived labor market status indicators and main job indicators we measure occupational mobility by comparing the reported occupation at re-employment with all those occupations the individual had performed in past jobs. Since the occupational data is collected only when the worker is employed, this procedure is valid only for job changes (with an intervening unemployment spell) after the first observed employment spell. For these cases, we assume that after an employment spell, the unemployed worker retains the occupation of the last job and stays with it until he/she re-enters employment, were the worker might perform a new occupation. Under this procedure we have allowed the unemployed worker to keep his/her occupation when he/she undergoes an intervening spell of non-participation that leads back to unemployment. If this spell of non-participation leads directly to employment, however, we do not count this change as it does not involve an unemployment to employment transition. We also have allowed the worker to retain his/her occupation if the employment spell is followed by a spell of non-participation that leads into unemployment. In summary, the worker retains his/her occupation for transitions of the type: E-U-E, E-U-NP-U-E, E-NP-U-E or combinations of these; and does not retain his/her occupation for transitions of the type: E-NP-E, E-U-NP-E or combinations of these. For unemployment spells that precede the first employment spell we impute the occupation of the first observed job. Hence these transitions will always be unemployment to employment transitions without an occupational change.

We construct monthly time series for the unemployment rate, employment to unemployment transition rate (job separation rate), unemployment to employment transition rate (job finding rate), and the components of the decomposition of the job finding rate described in the main text. Since there are months for which the SIPP does not provide data and we do not take into account months with less than 4 rotation groups, we have breaks in our time series. To cover the missing observations we interpolate the series using the TRAMO (Time Series Regression with ARIMA Noise, Missing Observations and Outliers) procedure developed by Gomez and Maravall (1999).[49] The periods with breaks are between 1989Q3-1989Q4, 1995Q4-1996Q1, 1999Q4-2000Q4, 2003Q4-2004Q1 and 2007Q4-2008Q2.

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[48] For the 1990-1993 panels we correct the job identifier variable following the procedure suggested by Stinson (2003).

[49] See also Fujita, Nekarda and Ramey (2007) for a similar procedure using the SIPP.

Given the interpolated series, we seasonally adjust them using the Census Bureau X12 program. The cyclical components of these series are obtained by detrending the log of each of these series based on quarterly averages and using the HP filter with smoothing parameter 1600. Our working series are not adjusted for time aggregation error. The main reason for this choice is that when using the now "standard" method to correct for time aggregation bias proposed by Shimer (2012) and extended by Elsby, Micheals and Solon (2009) and Fujita and Ramey (2009), one can only get closed form solution for the adjusted job finding and separation rates when only considering changes between two states (for example, employment and unemployment). Correcting for time aggregation when taking into account for 3-digit occupational changes then becomes extremely cumbersome. Using Fujita and Ramey's (2009) extension, however, we find that time aggregation has little effect on the cyclical behaviour of the aggregate job finding and separation rates in the SIPP.[50]

E Occupational Mobility by Gender, Age and Education

In this Appendix we analyse occupational mobility through unemployment by conditioning the samples on different demographic characteristics. Table A1 shows the job finding rates and job finding probabilities with and without occupational change and composition effects by gender. In particular, the first row of Table A1 shows that occupational mobility is important for both men and women. The job finding rate with occupational mobility, $focc$, for both categories explains on average around 45 percent of their respective job finding rate, f. When considering the job finding rate without an occupational change, $fnocc$, we find that it represents around 53 percent of their respective job finding rates. In terms of the probabilities of finding a job in a different or in the same occupation, $Pocc$ and $Pnocc$, the first row of Table A1 shows that women exhibit higher probabilities in both cases. However, the composition effects, $Cocc$ and $Cnocc$, are the same for both groups. Table A1 also shows that the degree of procyclicality of the job finding rate with occupational mobility and the composition effect, $Cocc$, are higher for women than for men, while the opposite is true when considering the job finding rate without an occupational change and the probabilities of finding a job with and without an occupational change.

Table A2 considers different age groups. Here we divide the sample into a "young" group that includes those workers between 16 and 30 years of age; a "prime" group corresponding to those between 31 and 50 years of age and an "old" group of workers that are between 51 and 65 years old. Occupational mobility also seems an important aspect of the job finding process for all these workers. The first row of Table A2 shows that the importance of $focc$ in f and $Pocc$ decreases with the age groups, while the importance of $fnocc$ in f and $Pnocc$ increases with the age groups. This evidence seems consistent with the idea that young workers find it less costly to move occupation possibly because of their relative lower occupational specific human capital levels. The composition effects, however, take very similar average values for "young" and "prime" age workers, while "older" workers exhibit lower average values of $Cocc$. Table A2 also shows that when comparing these measures with output per worker, $focc$, $fnocc$, $Pocc$ and $Cocc$ exhibit a

[50]Fujita, Nekarda and Ramey (2007) arrived to a similar conclusion when analysing aggregate job finding and separations rates using the SIPP for the period 1983-2003.

higher degree of procyclicality for the "prime" group of workers, while $Pnocc$ exhibits a higher degree of procyclicality for the "old" group of workers.

Table A3 divides the sample into different educational categories: (i) workers with less than a high school degree, (ii) with a high school degree, (iii) with some college education and (iv) workers with a college degree. Once again occupational mobility through unemployment is important for all these workers. The first row of Table A3 shows that the importance of $focc$ ($fnocc$) in f and of $Cocc$ ($Cnocc$) increases (decreases) with the level of education, although for those workers with college degrees these measures have lower average values than for those workers with some college. In terms of the probability of finding a job with an occupational change, $Pocc$, there is not much difference in the probability of an occupational change across educational groups. Table A3 also shows that $focc$ and $Pnocc$ exhibit the highest degree of procyclicality for the workers with college degrees, $fnocc$ exhibits the highest degree of procyclicality for those workers with high school degrees and $Pocc$ and $Cocc$ exhibit the highest degree of procyclicality when considering workers with less than high school education.

In summary, the patterns observed in Table 1 seem to be reproduced when considering each demographic group. Across each of these groups we find that the job finding rate with an occupational mobility is an important component of the aggregate job finding rate for each relevant category. In terms of the job finding probability with an occupational change, our findings suggest that these probabilities mostly differ across age groups and gender, but they do not differ across educational groups.[51] Further, Tables A1-A3 suggest that the degree of procyclicality or countercylicality of $focc$, $fnocc$, $Pocc$, $Pnocc$, $Cocc$ and $Cnocc$ found in Table 1, do not seems to be strongly driving by particular demographic groups. In most cases and consistent with the results of Table 1, we also observe that $focc$ and $Pocc$ have a higher degree of procyclicality than $fnocc$ and $Pnocc$. Finally, Tables A1-A3 show that the unemployment rate and the job finding and separation rates follow the expected patterns. Namely, (i) the unemployment rate decreases with age groups and educational categories; (ii) the job finding rate increases with age groups and educational categories; (iii) and the job separation rate decreases with age groups and educational categories.

Further NOTES – to be commented out for publication

In calculating job finding rates or unemployment outflow rates conditional on being an occupational movers or stayer, we require at least *8* more periods in sample. One period before the end of the sample period, all unemployed in these groups have to flow out (because otherwise we cannot assign them to either group)

[51]This result is consistent with the findings of Longhi and Taylor (2011) for the UK, who consider probit models to estimate the probability of an occupational change through unemployment conditioning on different demographic categories.

Table A1.a: Job Finding Rates and Occupational Change for Male Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.158	0.071	0.084	0.304	0.519	0.377	0.480	0.009	0.06		
Std. Dev	0.088	0.168	0.119	0.108	0.111	0.098	0.105	0.132	0.141	0.009	0.016
Autocorr.	0.643	0.759	0.658	0.759	0.734	0.653	0.780	0.834	0.902	0.691	0.871
Correlation Matrix											
frate	1.000	0.364	0.659	0.509	0.030	0.560	-0.040	-0.306	-0.680	0.337	0.617
focc		1.000	-0.217	0.637	0.778	0.546	-0.792	-0.688	-0.549	0.402	0.413
fnocc			1.000	0.000	-0.374	0.165	0.442	0.148	-0.279	0.100	0.269
Pocc				1.000	0.327	0.822	-0.386	-0.794	-0.713	0.597	0.752
Cocc					1.000	0.385	-0.976	-0.518	-0.237	0.206	0.088
Pnocc						1.000	-0.418	-0.640	-0.629	0.483	0.592
Cnocc							1.000	0.524	0.235	-0.228	-0.130
Srate								1.000	0.731	-0.663	-0.687
Urate									1.000	-0.535	-0.815
Outpw										1.000	0.828
Output											1.000

Table A1.b: Job Finding Rates and Occupational Change for Female Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.157	0.072	0.084	0.327	0.517	0.400	0.483	0.007	0.052		
Std. Dev	0.125	0.179	0.107	0.107	0.098	0.089	0.086	0.125	0.115	0.009	0.016
Autocorr.	0.861	0.756	0.791	0.763	0.474	0.776	0.649	0.755	0.885	0.691	0.871
Correlation Matrix											
frate	1.000	0.479	0.694	0.605	0.063	0.657	-0.102	-0.032	-0.603	0.192	0.552
focc		1.000	0.102	0.549	0.687	0.509	-0.682	-0.638	-0.633	0.495	0.531
fnocc			1.000	0.312	-0.210	0.477	0.255	0.208	-0.466	-0.021	0.329
Pocc				1.000	0.110	0.742	-0.098	-0.446	-0.561	0.481	0.658
Cocc					1.000	0.152	-0.959	-0.500	-0.241	0.423	0.275
Pnocc						1.000	-0.197	-0.344	-0.668	0.338	0.587
Cnocc							1.000	0.513	0.268	-0.409	-0.281
Srate								1.000	0.567	-0.561	-0.526
Urate									1.000	-0.438	-0.746
Outpw										1.000	0.828
Output											1.000

Table A2.a: Job Finding Rates and Occupational Change for Young Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.174	0.078	0.092	0.339	0.511	0.412	0.483	0.012	0.085		
Std. Dev	0.088	0.164	0.103	0.096	0.105	0.080	0.086	0.123	0.109	0.009	0.016
Autocorr.	0.755	0.773	0.761	0.737	0.705	0.687	0.761	0.810	0.875	0.691	0.871
Correlation Matrix											
frate	1.000	0.329	0.569	0.660	-0.036	0.481	-0.014	-0.058	-0.513	0.242	0.573
focc		1.000	-0.274	0.625	0.764	0.473	-0.678	-0.628	-0.519	0.396	0.383
fnocc			1.000	0.085	-0.451	0.102	0.586	0.339	-0.195	-0.063	0.155
Pocc				1.000	0.288	0.746	-0.311	-0.549	-0.635	0.484	0.690
Cocc					1.000	0.289	-0.864	-0.528	-0.250	0.279	0.122
Pnocc						1.000	-0.316	-0.556	-0.606	0.427	0.605
Cnocc							1.000	0.535	0.206	-0.301	-0.206
Srate								1.000	0.650	-0.597	-0.590
Urate									1.000	-0.494	-0.766
Outpw										1.000	0.828
Output											1.000

Table A2.b: Job Finding Rates and Occupational Change for Prime Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.146	0.064	0.080	0.293	0.516	0.367	0.481	0.006	0.044		
Std. Dev	0.102	0.191	0.121	0.118	0.105	0.108	0.082	0.142	0.153	0.009	0.016
Autocorr.	0.796	0.722	0.726	0.733	0.442	0.673	0.787	0.769	0.919	0.691	0.871
Correlation Matrix											
frate	1.000	0.483	0.696	0.496	0.239	0.630	-0.402	-0.380	-0.815	0.337	0.710
focc		1.000	-0.026	0.439	0.781	0.452	-0.737	-0.672	-0.657	0.447	0.490
fnocc			1.000	0.096	-0.156	0.377	0.207	-0.007	-0.438	0.055	0.349
Pocc				1.000	0.184	0.715	-0.236	-0.593	-0.524	0.555	0.642
Cocc					1.000	0.272	-0.772	-0.541	-0.389	0.382	0.342
Pnocc						1.000	-0.347	-0.563	-0.653	0.391	0.567
Cnocc							1.000	0.507	0.507	-0.330	-0.428
Srate								1.000	0.693	-0.653	-0.653
Urate									1.000	-0.486	-0.782
Outpw										1.000	0.828
Output											1.000

Table A2.c: Job Finding Rates and Occupational Change for Old Workers, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.118	0.049	0.066	0.272	0.479	0.352	0.513	0.004	0.035		
Std. Dev	0.126	0.218	0.151	0.174	0.156	0.134	0.110	0.175	0.157	0.009	0.016
Autocorr.	0.604	0.665	0.566	0.584	0.697	0.498	0.686	0.702	0.862	0.691	0.871
Correlation Matrix											
frate	1.000	0.524	0.682	0.537	0.091	0.612	-0.144	-0.283	-0.577	0.164	0.423
focc		1.000	0.108	0.594	0.530	0.392	-0.462	-0.593	-0.527	0.353	0.400
fnocc			1.000	0.119	-0.099	0.394	0.149	0.141	-0.158	-0.121	-0.029
Pocc				1.000	-0.015	0.505	0.070	-0.402	-0.513	0.313	0.479
Cocc					1.000	-0.004	-0.888	-0.292	-0.067	0.076	-0.040
Pnocc						1.000	-0.011	-0.373	-0.502	0.460	0.543
Cnocc							1.000	0.301	0.072	-0.030	0.014
Srate								1.000	0.656	-0.512	-0.560
Urate									1.000	-0.476	-0.743
Outpw										1.000	0.828
Output											1.000

Table A3.a: Job Finding Rates and Occupational Change for Workers with College, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.179	0.080	0.097	0.313	0.512	0.405	0.488	0.004	0.028		
Std. Dev	0.121	0.168	0.140	0.145	0.115	0.140	0.126	0.175	0.175	0.009	0.016
Autocorr.	0.763	0.665	0.579	0.615	0.678	0.759	0.693	0.682	0.866	0.691	0.871
Correlation Matrix											
frate	1.000	0.595	0.749	0.391	0.051	0.579	-0.068	-0.463	-0.776	0.303	0.608
focc		1.000	0.180	0.419	0.495	0.538	-0.487	-0.406	-0.623	0.502	0.547
fnocc			1.000	0.135	-0.216	0.371	0.223	-0.380	-0.496	0.036	0.267
Pocc				1.000	-0.328	0.336	0.338	-0.420	-0.399	0.482	0.500
Cocc					1.000	0.239	-0.972	-0.004	-0.146	0.100	0.053
Pnocc						1.000	-0.286	-0.501	-0.708	0.539	0.732
Cnocc							1.000	-0.013	0.152	-0.101	-0.065
Srate								1.000	0.600	-0.516	-0.500
Urate									1.000	-0.523	-0.812
Outpw										1.000	0.828
Output											1.000

Table A3.b: Job Finding Rates and Occupational Change for Workers with Some College, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.181	0.084	0.094	0.327	0.529	0.419	0.463	0.007	0.045		
Std. Dev	0.107	0.169	0.139	0.096	0.115	0.108	0.117	0.132	0.133	0.009	0.016
Autocorr.	0.756	0.784	0.784	0.571	0.765	0.718	0.764	0.772	0.902	0.691	0.871
Correlation Matrix											
frate	1.000	0.367	0.593	0.605	-0.078	0.621	-0.044	-0.132	-0.627	0.248	0.589
focc		1.000	-0.188	0.653	0.750	0.666	-0.743	-0.597	-0.551	0.407	0.463
fnocc			1.000	0.185	-0.528	0.155	0.586	0.107	-0.322	0.052	0.264
Pocc				1.000	0.244	0.727	-0.308	-0.518	-0.624	0.420	0.610
Cocc					1.000	0.362	-0.904	-0.551	-0.228	0.223	0.097
Pnocc						1.000	-0.427	-0.532	-0.664	0.392	0.588
Cnocc							1.000	0.532	0.238	-0.296	-0.245
Srate								1.000	0.633	-0.622	-0.621
Urate									1.000	-0.482	-0.772
Outpw										1.000	0.828
Output											1.000

Table A3.c: Job Finding Rates and Occupational Change for Workers with High School, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.154	0.067	0.084	0.304	0.511	0.376	0.488	0.008	0.062		
Std. Dev	0.101	0.161	0.102	0.117	0.053	0.092	0.052	0.129	0.130	0.009	0.016
Autocorr.	0.780	0.710	0.727	0.649	0.303	0.657	0.367	0.812	0.877	0.691	0.871
Correlation Matrix											
frate	1.000	0.444	0.732	0.638	0.056	0.437	-0.048	-0.282	-0.709	0.313	0.651
focc		1.000	0.036	0.553	0.457	0.413	-0.447	-0.712	-0.577	0.470	0.458
fnocc			1.000	0.195	-0.258	0.278	0.265	0.020	-0.468	0.110	0.386
Pocc				1.000	0.130	0.693	-0.116	-0.598	-0.607	0.414	0.603
Cocc					1.000	0.257	-0.986	-0.311	-0.094	0.208	0.105
Pnocc						1.000	-0.248	-0.600	-0.540	0.413	0.517
Cnocc							1.000	0.273	0.071	-0.198	-0.081
Srate								1.000	0.674	-0.590	-0.628
Urate									1.000	-0.449	-0.768
Outpw										1.000	0.828
Output											1.000

Table A3.d: Job Finding Rates and Occupational Change for Workers with Less HS, 1986 - 2009

	frate	focc	fnocc	Pocc	Cocc	Pnocc	Cnocc	Srate	Urate	Outpw	Output
Mean (levels)	0.130	0.049	0.079	0.310	0.441	0.380	0.559	0.014	0.121		
Std. Dev	0.101	0.183	0.100	0.122	0.113	0.095	0.085	0.133	0.094	0.009	0.016
Autocorr.	0.670	0.741	0.647	0.758	0.654	0.574	0.681	0.656	0.855	0.691	0.871
Correlation Matrix											
frate	1.000	349	0.662	0.557	-0.028	0.463	0.042	0.113	-0.510	0.072	0.419
focc		1.000	0.026	0.470	0.653	0.273	-0.645	-0.454	-0.488	0.308	0.292
fnocc			1.000	0.149	-0.292	0.227	0.385	0.170	-0.290	-0.081	0.124
Pocc				1.000	0.253	0.746	-0.272	-0.370	-0.648	0.559	0.768
Cocc					1.000	0.089	-0.971	-0.369	-0.246	0.331	0.236
Pnocc						1.000	-0.096	-0.330	-0.546	0.305	0.550
Cnocc							1.000	0.356	0.224	-0.331	-0.240
Srate								1.000	0.520	-0.503	-0.420
Urate									1.000	-0.474	-0.727
Outpw										1.000	0.828
Output											1.000

	High School			College+		
	young	prime	old	young	prime	old
occ. stay after occ. stay	0.396	0.318	0.279	0.291	0.373	0.133*
occ. stay after occ. move	0.388	0.293	0.276	0.472	0.328	0.324*
occ. move after occ. move	0.286	0.307	0.273	0.370	0.357	0.357*
occ. move after occ. stay	0.309	0.244	0.256	0.271	0.201	0.277*

Table 9: Re-employment rates of the repeat unemployed, by initial and subsequent occupational moving and staying, by schooling and age group (*=very few observations)