Group decision making

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very preliminary

Abstract

1 Introduction

We examine collective decision making in large groups. In some situations, decision rules take the form of a majority or supermajority rule in which each member has a vote (i.e. one class voting, unicameral legislature). In some other situations, members are grouped into classes and the proposal put to a vote passes if it is not vetoed by a majority of members within a particular class (i.e. two or several class voting, multicameral legislative process).

The purpose of this paper is to compare the welfare and distributional properties of various decision rules such as the ones just described. We analyze settings where groups are composed of subgroups with conflicting interests, and where preferences within each subgroup are heterogenous: one objective of the paper will also be to understand whether and when subgroup heterogeneity is an asset for that subgroup.

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We make these comparisons using a collective search model where proposals are drawn randomly in any period, and where a proposal is adopted if it gets the required support.

We show that *whatever the majority requirement considered*, one-class voting typically generates inefficiencies, while an appropriate multicameral design can restore efficiency. With one-class voting, inefficiencies either take the form of delay in reaching agreements (this occur when the majority requirement is set high enough so as to prevent a minority group from being hurt), or inefficiencies take the form of reaching ex post inefficient agreements that hurt a minority (a form of "tyranny of the majority"), with the consequence that all sub-groups, for fear of being in the losing minority, end up accepting inefficient proposals.

There is however one instance in which one-class voting generates *only little* inefficiencies. This occurs when there are two sub-groups of similar size, and when transfer mechanisms within each subgroup ensure that benefits are spread across many subgroup members.

Concerning distribution, we find that with one-class voting (and two subgroups), whether homogenous preferences is an asset for a subgroup depends on the strength of the majority requirement.

When the majority requirement is not too large, the subgroup with more homogenous preferences obtains a larger share of the surplus, because proposals that favor it pass more easily: it is sufficient a small fraction of "dissidents" from the other group happen to like the proposal, and this event is not rare when the other group has heterogenous preferences.

When the majority requirement is large, the opposite is true. The subgroup with more heterogenous preferences obtains a larger share of the surplus because proposals that pass have to please a substantial fraction of that group, including those members that may have preferences opposite to those of the other group.

Finally, with multi-class voting, we find that sub-group heterogeneity is always an asset: though it increases overall delays in reaching agreement, the more heterogenous groups gets a larger share of the surplus, because obtaining the assent of such a subgroup is more difficult, so proposals that on average favor such a subgroup pass more easily.

One corollary of these results is that when there are two groups of similar size, one-class voting (with a not too large majority requirement) foster transfer mechanisms that reduce conflicts of interests within subgroups. Such incentives do not exit in other circumstances (multi-class voting or large majority requirement).

2 Basic Model

We consider a group consisting of n members, labeled $i = 1, ..., n$. At any date $t = 1, ...$, if a decision has not been made yet, a new proposal is drawn and examined. A proposal is denoted u, where $u = (u_i)_{i \in \{1,...,n\}}$ is a vector in R^n that describes the utility u_i that member i gets if the proposal u is adopted. The set of possible proposals is denoted U, and it is assumed to be a compact convex subset of $[\underline{u}, \overline{u}]^n$. We also assume that proposals at the various dates $t = 1, ..$ are drawn *independently* from the same distribution with continuous density $f(\cdot) \in \Delta(U)$. We shall be more specific about that distribution over proposals at the end of this Section.

Upon arrival of a new proposal u, each member decides whether to accept that proposal. The game stops whenever the current proposal receives sufficiently strong support. We call the rule that specifies whether support is strong enough *a decision rule.* We distinguish various types of decision rules. First we consider rules where each member has a vote and where support is characterized by a simple majority or supermajority rule. These rules will be referred to as one-class voting rules. Second, we consider rules where each subgroup constitute a class, and where a proposal is accepted if there is not a majority within a class that rejects it. These rules will be referred to as multiple-class voting rules.

But more general rules may also be considered, whereby acceptance of a proposal may depend on (i) the total number of members that accept it (ii) the number of members in each subgroup that accept it (iii) the number of subgroups that accept it.

In its form general form, a decision rule is formalized as follows. A vote or decision to support for member i is denoted $z_i = 0, 1$ where $z_i = 1$ stands for support and $z_i = 0$ for no support, and a decision rule is a mapping $\rho(.)$ from the vector of individual votes $z = (z_i)_i$ to $\{0, 1\}$, where $\rho(z) = 1$ stands for support.

We normalize to 0 the payoff that parties obtain under perpetual disagreement, and we let δ denote the common discount factor of the committee members. That is, if the proposal u is accepted at date t, the date 0 payoff of member i is $\delta^t u_i$.

Strategies and equilibrium. In principle, a strategy specifies an acceptance rule that may at each date be any function of the history of the game. We will however restrict our attention to *stationary* equilibria of this game, where each member adopts the same acceptance rule at all dates.[1]

Given any stationary acceptance rule σ_{-i} followed by members j, $j \neq i$, we may define the expected payoff $\bar{v}_i(\sigma_{-i})$ that member i derives given σ_{-i} from following his (best) strategy. An optimal acceptance rule for member i is thus to accept the proposal u if and only if

$$u_i \geq \delta \bar{v}_i(\sigma_{-i}),$$

which is stationary as well (this defines the best-response of member i to σ_{-i}).

[1]To avoid coordination problems that are common in voting (for example, all players always voting "no"), we will also restrict attention to equilibria that employ no weakly dominated strategies (in the stage game). These coordination problems could alternatively be avoided by assuming that votes are sequential.

Stationary equilibrium acceptance rules are thus characterized by a vector $v = (v_1, .., v_n)$ such that member i votes in favor of u if $u_i \geq \delta v_i$ and votes against it otherwise. For any decision rule $\rho(.)$ and value vector v, it will be convenient to refer to $A_{v,\rho}$ as the corresponding *acceptance set*, that is, the set of proposals that get support given that each i supports u if and only if $u_i \geq \delta v_i$:

$$A_{v,\rho} = \{u \in U, \text{ for } z_i = 1_{u_i \geq \delta v_i} \text{ and } z = (z_i)_i, \ \rho(z) = 1\}. \tag{1}$$

Equilibrium consistency then requires that

$$v_i = \Pr(u \in A_{v,\rho}) E[u_i \mid u \in A_{v,\rho}] + [1 - \Pr(u \in A_{v,\rho})] \, \delta v_i \tag{2}$$

or equivalently

$$v_i = \frac{\Pr(u \in A_{v,\rho})}{1 - \delta + \delta \Pr(u \in A_{v,\rho})} E[u_i \mid u \in A_{v,\rho}]. \tag{3}$$

A stationary equilibrium is characterized by a vector v and an acceptance set $A_{v,\rho}$ that satisfy (1)-(2). It always exists, as shown in Compte and Jehiel (2004-09).

Preferences within and across subgroups.

We now specialize our general framework. We assume that there are K subgroups in the population, labelled $k = 1, ..., K$. Proposals affect subgroups differently, and there is also some heterogeneity in each subgroup.

Formally, a proposal is characterized by a vector $x \in (x_1, ..., x_K) \in X$, where X is a compact convex subset of R^K, and proposals are drawn from X according to some density g. The utility that some individual i in subgroup k derives is

$$u_i^{(k)} = x_k + \varepsilon_i^k$$

where ε_i^k is assumed to be a random variable independent of x. We shall denote by F_k its cumulative density. We shall assume that F_k has a density

f_k that is single peaked. Without loss of generality we also assume that $E\varepsilon_i^k = 0$. We shall also denote by $\bar{\eta}$ an upperbound on the support of $\mid \varepsilon_i^k \mid$, make the minimal assumption that X contains the vector $(\bar{\eta}, ..., \bar{\eta})$.

For each subgroup, two features of interest will be *subgroup homogeneity* (which depends on how concentrated the density f_k is) and *skewdness*, which we summarize by the scalar

$$\gamma^k = 1 - F_k(0),$$

that is, the fraction of individuals in subgroup k that get an above average payoff – as compared to own subgroup). This parameter γ^k can be interpreted as an a priori degree of popularity of proposals, within group k, and it will thus affect the easiness with which group k will vote in favor of proposals close to the marginal ones.

Transfer mechanisms within subgroups will not be modelled. However, our view is that in the absence of transfer mechanisms within the subgroup, γ^k typically takes low values (below 1/2), and transfer mechanisms with subgroup k has the effect of increasing γ^k. Some of our result will be interpreted with this view in mind.

Finally, we denote by α_k the fraction of individuals that belong to subgroup k. Throughout out the paper we shall be interested in the set of equilibrium values that obtains in limit case where the group size n is arbitrarily large. We shall refer to such values as *limit equilibrium values.* Though our analysis can be done for impatient players, our focus will be on cases where players are patient (δ close to 1).

3 One-class voting: inefficiency results

Two types of inefficiencies may arise. Inefficiencies due to delays in reaching agreement. Inefficiencies resulting from agreement on a Pareto inferior outcome. In this section, we focus on one class voting rule and derive conditions under which inefficiencies arise, *whatever the majority requirement considered.*

Formally, we denote by β the fraction of individuals that is required for a proposal to be accepted. We refer to this rule as a $\beta-$majority rule. We provide a simple condition under which there exist no $\beta-$majority rules that generates efficient decision making.

> **Proposition 1:** *Assume that* $\max 1-\alpha_k > \max \gamma_k$*. Then, there exists* $\bar{\delta}$ *such that for all* $\delta > \bar{\delta}$*, and for any* $\beta-$*majority rule, limit equilibrium values remain bounded away from the Pareto frontier of* X*.*

In case groups are of comparable size, inefficiencies thus arise when $\max \gamma_k < 1 - 1/K$. With more than two groups, the condition is thus easily satisfied. With two groups of equal size, inefficiencies arise when proposals have low "a priori" popularity, that is, given our earlier interpretation, when there are little transfer mechanisms within subgroups.

Proof: Consider n large and denote by v_k the expected equilibrium value for an individual in group k. Consider now any draw $x = (x_k)_{k \in K}$. The individuals in group k that accept x are those for which

$$x_k + \varepsilon_i \geq \delta v_k.$$

For a large n, there is thus a fraction approximately equal to $1 - F_k(\delta v_k - x_k)$ that accept it. The set of proposals that pass, which we denote by A, is thus:

$$A = \{(x_k)_k, \sum_k \alpha_k (1 - F_k(\delta v_k - x_k)) > \beta\}$$

For almost efficient decision making, the set A should be $\nu-$close to the equilibrium vector v, for some ν close to 0. This thus requires that, for δ close enough to 1

$$\sum_k \alpha_k (1 - F_k(\nu)) > \beta.$$

For ν close to 0, the left hand side is close to $\sum_k \alpha_k \gamma^k$, hence since $\max \gamma^k < \max 1 - \alpha_k$, we obtain that almost efficient decision making requires that

$$\beta < \max 1 - \alpha_k.$$

But this implies that a proposal passes when all subgroups but one unanimously agree to it. So for any candidate $v = (v_k)_k$, the set A contains all $x = (x_k)_k$ such that $x_k > v_k + \bar{\eta}$ for all k but one. Such draws always exist (because X is a convex subset of R^K with a non-empty interior) and include draws that are far away from the efficient frontier. **Q.E.D.**

Intuitively, there may be two forms of inefficiencies. The first form typically arises when the majority requirement is small: then each individual has a low acceptance threshold because there is a high chance that he will not be part of the majority that accepts, and as a result, the agreement set is large and include Pareto inferior outcomes. The second form arises when the majority requirement is too large: then each individual sees little risk that the outcome will hurt him, and he prefers to patiently wait for a nice draw, and as a result, inefficient delays arise.

The logic of the argument in the proof of the Proposition is that for a majority rule $\beta < \max 1 - \alpha_k$, it is sufficient that all subgroups but one unanimously agrees to a proposal to pass it. As a consequence, whatever the candidate equilibrium values v, the agreement set must be large, and the first type of inefficiencies applies. Now for majority rules $\beta > \max \gamma^k$, draws x close to the candidate equilibrium values v cannot pass because they do not get enough support: only draws that are strictly Pareto superior to v may pass, which requires that v is bounded away from the frontier (inefficient delays must arise in that case).

Under the assumptions of the proposition, $\max \gamma_k < \max 1 - \alpha_k$, so inefficiencies must arise whatever the majority rule.

4 Multiple-class voting: an efficiency result

In this Section, we show that efficiency may be restored when one considers multiple-class voting. Specifically, we examine rules where support by subgroup k obtains when a fraction β_k of its members supports it, and where a proposal is adopted when all subgroups support it. So these rules require sufficient support within each subgroup (characterized by the vector $(\beta_1, ..., \beta_K)$), and unanimity across subgroups.[2]

We show below that there always exist a vector $(\beta_1, ..., \beta_K)$ that induces approximate efficiency.

> **Proposition 2**: *For any* $\xi > 0$, there exists $\eta > 0$ *such that the multiple-class voting rule characterized by* $(\beta_1, ..., \beta_K)$ *where* $\beta_k = 1 - F_k(\eta)$ *generates* $\xi-$*efficient decisions when individuals are patient enough.*

Proof: Consider n large and denote by v_k the expected equilibrium value for an individual in group k. Consider now any draw $x = (x_k)_k$. The individuals in group k that accept x are those for which

$$x_k + \varepsilon_i \geq \delta v_k.$$

For a large n, there is thus a fraction approximately equal to $1-F_k(\delta v_k - x_k)$ that accept it. Subgroup k supports proposal x with probability arbitrarily close to 1 (as n gets large) when $1 - F_k(\delta v_k - x_k) > \beta_k = 1 - F_k(\eta)$ or equivalently when $x_k > \delta v_k - \eta$, and arbitrarily close to 0 (as n gets large) when $x_k < \delta v_k - \eta$. The set of proposals that pass, which we denote by A_v, is thus:

$$A_v = \{x = (x_1, ..., x_K) \in X,\ x_k > \delta v_k - \eta,\ \text{for all } k\}$$

[2]Alternatively, under rule $(\beta_1, ..., \beta_K)$, a proposal may be vetoed by subgroup k if and only if there is a fraction $1 - \beta_k$ that of its members that opposes the proposal.

Note now for any candidate equilibrium vector $v \in X$, $\Pr A_v$ is bounded away from 0, so when players are patient enough, v_k should be close to $E[x_k \mid x \in A_v]$.

Now assume by contradiction that v were away from the frontier by more than $\xi = \eta^{1/2}$, then we would have $E[x_k \mid x \in A_v] > v_k + O(\xi - \eta)$, so for η small enough, v_k could not be close to $E[x_k \mid x \in A_v]$. Contradiction. Q.E.D.

5 The effect of size and heterogeneity

5.1 One-class voting

We examine below how the size of a subgroup as well as its homogeneity affect its strength, under one-class voting. Our main finding is that size matters and helps, and that homogeneity is an asset only if the majority requirement is not too strong.

We consider the case of two subgroups and focus on the case where the distributions ε_i are centered and where a single subgroup cannot on its own enforce a proposal. We will further assume that X is symmetric so that differences in expected payoffs only stem from asymmetries in size or differences in the distributions F_k.

Formally, we assume:

> **A1:** f_k is centered on 0 for each k, and $\beta > \max \alpha_k$. Besides X is symmetric, with a Pareto frontier parameterized by $g(x) \equiv 0$.

As explained earlier, the assumption $\beta > \max \alpha_k (\geq 1/2)$ implies that inefficient delays must occur in equilibrium, with the set of accepted proposals thus concentrated around some x^* on the Pareto Frontier of X, and with $v^* = \lambda^* x^*$ for some λ^*. Our first objective is to characterize x^*, v^* and λ^*. We will next make comparative statics as a function of size and heterogeneity.

5.1.1 Characterization.

For any λ, define

$$A(\lambda, x^*) = \{(x_1, x_2), \sum_k \alpha_k (1 - F((\lambda x_k^* - x_k)) \geq \beta\}$$

Assuming that the proposals accepted in equilibrium are concentrated around x^*, and assuming that $\lambda^* = \lambda$, the set $A(\lambda, x^*)$ characterizes the set of proposals that passes given the majority requirement. Consistency thus requires that

$$A(\lambda, x^*) \cap X = \{x^*\} \tag{4}$$

Accordingly, in what follows, we define λ^* as the highest value of λ such that $B(\lambda) \cap X \neq \emptyset$, and denote by $x^* = (x_1^*, x_2^*)$ that point of intersection.[3]

We have the following Proposition.

Proposition 3: Limit equilibrium values satisfy $v^* = \lambda^* x^*$ where x^* and λ^* are set so (4) holds.

Since at the solution x^*, the sets $A(\lambda^*, x^*)$ and X must have the same tangent, an immediate corollary of Proposition 3 is the following proposition:

Proposition 4: *We consider very patient players. Limit equilibrium values satisfy $v^* = \lambda^* x^*$ where the solution (x^*, λ^*) is characterized by the equations:*

$$g(x^*) = 0 \tag{5}$$

$$\sum_k \alpha_k (1 - F_k((\lambda^* - 1)x_k^*)) = \beta \tag{6}$$

$$\frac{g_1'(x^*)}{g_2'(x^*)} = \frac{\alpha_1 f_1((\lambda^* - 1)x_1^*)}{\alpha_2 f_2((\lambda^* - 1)x_2^*)} \tag{7}$$

[3] λ^* and x^* are uniquely defined because X is a convex set and because under A1, $B(\lambda)$ is also a convex set (also recall that the distributions f_k are single peaked).

5.1.2 Comparative Statics

We now wish to use Proposition 4 to illustrate the effect of size and homogeneity. Throughout the rest of this Section, we set:

$$g(x) = (x_1)^a + (x_2)^a - 1, \text{ with } a > 1$$

We also assume that densities take the form:

$$f_k(\varepsilon) = \frac{1}{b_k}(1 - \frac{|\varepsilon|}{b_k}), \text{ with } b_k > 0,$$

where b_k is thus a measure of the dispersion of the preferences within subgroup k.

Set $y_k = (1 - \lambda^*)x_i^*$. With this change of variable, and using Equations (5) and (7), we are looking for (y_1, y_2) such that

$$(\frac{y_1}{y_2})^{a-1} = \frac{\alpha_1}{\alpha_2}\frac{f_1(-y_1)}{f_2(-y_2)} \quad (8)$$

$$\alpha_1 F_1(-y_1) + \alpha_2 F_2(-y_2) = 1 - \beta \quad (9)$$

We can then derive λ^* from

$$1 - \lambda^* = ((y_1)^a + (y_2)^a)^{1/a}$$

Assume $b_1 > b_2$. If $\alpha_1 < \alpha_2 \frac{b_1}{b_2}$, then we can define $y^* > 0$ such that

$$\alpha_1 f_1(-y^*) = \alpha_2 f_2(-y^*),$$

We then let

$$\beta^* = \sum_k \alpha_k (1 - F_k(-y^*)).$$

Otherwise, note that we have $\alpha_1 f_1(-y) > \alpha_2 f_2(-y)$ for all $y \in (-b_2, b_2)$.

We have the following Proposition:

Proposition 5: *Assume $b_1 > b_2$. If $\alpha_1 > \frac{b_1}{b_2}\alpha_2$, then $x_1^* > x_2^*$.*
If $\alpha_1 < \frac{b_1}{b_2}\alpha_2$, then

(i) at majority rule β^*, $x_1^* = x_2^* = x_0^*$, *and* $\lambda^* = \lambda_0$, *where* $x_0^* \equiv (1/2)^{1/a}$ and $\lambda_0 \equiv 1 - y^*/x_0^*$.

(ii) for any more stringent majority rule $\beta > \beta^*$, *we have:*

$$x_1^* > x_0^* > x_2^* \text{ and } \lambda^* < \lambda_0,$$

(iii) for any less stringent majority rule $\beta < \beta^*$, *we have*

$$x_1^* < x_0^* < x_2^* \text{ and } \lambda^* > \lambda_0$$

Proof: Consider $\beta > \beta^*$, and assume by contradiction that $y_1 < y_2$. Then (8) implies $\alpha_1 f_1(-y_1) < \alpha_2 f_2(-y_2)$. Since $0 < y_1 < y_2$, we have $\alpha_1 f_1(-y_2) < \alpha_1 f_1(-y_1)$, hence $\alpha_1 f_1(-y_2) < \alpha_2 f_2(-y_2)$, which requires $y_2 < y^*$, hence

$$y_1 < y_2 < y^*.$$

(9) then implies:

$$1 - \beta = \alpha_1 F_1(-y_1) + \alpha_2 F_2(-y_2) > \sum_k \alpha_k F_k(-y^*) = 1 - \beta^*.$$

Contradiction. So $y_1 > y_2$, hence by a similar argument, $y_1 > y_2 > y^*$; Also note that we must thus have $x_1^* > x_2^*$, hence $x_1^* > (1/2)^{1/a} > x_2^*$, hence $y^* = (1 - \lambda_0^*)(1/2)^{1/a} < y_2 = (1 - \lambda^*)x_2^* < (1 - \lambda^*)(1/2)^{1/a}$, which implies that $\lambda^* < \lambda_0^*$. This proves claim (ii). Claim (iii) is proved similarly.

To prove claim (i), observe that $y_1 < y_2$ implies $\alpha_1 f_1(-y_2) < \alpha_2 f_2(-y_2)$ which cannot happen when $\alpha_1 > \frac{b_1}{b_2}\alpha_2$. Q.E.D.

The following figures plot the ratio v_1/v_2 and λ^* as a function of the majority rule β for specific values of the parameters α_k and b_k.

5.2 Multiple-class voting

We now examine the effect of size and heterogeneity in the case of multiple-class voting. We have seen that under an appropriate choice of the vector

$(\beta_k)_k$ efficiency could be restored. We now fix $\beta_k = 1/2$ for all k, and examine the effect of size and heterogeneity across subgroups, assuming that

$$\gamma^k < 1/2 \text{ for all } k.$$

This assumption ensures that support for a proposal is not easy to obtain, and that some delay in reaching agreement will obtain. We are interested in which subgroups obtain more favorable treatment.

Our main finding is that the outcome is independent of size and that it is more favorable to subgroups for which support is more difficult to obtain.

Formally we define μ^k as the threshold for which

$$\Pr(\varepsilon_i^k > -\mu^k) = \beta_k (= 1/2).$$

Note that since $\gamma^k < 1/2$, μ^k must be positive: $-\mu^k$ corresponds to the idiosyncratic element of the median group member, that is, it characterizes the extent to which the distribution over idiosyncratic elements ε_i^k is skewed to the left.

Next denote by $x^* = (x_k^*)_k$ the point of the Pareto frontier of X characterized by:[4]

$$\frac{x_k^*}{x_1^*} = \frac{\mu^k}{\mu^1}.$$

We have the following Proposition:

> **Proposition 6**. *There exists $\bar{\delta}$ such that for all $\delta > \bar{\delta}$, and for any $\beta-$majority rule, limit equilibrium values remain close to the point $v^* = \lambda^* x^*$* with $\lambda^* = 1 - \frac{\mu^k}{x_k^*}$.

This Proposition illustrates that high values of μ^k induce large delays in reaching agreement, but it also illustrates that compared to other subgroups,

[4]Note that our assumption that X contains $(\overline{\eta}, ..., \overline{\eta})$ implies that the ratios η^k / x_k^* are below 1.

the outcome is more favorable to those having a high parameter μ^k, that is, those groups for which the median member is most hurt.

Proof: Consider a candidate equilibrium value v. Consider any draw x. Following the proof of Proposition 2, subgroup k supports proposal x with probability arbitrarily close to 1 (as n gets large) when $1-F_k(\delta v_k - x_k) > \beta_k$ or equivalently when

$$x_k > \delta v_k + \mu_k$$

The agreement set thus coincides with:

$$A_v = \{x \in X, \quad x_k > \delta v_k + \mu_k \text{ for } k = 1, 2\}$$

Let $\lambda_v = \frac{\Pr A_v}{1-\delta+\delta \Pr A_v}$. In equilibrium, it should be that

$$v_k = \lambda_v E[x_k \mid x \in A_v]$$

Since $\mu_k > 0$ for all k, then, for patient individuals, we must have $\lambda_v < 1$ and $\Pr A_v$ must be comparable to $1 - \delta$, hence, for patient individuals, we must have $E[x_k \mid x \in A_v] \simeq v_k + \mu_k$, with the vector $(v_k + \mu_k)_k$ lying on the Pareto frontier of X. This further implies that

$$\frac{v_k}{v_1} = \frac{v_k + \mu_k}{v_1 + \mu_1}$$

hence

$$\frac{v_k}{v_1} = \frac{\mu_k}{\mu_1} = \frac{v_k + \mu_k}{v_1 + \mu_1},$$

which in turn implies that $(v_k + \mu_k)_k = (x_k^*)_k$. Q.E.D.

5.3 When a constitutional agreement is lacking

In the previous section the rule was fixed ($\beta_k = 1/2$) This subsection can be viewed as a dual version of the previous one: we fix γ^k, and give each subgroup the option to independently decide the majority requirement β_k to be applied to their own subgroup.

Our main finding is that inefficiencies arise again, as each subgroup is tempted to increase its majority requirement β_k. Such an increase tilts the outcome in a way that is favorable to members of subgroup k, but it generates efficiency losses in terms of delays. This section explains the trade-off between these two motives, and the equilibrium choice $(\beta_k^*)_k$.

Formally, we assume that in a first stage, subgroups simultaneously choose the majority requirement β_k that applies within their own group, and that in a second stage, our previous collective search game is played. As in the previous subsection, it will be convenient to let

$$\mu_k \equiv -F_k^{-1}(1-\beta_k).$$

Also, for simplicity, we make the following assumption:

> **A2**: X is the simplex: $X = \{x = (x_1, ..., x_K), \sum_{k \in K} x_k \leq 1, x_k \geq 0 \text{ for all } k\}$.

We have the following Proposition:

Proposition 7: *The two stage game has a unique pure strategy equilibrium. Each subgroup chooses β_k so that $\mu_k = \mu^* = \frac{K-1}{K^2}$, and the resulting equilibrium payoffs are $v_k = \frac{1}{K} - \gamma^* = \frac{1}{K^2}$.*

Intuitively, each subgroup has an incentive to increase its majority requirement because a higher majority requirement tilts the outcome in its favor. But a higher majority requirement also induces further inefficiencies (in the form of delays), hence there is a limit to that increase. Nevertheless, substantial inefficiencies result.

Proof: We first derive the continuation equilibrium outcome in the subgame where each subgroup k has chosen β_k respectively. This is done exactly as in Proposition 6. Calling x^* the point of the Pareto frontier for which $\frac{x_k^*}{x_1^*} = \frac{\mu_k}{\mu_1}$, that is: $x_k^* = \mu_k / \sum_j \mu_j$, the equilibrium continuation value v^* satisfies:

$$v^* = \lambda x^* \text{ with } \lambda = 1 - \frac{\mu^k}{x_k^*}.$$

implying that:

$$v_1 = (1 - \sum_k \mu_k) \frac{\mu_1}{\sum_k \mu_k} = \frac{\mu_1}{\sum_k \mu_k} - \mu_1$$

Choosing β_k is equivalent to choosing μ_k and the derivation of equilibrium choices of μ_k follows from standard computations. Q.E.D

Note that the equilibrium outcome does not depend on the distributions F_k. It depends however on the shape of X. Assuming that the frontier of X can be parameterized by $g_a(x) = 0$ with

$$g_a(x) = \sum_k (x_k)^a - K^{1-a}$$

for some $a > 1$, the following figure plots the sum of expected payoffs as a function of a. As a increases, transferability is reduced and the efficiency loss is reduced.