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Gambling for Redemption and Self-Fulfilling Debt Crises*

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ABSTRACT

We develop a model for analyzing the sovereign debt crises of 2010–2012 in the Eurozone. The government sets its expenditure-debt policy optimally. The need to sell large quantities of bonds every period leaves the government vulnerable to self-fulfilling crises in which investors, anticipating a crisis, are unwilling to buy the bonds, thereby provoking the crisis. In this situation, the optimal policy of the government is to reduce its debt to a level where crises are not possible. If, however, the economy is in a recession where there is a positive probability of recovery in fiscal revenues, the government may optimally choose to "gamble for redemption," running deficits and increasing its debt, thereby increasing its vulnerability to crises.

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1. Introduction

This paper develops a model of the sorts of sovereign debt crisis that we have witnessed during 2010–2012 in such European countries as Greece, Ireland, and Portugal and that are ongoing in such countries as Cyprus, Italy, and Spain. The government sets its expenditure-debt policy optimally, given a fixed probability of a recovery in fiscal revenues. In doing so, the government can optimally choose to "gamble for redemption," and the economy can be optimally driven into a situation of increasing vulnerability to speculative attacks in the form of self-fulfilling debt crises. We provide a theory of sovereign debt crises in which both borrowers and lenders behave optimally, but where countries borrow so much, and lenders are willing to lend them that much, as to make a default unavoidable. Our theory contrasts alternative explanations based on misperceptions or other forms of irrationality.

Our paper provides a benchmark for analyzing government behavior in the face of fiscal pressures and a tool for testing the implications of alternative policy responses. In our analysis, as in Cole and Kehoe (1996, 2000), we characterize in a simple Markov structure the time consistent policy of a strategic government that is faced with non-strategic bond holders.

Government Debt

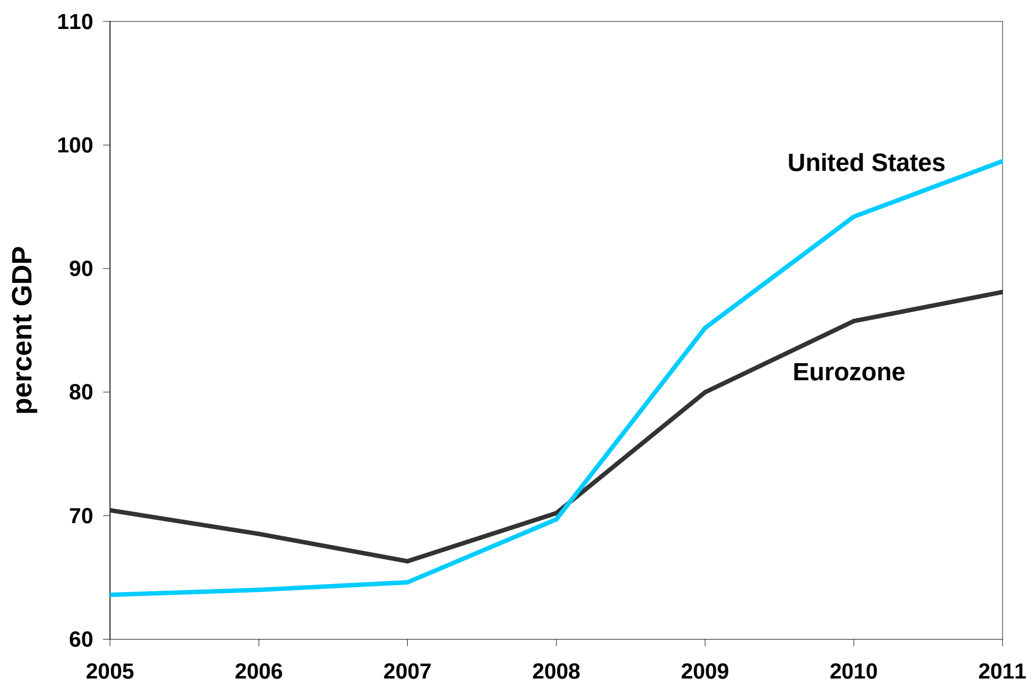

Figure 1

The recent worldwide recession — which continues in many countries — and the policies intended to overcome it have generated very large government budget deficits and increases in government debt over the entire developed world. As Figure 1 shows, in the Unites States government debt has gone up since 2005, reaching 98.7 percent of GDP in 2011 and is expected to reach 104.8 percent by 2012, spurring a very heated debate in July 2011 about changing the debt ceiling. In the Eurozone the level of government debt was going down systematically since the mid-1990s reaching a minimum in 2007 of 66.3 percent of GDP. Since then, it has risen to 88.1 percent in 2011 and is expected to reach 92.2 percent in 2012.

The pattern observed for the entire Euro area in Figure 1 is common across most European countries, but it has been more extreme in countries like Greece, whose debt is up to 165.4 percent of GDP in 2011. Figure 2 plots debt to GDP ratios for the most troubled European economies and Germany.

Government Debt in Selected European Countries

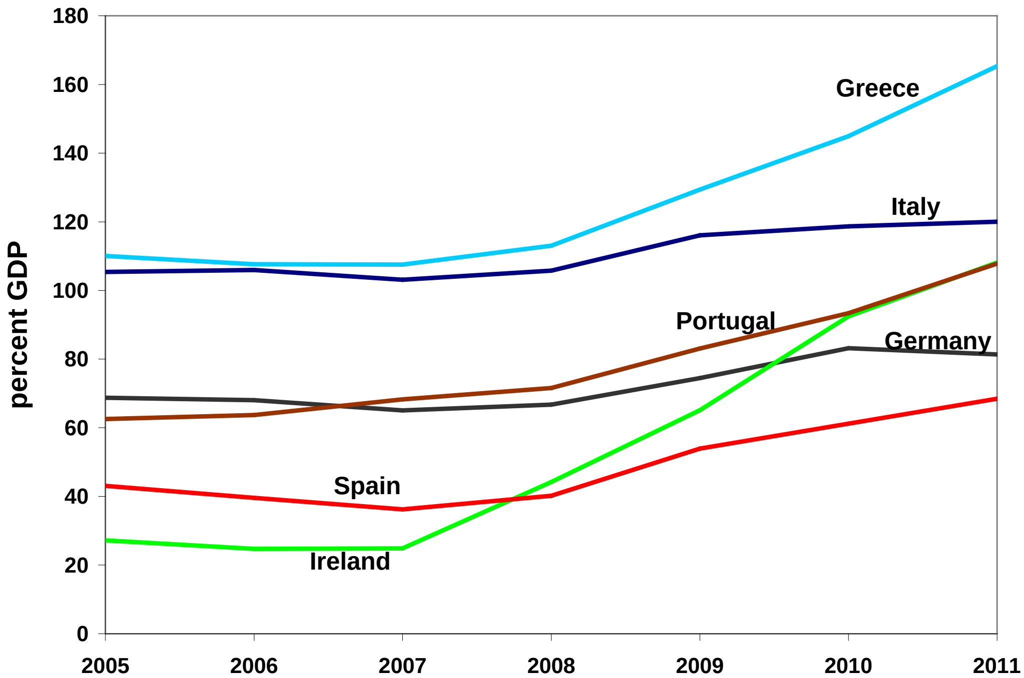

Figure 2

In our model, the crucial element putting countries at risk of suffering a self-fulfilling debt crisis is the drop in government revenues that occurs as the result of a recession in the private sector of the economy. Figure 3 shows that the worldwide recession that started in 2008 is still ongoing in Greece, Ireland, Italy, Portugal, and Spain.

Real GDP in Selected European Countries

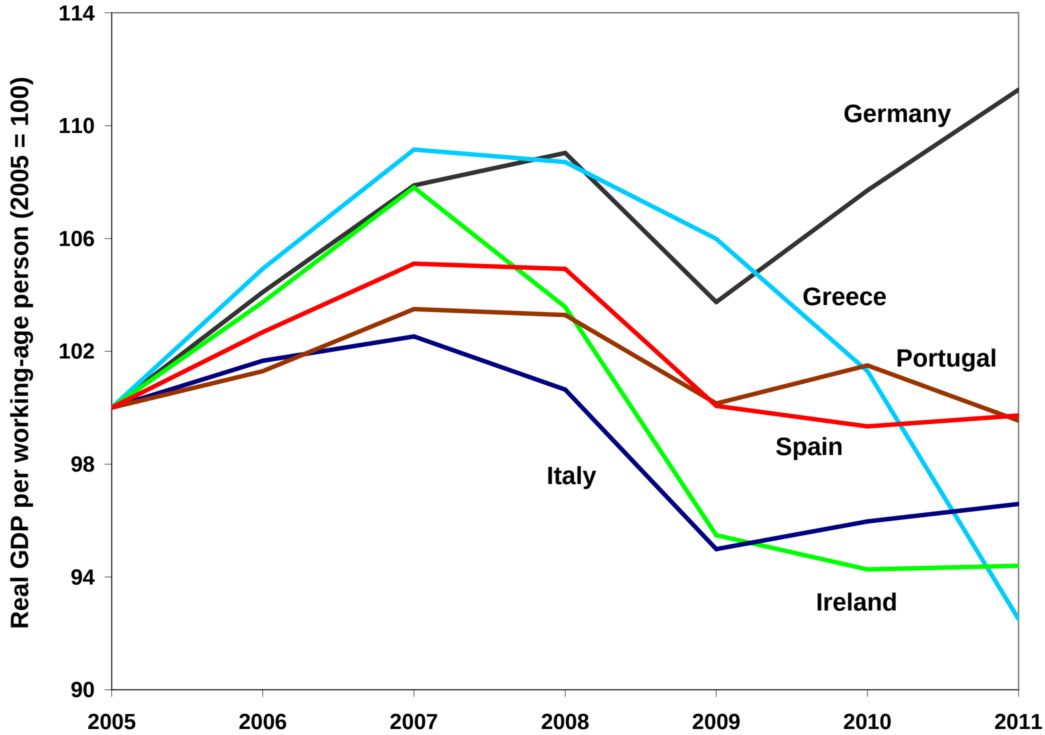

Figure 3

Notice that the drops in real government revenues (deflated by the GDP deflator) — presented in figure 4 — are also very large. As a consequence the yields in government bonds starting diverging. While the return on German bonds decreased, the risk premia of the troubled economies experienced very large and sudden increases (see Figure 5), reflecting an increased perception of the probability of a default.

Our analysis shows that — under certain conditions, which correspond to parameter values in our model — it is optimal for the government to "gamble for redemption." By this, we mean that the government does not undertake painful adjustments to reduce spending, hoping for a recovery of government revenues, and debt continues to increase. Indeed, the government strategy follows a martingale gambling strategy that will send the economy into the crisis zone if the recovery does not happen soon enough. Under other conditions, however, the government gradually reduces the level of debt to exit the crisis zone and avert the possibility of a liquidity crisis, as in Cole and Kehoe (1996, 2000).

Real Government Revenue

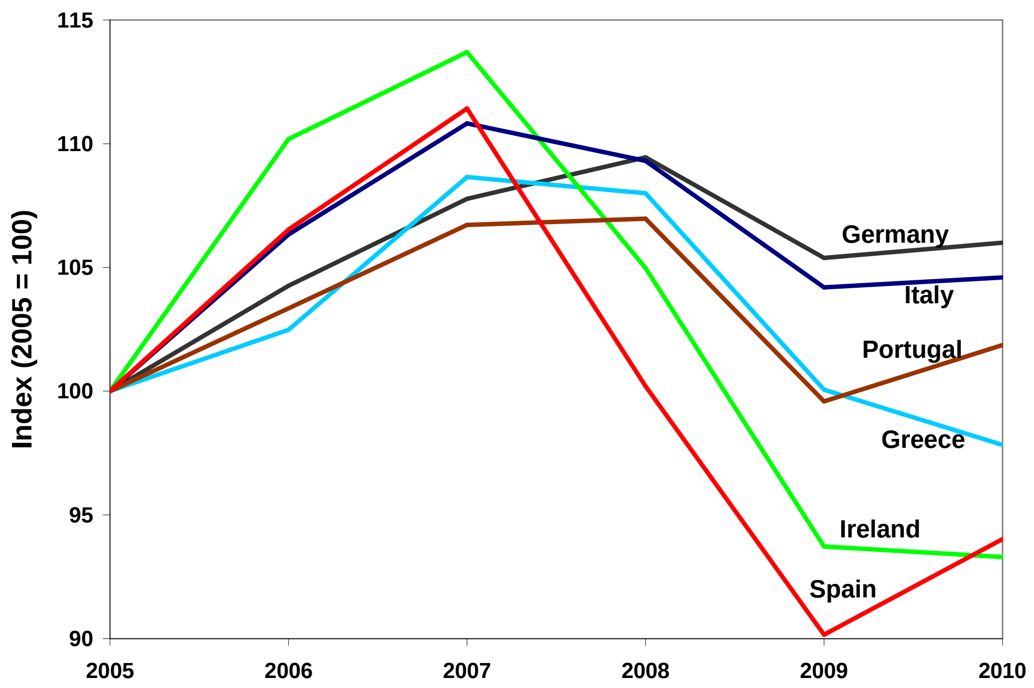

Figure 4

In our environment not running down debt (or running it up until default is unavoidable) can be part of the optimal strategy under some circumstances. In contrast, Reinhart and Rogoff (2009) argue that the reason some countries fail to adjust and are vulnerable to a potential crisis is because somehow both the governments and their lenders are fooling themselves into thinking that "this time is different" (this is indeed the title of their book). As such, countries vulnerability would be the result of self-delusion and lack of rationality. In contrast to their view, we provide a model in which such apparently irrational behavior can be an optimal response to either fundamentals or to expectations of a bailout by both borrowing governments and lenders that perfectly understand the risks of a crisis.

This paper is most related to those of Cole and Kehoe (1996, 2000). Cole and Kehoe provide a dynamic stochastic general equilibrium model of a country subject to the possibility of a self-fulfilling debt crisis in every period. The substantial difference between their framework and ours is that, in their framework, debt crises are liquidity crisis due solely to the inability to roll over debt that could arise under certain conditions. As such, a decisive action by a third party providing a loan or a bailout would be enough to avert the problem. Indeed, Cole and Kehoe (1996, 2000) intended their model to shed light on the financial troubles of Mexico in 1994–95, and, in that occasion, the decisive action of the Clinton administration on January 31, 1995 was enough to avert the crisis. European Union rescue packages for countries Greece, Ireland, and Portugal have not had the same healing properties. In fact, quite the opposite seems to be the case, with the spreads on bonds, relative to Germany's, continuing to rise despite the announcements and first implementations of the rescue packages. This indicates more fundamental solvency problems than those present in a standard liquidity crisis of the type studied in Cole and Kehoe, as discussed by Chamley and Pinto (2011) for the Greek case. Our model accommodates this issue.

Yields on 5 year government bonds

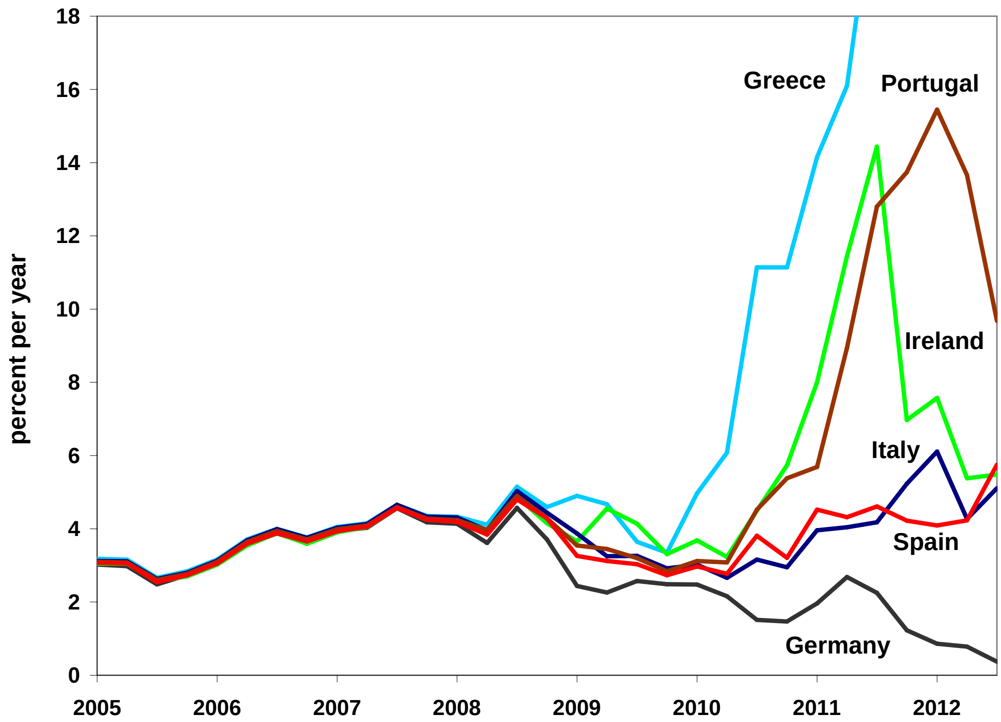

Figure 5

Aguiar and Gopinath (2006) and Arellano (2008) focus on default incentives on international borrowing over the cycle, but abstract from the possibility of self-fulfilling debt crises. Similar frameworks have been used to analyze currency crisis following Calvo (1988). Da

Rocha et al (2010) and Araujo et al (2011) extend the Cole-Kehoe framework to include different monetary and currency regimes.

Our model establishes conditions under which a debt crisis can occur, and how that possibility shapes optimal government's behavior, but is silent about why at a particular point in time a crisis might or might not occur. Indeed, once the government is in the crisis zone (and we show under what conditions a government will find it optimal to enter it) a potential crisis is triggered by a non-fundamental random variable: a sunspot. A very interesting related strand of the literature, starting with the work of Morris and Shin (1998), would relate the probability of such an event happening to lack of perfect knowledge within a global game framework. While this strand of the literature focuses on why a crisis might happen at a particular point in time, we focus on the conditions under which a benevolent government might find it optimal to subject itself to such a vulnerable position.

Bailouts in the form of issuing credit at below market price to troubled economies — as those used in the presence of liquidity crisis — might or might not work. In fact, we develop a calibrated example in which the probability of a bailout makes it more attractive for a government to enter the crisis zone, becoming more vulnerable to a crisis.

Our model provides sharp predictions of when a third party bailout helps avert a liquidity crisis, as in the case of the Mexican crisis of 1994–95, and when such a strategy would only make things worse. Since the response of the government is always optimal, however, the only policy prescription relate to the optimal form of the bailout for the intervening third party. A different scenario arises if the government is impatient, as in Cole and Kehoe (1996), or if the government assigns a different probability to a recovery than bankers, or if the government is trying to trick bankers or third parties capable of implementing a bailout with respect to the probability of a recovery. It might be the case that the government is gambling for redemption, while it should be making sacrifices and decreasing its debt position. A complete analysis of a game between two strategic players — the government of the country and the third party, in the case of the European countries that we have discussed, the European Union or the European Central Bank — is beyond the scope of this paper.

2. General model

The state of the economy in every period $s=(B,a,z_{-1},\zeta)$ is the level of government debt B, whether or not the private sector is in normal conditions $a=1$ or in a recession $a=0$, whether or not default has occurred in the past z_{-1}, and the value of the sunspot variable ζ.

The country's GDP is

$$y(a,z)=A^{1-a}Z^{1-z}\bar{y}$$

where $1>A,Z>0$. Before period 0, $a=1$, $z=1$. In period 0, a unexpectedly becomes $a_0=0$ and GDP drops from $y=\bar{y}$ to $y=A\bar{y}<\bar{y}$. In every period t, $t=1,2,...$, a_t becomes 1 with probability p, $1>p>0$. Once $a_t=1$, it stays equal to 1 forever. The drop in productivity by the factor Z is the country's default penalty. Once $z_t=0$, it stay equal to 0 forever. Here the default penalty occurs in the same period as the crisis. We could have it occur in the next period, in which case

$$y(a,z_{-1})=A^{1-a}Z^{1-z_{-1}}\bar{y}.$$

While the results in our numerical experiments would change, this alternative assumption has no qualitative effect on our results.

Government tax revenue is $\theta y(a,z)$ where we assume, to keep things simple, that the tax rate θ is fixed. The government's problem is to choose c,g,B',z to solve

$$V(s;p,\pi)=\max u(c,g)+\beta EV(s';p,\pi)$$

$$\text{s.t. } c=(1-\theta)y(a,z)$$

$$g+zB=\theta y(a,z)+q(B',s;p,\pi)B'$$

$$z=0 \text{ if } z_{-1}=0.$$

Here $z=1$ is the decision not to default, and $z=0$ is the decision to default. We later specify the precise timing of the interaction of the government and international bankers, which determines the price of bonds $q(B',s;p,\pi)$ and the default decision z.

In general, we assume that, for any b such that $A\bar{y}-b$ is an element of the feasible set of levels for government expenditures g,

$$u_g((1-\theta)A\bar{y}, \theta A\bar{y}-b) > u_g((1-\theta)\bar{y}, \theta\bar{y}-b).$$

This assumption provides the government with the incentive to transfer resources into the current period during a recession from future periods in which the economy has recovered. It is satisfied by any concave utility function separable in c and g. It is also satisfied by functions like $\log(c+g-\bar{c}-\bar{g})$.

In every period a random variable ζ_t is drawn from the uniform distribution on $[0,1]$. If $\zeta_t > 1-\pi$, international bankers expect there to be a crisis and do not lend to the government if such a crisis would be self-fulfilling. This allows us to set the probability of a self-fulfilling crisis at an arbitrary level π, $1 \geq \pi \geq 0$, if the level of debt is high enough for such a crisis to be possible. The timing within each period is like that in Cole and Kehoe (1996, 2000):

1. ζ_t is realized, the aggregate state is $s_t = (B_t, a_t, z_{t-1}, \zeta_t)$, and the government chooses how much debt B_{t+1} to sell.

2. Each of a continuum of measure one of international bankers chooses how much debt b_{t+1} to purchase. (In equilibrium, of course, $b_{t+1} = B_{t+1}$.)

3. The government makes it default decision z_t, which determines y_t, c_t, and g_t.

The crucial elements of this timing are that, first, the government chooses how much debt B_{t+1} it wants to sell and this determines the price of the debt if there is no crisis, and only later does the government choose whether to default or not. That is, the price of the debt depends on the amount of new debt being offered B_{t+1}, but whether or not a crisis takes place depends on the amount of debt that has to be repaid B_t.

2.1. Bond prices

International bankers are risk neutral with discount factor β so that the bond prices $q(B',s;p,\pi)$ are determined by the probability of default in the next period. There is a continuum of measure one of bankers. Each solves the dynamic programming problem

$$W(b,B',s;p,\pi)=\max x+\beta EW((b',B'',s';p,\pi)$$
$$x+q(B',s;p,\pi)b'=w+z(B',s,q(B',s;p,\pi);p,\pi)b$$
$$x\geq 0\,,\ b\geq -A\,.$$

The constraint $b\geq -A$ eliminates Ponzi schemes, but A is large enough so that the constraint does not otherwise bind. We assume that the banker's endowment of consumption good w is large enough to rule out corner solutions in equilibrium.

There are four cutoff levels of debt: $\bar{b}(a;p,\pi)$, $\bar{B}(a;p,\pi)$, $a=0,1$:

1. If $B\leq\bar{b}(0;p,\pi)$, the government does not default when the private sector is in a recession even if international bankers do not lend, and, if $B>\bar{b}(0;p,\pi)$, the government defaults when the private sector is in a recession if international bankers do not lend.

2. If $B\leq\bar{b}(1;p,\pi)$, the government does not default when the private sector is in normal conditions even if international bankers do not lend, and, if $B>\bar{b}(1;p,\pi)$, the government defaults when the private sector is in normal conditions if international bankers do not lend.

3. If $B\leq\bar{B}(0;p,\pi)$, the government does not default when the private sector is in a recession if international bankers lend, and, if $B>\bar{B}(0;p,\pi)$, the government defaults when the private sector is in a recession even if international bankers lend.

4. If $B\leq\bar{B}(1;p,\pi)$, the government does not default when the private sector is in a normal conditions if international bankers lend, and, if $B>\bar{B}(1;p,\pi)$, the government defaults when the private sector is in normal conditions even if international bankers lend.

The assumption that once $z_t = 0$, it stay equal to 0 forever says that a country that defaults is permanently excluded from international borrowing or lending. This assumption can be modified at the cost of complicating the analysis. The assumption has two consequences for the relation of the bond price q to the current state s: First, once default has occurred, international bankers do not lend:

$$q(B',(B,a,0,\zeta);p,\pi)=0.$$

Second, during a crisis, international bankers do not lend:

$$q(B',(B,a,1,\zeta);p,\pi)=0$$

if $B > \bar{b}(a;p,\pi)$ and $\zeta > 1-\pi$. Otherwise, the bond price q depends on the amount of bonds B' that the government offers for sale.

We will show that $\bar{b}(0;p,\pi) < \bar{b}(1;p,\pi)$, $\bar{b}(0;p,\pi) < \bar{B}(0;p,\pi)$, $\bar{b}(1;p,\pi) < \bar{B}(1;p,\pi)$, and $\bar{B}(0;p,\pi) < \bar{B}(1;p,\pi)$. Here, we first consider the case where $\bar{b}(1;p,\pi) < \bar{B}(0;p,\pi)$, that is, where

$$\bar{b}(0;p,\pi) < \bar{b}(1;p,\pi) < \bar{B}(0;p,\pi) < \bar{B}(1;p,\pi).$$

The first order condition for to the international bankers' utility maximization problem implies that

$$q(B',s;p,\pi)=\beta E z(B'(s'),s',q(B'(s'),s';p,\pi);p,\pi),$$

which implies that

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B' \le \bar{b}(0;p,\pi)\\ \beta\big(p+(1-p)(1-\pi)\big) & \text{if } \bar{b}(0;p,\pi) < B' \le \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \le \bar{B}(0;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{B}(0;p,\pi) < B' \le \bar{B}(1;p,\pi)\\ 0 & \text{if } \bar{B}(1;p,\pi) < B'\end{cases}$$

$$q(B',(B,1,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B' \le \bar{b}(1;p,\pi)\\ \beta(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \le \bar{B}(1;p,\pi)\\ 0 & \text{if } \bar{B}(1;p,\pi) < B'\end{cases}.$$

Figure 6 shows the price schedule.

Bond prices as a function of bonds offered for sale and conditions in the private sector

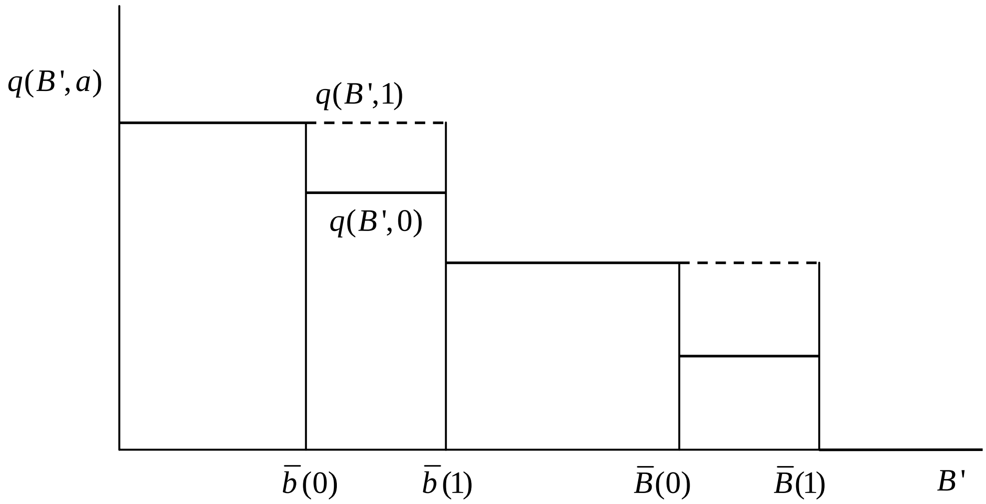

Figure 6

Now consider the case where $\bar{B}(0;p,\pi) < \bar{b}(1;p,\pi)$, that is, where

$$\bar{b}(0;p,\pi) < \bar{B}(0;p,\pi) < \bar{b}(1;p,\pi) < \bar{B}(1;p,\pi).$$

Here the solution to the international bankers' utility maximization problem implies that

$$q(B',(B,0,1,\zeta);p,\pi) = \begin{cases} \beta & \text{if } B' \le \bar{b}(0;p,\pi) \\ \beta\big(p+(1-p)(1-\pi)\big) & \text{if } \bar{b}(0;p,\pi) < B' \le \bar{B}(0;p,\pi) \\ \beta p & \text{if } \bar{B}(0;p,\pi) < B' \le \bar{b}(1;p,\pi) \\ \beta p(1-\pi) & \text{if } \bar{b}(1;p,\pi) < B' \le \bar{B}(1;p,\pi) \\ 0 & \text{if } \bar{B}(1;p,\pi) < B' \end{cases}$$

There is yet a third case, which we will show is not degenerate, where

$$\bar{b}(0;p,\pi) < \bar{b}(1;p,\pi) = \bar{B}(0;p,\pi) < \bar{B}(1;p,\pi).$$

Here

$$q(B',(B,0,1,\zeta);p,\pi)=\begin{cases}\beta & \text{if } B'\le \bar{b}(0;p,\pi)\\ \beta\left(p+(1-p)(1-\pi)\right) & \text{if } \bar{b}(0;p,\pi)<B'\le \bar{b}(1;p,\pi)\\ \beta p(1-\pi) & \text{if } \bar{b}(1;p,\pi)<B'\le \bar{B}(1;p,\pi)\\ 0 & \text{if } \bar{B}(1;p,\pi)<B'\end{cases}.$$

Since the second and the third cases, where $\bar{B}(0;p,\pi)\le \bar{b}(1;p,\pi)$, are only possible for very low values of A, we will focus on the first case.

2.2. Definition of equilibrium

An equilibrium is a value function for government $V(s)$ and policy functions $B'(s)$ and $z(B',s,q;p,\pi)$ and $g(B',s,q;p,\pi)$, a value function for bankers $W(b,B',s;p,\pi)$ and policy correspondence $b'(b,B',s;p,\pi)$, and a bond price function $q(B',s;p,\pi)$ such that

1. $V(s)$ and $B'(s)$ solve the government's problem at the beginning of the period:

$$\begin{gathered}V(B,a,z_{-1},\zeta;p,\pi)=\max u(c,g)+\beta EV(B',a',z,\zeta';p,\pi)\\ \text{s.t. } c=(1-\theta)y(a,z(B',s,q(B',s;p,\pi);p,\pi))\\ \begin{aligned}&g(B',s,q(B',s;p,\pi);p,\pi)+z(B',s,q(B',s;p,\pi);p,\pi)B\\ &\quad=\theta y(a,z)+q(B',s;p,\pi)B'\end{aligned}\end{gathered}.$$

2. $b'(b,B',s;p,\pi)$ solves the banker's problem and $q(B',s;p,\pi)$ is consistent with market clearing and rational expectations:

$$B'(s)\in b'(b,B',s;p,\pi)$$

$$q(B',s;p,\pi)=\beta Ez(B'(s'),s',q(B'(s'),s';p,\pi);p,\pi).$$

3. $z(B',s,q;p,\pi)$ and $g(B',s,q;p,\pi)$ solve the government's problem at the end of the period:

$$\begin{gathered}\max u(c,g)+\beta EV(B',a',z,\zeta';p,\pi)\\ \text{s.t. } c=(1-\theta)y(a,z)\\ g+zB=\theta y(a,z)+qB'\\ z=0 \text{ or } z=1\text{, but } z=0 \text{ if } z_{-1}=0.\end{gathered}$$

Notice that, when the government solves its problem at the beginning of the period, it takes as given the optimal responses both of international bankers and itself later in the period. In particular, the government cannot commit to repaying its debt and not defaulting later in the period. Furthermore, since the occurrence of a crisis depends on the amount of debt to be repaid B, not the amount of debt offered for sale B', once a sunspot has occurred that signals that a self-fulfilling crisis will take place that period, there is nothing that the government can do to avoid it.

It is also worth noting that this model has many equilibria. Our definition of equilibrium restricts our attention to equilibria with a simple Markov structure. Many other possibilities are possible. If we include the date in the state $s = (B, a, z_{-1}, \zeta, t)$, for example, we could allow crises to occur only in even periods t or in periods that are prime numbers, or we could allow the probability of a crisis π to be time varying in other ways. The advantage of our simple Markov structure is that it makes it easy to characterize and compute equilibria.

3. Self-fulfilling liquidity crises

Although we are able to prove some propositions about the equilibria of the general model, we need to resort to numerical examples to illustrate the possibilities and to do comparative statics analysis. Before turning to the results for the general model, we study two special cases, where we can provide analytical characterizations of the equilibria in which we are interested. The first is the case where $a = 1$, that is, where the private sector has recovered and where there is no incentive for the government to gamble for redemption. This is the model of Cole-Kehoe (1996, 2000) without private capital. Notice that we can easily modify the analysis of this case to study the limiting case where $a = 0$ and $p = 0$, that is, where there is a recession but no possibility for recovery, simply by replacing $\bar{y}$ with $A\bar{y}$ in what follows. In this case, where self-fulfilling crises are possible, but where there is no incentive for the government to gamble for redemption, the optimal strategies of the government involve either leaving debt constant or running it down to eliminate the possibility of a crisis. In the next section, we consider the other extreme case, where recovery is possible but not self-fulfilling crises.

We start by assuming that $\pi = 0$. Notice that, since a recovery has already occurred in the private sector, p is irrelevant. To derive the optimal government policy, we solve

$$\max \sum_{t'=t}^{\infty} \beta^s u(c_{t'}, g_{t'})$$

$$\text{s.t. } c_{t'} = (1-\theta)\bar{y}$$

$$g_{t'} + B_{t'} = \theta\bar{y} + \beta B_{t'+1}$$

$$B_{t'} = B$$

$$B_{t'} \leq \bar{B}(1; p, 0).$$

The first order conditions are

$$\beta^{t'} u_g((1-\theta)\bar{y}, g_{t'}) = \lambda_{t'}$$

$$\lambda_{t'+1} = \beta\lambda_{t'}.$$

The transversality condition is

$$\lim_{t'\to\infty} \lambda_s B_{t'+1} \geq 0.$$

The first order conditions imply that

$$u_g((1-\theta)\bar{y}, g_{t'}) = u_g((1-\theta)\bar{y}, g_{t'-1})$$

$$g_{t'+1} = g_{t'} = \hat{g}.$$

Consequently,

$$B_{t'+1} = \frac{1}{\beta}\left(\hat{g} + B_{t'} - \theta\bar{y}\right).$$

Suppose that

$$\hat{g} = \theta\bar{y} - (1-\beta)B.$$

Then $B_s = B$. Otherwise $B_{t'}$ is explosive. It is easy to show that too low of a $\hat{g}$ results in a path for $B_{t'}$ that violates the transversality condition. Too high of a $\hat{g}$ results in a path for $B_{t'}$ that hits $\bar{B}(1; p, 0)$. It is easy to prove that this cannot be optimal.

We can calculate the value of being in state $s = (B, a, z_{-1}, \zeta) = (B,1,1,\zeta)$ as

$$V(B,1,1,\zeta; p, 0) = \frac{u((1-\theta)\bar{y}, \theta\bar{y} - (1-\beta)B)}{1-\beta}.$$

The calculation of utility when default has occurred, when $z=0$, is mechanical. In this case $B=0$ and

$$c=(1-\theta)Z\overline{y}$$

$$g=\theta Z\overline{y}\,.$$

Notice that, once a default has occurred, ζ and π are irrelevant. Consequently, when $s=(B,a,z_{-1},\zeta)=(B,1,0,\zeta)$

$$V(B,1,0,\zeta;p,0)=\frac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta},$$

Let us calculate $\overline{b}(1;p,0)$. Let $V_n(B,a,q;p,\pi)$ be the value of not defaulting. The utility of repaying B even if the international bankers do not lend is

$$V_n(B,1,0;p,0)=u((1-\theta)\overline{y},\theta\overline{y}-B)+\frac{\beta u((1-\theta)\overline{y},\theta\overline{y})}{1-\beta},$$

while the utility of defaulting $V_d(B,a,q;p,\pi)$ is

$$V_d(B,1,0;p,0)=\frac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}.$$

Consequently, $\overline{b}(1;p,0)$ is determined by the equation

$$V_n(\overline{b}(1;p,0),1,0;p,0)=V_d(\overline{b}(1;p,0),1,0;p,0)$$

$$u((1-\theta)\overline{y},\theta\overline{y}-\overline{b}(1;p,0))+\frac{\beta u((1-\theta)\overline{y},\theta\overline{y})}{1-\beta}=\frac{u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}$$

$$u((1-\theta)Z\overline{y},\theta Z\overline{y})-u((1-\theta)\overline{y},\theta\overline{y}-\overline{b}(1;p,0))=\frac{\beta}{1-\beta}\big(u((1-\theta)\overline{y},\theta\overline{y})-u((1-\theta)Z\overline{y},\theta Z\overline{y})\big).$$

Let us now characterize $\overline{B}(1;p,0)$. We first find the stationary upper limit on debt $\overline{B}^s(1;p,0)$ and then show that $\overline{B}(1;p,0)=\overline{B}^s(1;p,0)$. Suppose that international lenders set $q=\beta$ when $a=1$. Then, if the government decides to repay its debt

$$V_n(B,1,\beta;p,0)=\frac{u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B)}{1-\beta},$$

while the value of defaulting is

$$V_d(B,1,\beta;p,0) = u((1-\theta)Z\overline{y},\theta Z\overline{y}+\beta B)+\frac{\beta u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}.$$

Consequently, $\overline{B}^s(1;p,0)$ is determined by the equation

$$V_n(\overline{B}^s(1;p,0),1,\beta;p,0) = V_d(\overline{B}^s(1;p,0),1,\beta;p,0)$$

$$\frac{u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{B}^s(1;p,0))}{1-\beta} = u((1-\theta)Z\overline{y},\theta Z\overline{y}+\beta\overline{B}^s(1;p,0))+\frac{\beta u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta}$$

$$\begin{aligned} & u((1-\theta)Z\overline{y},\theta Z\overline{y}+\beta\overline{B}^s(1;p,0))-u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{B}^s(1;p,0)) \\ & =\frac{\beta}{1-\beta}\left(u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{B}^s(1;p,0))-u((1-\theta)Z\overline{y},\theta Z\overline{y})\right) \end{aligned}.$$

Suppose that, at $t=0$, the government finds itself with $B_0 > \overline{B}^s(1;p,0)$. Then it cannot be optimal to run down its debt to $B_1 \leq \overline{B}^s(1;p,0)$ in one period, setting

$$g_0 = \theta\overline{y} - B_0 + \beta B_1$$

in $t=0$ and

$$g_t = \theta\overline{y} - (1-\beta)B_1$$

thereafter. To demonstrate this, suppose, to the contrary, that

$$\begin{aligned} & u((1-\theta)\overline{y},\theta\overline{y}-B_0+\beta B_1)+\frac{\beta u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B_1)}{1-\beta} \\ & \geq u((1-\theta)Z\overline{y},\theta Z\overline{y}+\beta B_0)+\frac{\beta u((1-\theta)Z\overline{y},\theta Z\overline{y})}{1-\beta} \end{aligned}.$$

If we choose B_1 to maximize the left hand side of this inequality, the utility of not defaulting, we set $B_1 = B_0$. This implies that

$$\begin{aligned} & u((1-\theta)\overline{y},\theta\overline{y}-B_0+\beta B_0)+\frac{\beta u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B_0)}{1-\beta} \\ & \geq u((1-\theta)\overline{y},\theta\overline{y}-B_0+\beta B_1)+\frac{\beta u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B_1)}{1-\beta} \end{aligned}$$

for any $B_1 \neq B_0$. This, however, contradicts or assumption that $B_0 > \overline{B}^s(1; p, 0)$.

Let us now consider the case where $\pi > 0$. First, observe that

$$\overline{b}(1; p, \pi) = \overline{b}(1; p, 0)$$

Suppose that $B_0 > \overline{b}(1; p, \pi)$ and the government decides to reduce B to $\overline{b}(1; p, \pi)$ in T periods, $T = 1, 2, ..., \infty$. The first order conditions for the government's problem imply that

$$g_t = g^T(B_0; \pi).$$

The government's budget constraints are

$$\begin{aligned}
g^T(B_0; \pi) + B_0 &= \theta \overline{y} + \beta(1-\pi) B_1 \\
g^T(B_0; \pi) + B_1 &= \theta \overline{y} + \beta(1-\pi) B_2 \\
&\vdots \\
g^T(B_0; \pi) + B_{T-2} &= \theta \overline{y} + \beta(1-\pi) B_{T-1} \\
g^T(B_0; \pi) + B_{T-1} &= \theta \overline{y} + \beta \overline{b}(1; p, \pi).
\end{aligned}$$

Multiply each equation by $(\beta(1-\pi))^t$ and adding, we obtain

$$\sum_{t=0}^{T-1} (\beta(1-\pi))^t g^T(B_0; \pi) + B_0 = \sum_{t=0}^{T-1} (\beta(1-\pi))^t \theta \overline{y} + (\beta(1-\pi))^{T-1} \beta \overline{b}(1; p, \pi)$$

$$g^T(B_0; \pi) = \theta \overline{y} - \frac{1-\beta(1-\pi)}{1-(\beta(1-\pi))^T} \left(B_0 - (\beta(1-\pi))^{T-1} \beta \overline{b}(1; p, \pi) \right).$$

Notice that

$$g^\infty(B_0; \pi) = \lim_{T \to \infty} g^T(B_0; \pi) = \theta \overline{y} - (1-\beta(1-\pi)) B_0.$$

We can compute the value $V^T(B_0; \pi)$ of each of the policies of running down the debt in T periods, $T = 1, 2, ..., \infty$. Letting $V_t^T(B_0; \pi)$ be the value of the policy where there are still t periods to go in running debt, we can write

$$V_T^T(B_0; \pi) = u((1-\theta)\overline{y}, g^T(B_0; \pi)) + \beta(1-\pi) V_{T-1}^T(B_0; \pi) + \frac{\beta \pi u((1-\theta) Z\overline{y}, \theta Z\overline{y})}{1-\beta}$$

$$V_{T-1}^T(B_0; \pi) = u((1-\theta)\overline{y}, g^T(B_0; \pi)) + \beta(1-\pi) V_{T-2}^T(B_0; \pi) + \frac{\beta \pi u((1-\theta) Z\overline{y}, \theta Z\overline{y})}{1-\beta}$$

$$\vdots$$

$$V_2^T(B_0;\pi) = u((1-\theta)\bar{y}, g^T(B_0;\pi)) + \beta(1-\pi)V_1^T(B_0;\pi) + \frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta}$$

$$V_1^T(B_0;\pi) = u((1-\theta)\bar{y}, g^T(B_0;\pi)) + \frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta}.$$

Notice that g increases from $g^T(B_0;\pi)$ to $\theta\bar{y}$ in period T. To calculate $V^T(B_0;\pi)$, we use backwards induction

$$\begin{aligned} V_2^T(B_0;\pi) &= (1+\beta(1-\pi))u((1-\theta)\bar{y}, g^T(B_0;\pi)) \\ &+ \frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} \\ &+ \beta(1-\pi)\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta} \end{aligned}$$

$$\begin{aligned} V_3^T(B_0;\pi) &= (1+\beta(1-\pi)+(\beta(1-\pi))^2)u((1-\theta)\bar{y}, g^T(B_0;\pi)) \\ &+ (1+\beta(1-\pi))\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} \\ &+ (\beta(1-\pi))^2\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta} \end{aligned}$$

$$\vdots$$

$$\begin{aligned} V_T^T(B_0;\pi) &= (1+\beta(1-\pi)+(\beta(1-\pi))^2+\ldots+(\beta(1-\pi))^{T-1})u((1-\theta)\bar{y}, g^T(B_0;\pi)) \\ &+ (1+\beta(1-\pi)+(\beta(1-\pi))^2+\ldots+(\beta(1-\pi))^{T-2})\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} \\ &+ (\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta} \end{aligned}$$

and, of course, $V^T(B_0;\pi) = V_T^T(B_0;\pi)$:

$$\begin{aligned} V^T(B_0;\pi) &= \frac{1-(\beta(1-\pi))^T}{1+\beta(1-\pi)}u((1-\theta)\bar{y}, g^T(B_0;\pi)) + \frac{1-(\beta(1-\pi))^{T-1}}{1+\beta(1-\pi)}\frac{\beta\pi u((1-\theta)Z\bar{y}, \theta Z\bar{y})}{1-\beta} \\ &+ (\beta(1-\pi))^{T-2}\frac{\beta u((1-\theta)\bar{y}, \theta\bar{y})}{1-\beta} \end{aligned}$$

Notice that

$$V^{\infty}(B_0;\pi)=\frac{u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta(1-\pi))B_0)}{1+\beta(1-\pi)}+\frac{\beta\pi u((1-\theta)Z\bar{y},\theta Z\bar{y})}{(1-\beta)(1+\beta(1-\pi))}.$$

To find $\bar{B}(1;p,\pi)$, we solve

$$\begin{aligned}&\max\left[V^1(\bar{B}(1;p,\pi)),V^2(\bar{B}(1;p,\pi)),...,V^{\infty}(\bar{B}(1;p,\pi);\pi)\right]\\ &=u((1-\theta)Z\bar{y},\theta Z\bar{y}+\beta(1-\pi)\bar{B}(1;p,\pi)))+\frac{\beta u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta}\cdot\end{aligned}$$

Our arguments have produced the following analytical characterization of $V(B,1,1,\zeta;p,\pi)$:

$$V(B,1,1,\zeta;p,\pi)=\begin{cases}\dfrac{u((1-\theta)\bar{y},\theta\bar{y})}{1-\beta} & \text{if } B\le\bar{b}(1;p,\pi)\\ \max\left[V^1(B;\pi),V^2(B;\pi),...,V^{\infty}(B;\pi)\right] & \text{if } \bar{b}(1;p,\pi)<B\le\bar{B}(1;p,\pi),\ \zeta\le 1-\pi\\ \dfrac{u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta} & \text{if } \bar{b}(1;p,\pi)<B\le\bar{B}(1;p,\pi),\ 1-\pi<\zeta\\ \dfrac{u((1-\theta)Z\bar{y},\theta Z\bar{y})}{1-\beta} & \text{if } \bar{B}(1;p,\pi)<B\end{cases}$$

Some of the different possibilities for optimal government strategies — which vary with the initial debt — are illustrated in figure 7.

Optimal debt policy with self-fulfilling crises

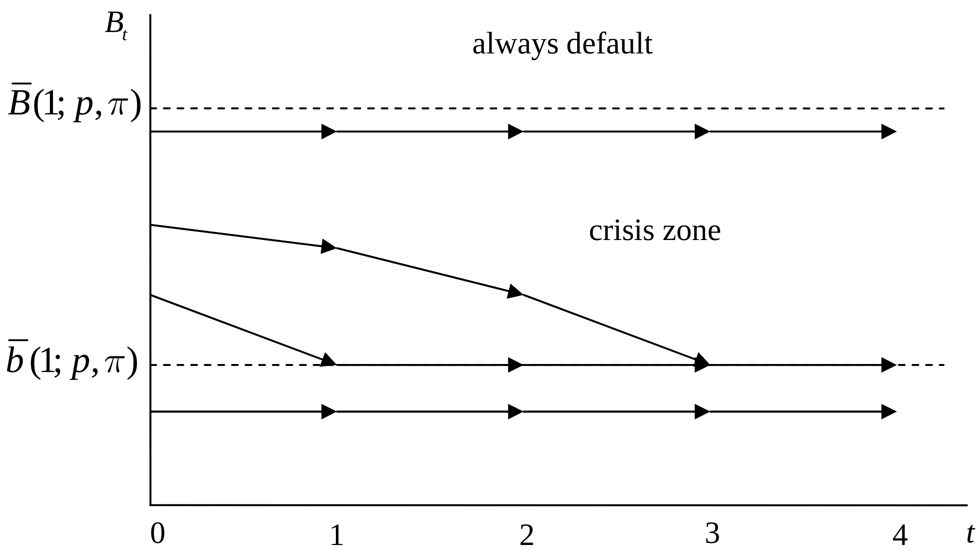

Figure 7

4. Gambling for redemption without self-fulfilling crises

Suppose now that $a = 0$ and $\pi = 0$. That is, no self-fulfilling crises are possible, but the private sector is in a recession and faces the probability p, $1 > p > 0$, of recovering in every period. We can also interpret this as the limiting case in which crises can occur, but the government and the international bankers assign probability $\pi = 0$ to them.

Uncertainty tree with recession path highlighted

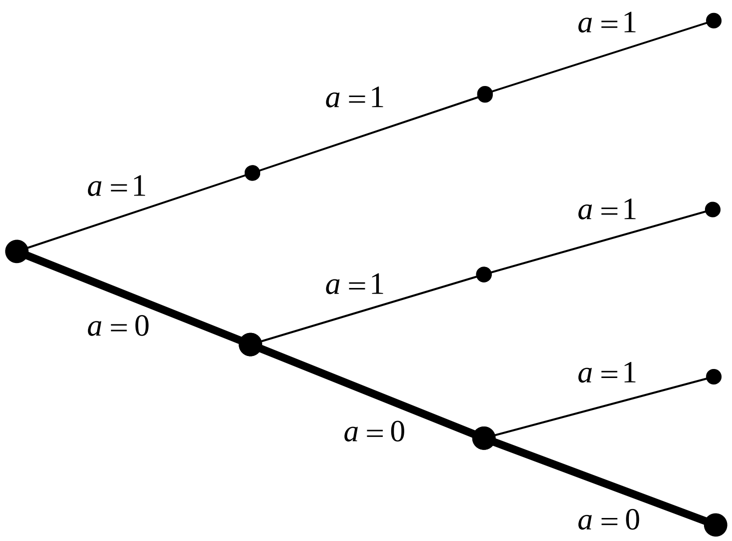

Figure 8

In this section, we argue that the optimal government policies is to increase its debt as long as $a=0$. In fact, if the country is unlucky in the sense that $a=0$ long enough, the government may choose to eventually default. Consequently, the upper limits on the debt, $\bar{B}(0;p,0)$ and $\bar{B}(1;p,0)$ are crucial for our analysis. Notice that, since

$$\bar{B}(1;p,0)=\bar{B}(1;0,0),$$

because the probability of recovery does not matter once it has occurred, we have already derived a condition that determines $\bar{B}(1;p,0)$.

$$\begin{aligned}&u((1-\theta)Z\bar{y},\theta Z\bar{y}+\beta\bar{B}(1;p,0))-u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(1;p,0))\\&=\frac{\beta}{1-\beta}\left(u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(1;p,0))-u((1-\theta)Z\bar{y},\theta Z\bar{y})\right)\end{aligned}.$$

To determine $\bar{B}(0;p,0)$, we suppose that the government runs up its debt to some level $B\leq\bar{B}(0;p,0)$, borrows $\bar{B}(0;p,0)<B'\leq\bar{B}(1;p,0)$ at price βp, then repays the next period if

the private sector recovers and on defaults otherwise. The value of borrowing $\overline{B}(1;p,0)$ at price βp, repaying the current debt, and then repaying in the next period if the private sector recovers and on defaulting otherwise is

$$
\begin{aligned}
V_n(B,0,1;p,0) &= u((1-\theta)A\overline{y},\theta A\overline{y}+\beta p\overline{B}(1;p,0)-B) \\
&+\beta(1-p)\left(\frac{u((1-\theta)AZ\overline{y},\theta AZ\overline{y})}{1-\beta(1-p)}+\frac{\beta p u((1-\theta)Z\overline{y},\theta Z\overline{y})}{(1-\beta)(1-\beta(1-p))}\right) \\
&+\frac{\beta p}{1-\beta}u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{B}(1;p,0))
\end{aligned}
$$

The value of borrowing $\overline{B}(1;p,0)$ at price βp and then defaulting is

$$
\begin{aligned}
V_d(B,0,1;p,0) &= u((1-\theta)AZ\overline{y},\theta AZ\overline{y}+\beta p\overline{B}(1;p,0)) \\
&+\beta(1-p)\left(\frac{u((1-\theta)AZ\overline{y},\theta AZ\overline{y})}{1-\beta(1-p)}+\frac{\beta p u((1-\theta)Z\overline{y},\theta Z\overline{y})}{(1-\beta)(1-\beta(1-p))}\right). \\
&+\frac{\beta p}{1-\beta}u((1-\theta)Z\overline{y},\theta Z\overline{y})
\end{aligned}
$$

The equation that determines $\overline{B}(0;p,0)$ is, therefore,

$$V_n(\overline{B}(0;p,0),0,1;p,0)=V_d(\overline{B}(0;p,0),0,1;p,0)$$

$$
\begin{aligned}
&u((1-\theta)A\overline{y},\theta A\overline{y}+\beta p\overline{B}(1;p,0)-\overline{B}(0;p,0))+\frac{\beta p}{1-\beta}u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)\overline{B}(1;p,0)) \\
&=u((1-\theta)AZ\overline{y},\theta AZ\overline{y}+\beta p\overline{B}(1;p,0))+\frac{\beta p}{1-\beta}u((1-\theta)Z\overline{y},\theta Z\overline{y})
\end{aligned}.
$$

The government may, in fact, choose a lower level of the debt than $\overline{B}(1;p,0)$, but $\overline{B}(0;p,0)$ is the highest level of the debt B' at which the government can borrow at price $q(B',s;p,\pi)=\beta$. Indeed, if the constraint $B'\le\overline{B}(1;p,0)$ does not bind, we can calculate the optimal B' by solving

$$
\begin{aligned}
\max\, & u((1-\theta)A\overline{y},\theta A\overline{y}+\beta pB'-B) \\
&+\beta(1-p)\left(\frac{u((1-\theta)AZ\overline{y},\theta AZ\overline{y})}{1-\beta(1-p)}+\frac{\beta p u((1-\theta)Z\overline{y},\theta Z\overline{y})}{(1-\beta)(1-\beta(1-p))}\right). \\
&+\frac{\beta p}{1-\beta}u((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B')
\end{aligned}
$$

The first order condition is

$$u_g((1-\theta)A\overline{y},\theta A\overline{y}+\beta pB'-B)=u_g((1-\theta)\overline{y},\theta\overline{y}-(1-\beta)B').$$

Letting $\hat{B}'(B)$ be the solution to this problem,

$$B'(B)=\min[\hat{B}'(B),\overline{B}(1;p,0)].$$

There are two cases:

1. The government chooses to never violate the constraint $B\leq\overline{B}(0;p,0)$, and the optimal debt converges to $\overline{B}(0;p,0)$ if $a=0$ for sufficiently long.

2. The government chooses to default in T periods if $a=0$ sufficiently long.

The crucial parameter in determining which of these two cases the economy is in is the default penalty factor Z. If Z is sufficiently low, the government will choose to never default. If Z is close to 1, it will optimally choose to default after a sufficiently long number of periods in which $a=0$.

4.1. Equilibrium with no default

Let us first consider the case where government chooses to never violate the constraint $B\leq\overline{B}(0;p,0)$. For this to be an equilibrium, two things must be true:

1. The expected discounted value of steady state utility at $B=\overline{B}(0;p,0)$ must be higher than that of defaulting after bankers have purchased $B'=\overline{B}(0;p,0)$ at price β,

$$
\begin{aligned}
&u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\bar{B}(0;p,0)) \\
&\quad+\beta(1-p)\left(\frac{u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\bar{B}(0;p,0))}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(0;p,0))}{(1-\beta)(1-\beta(1-p))}\right) \\
&\quad+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(0;p,0)) \\
&\quad\geq u((1-\theta)AZ\bar{y},\theta AZ\bar{y}+\beta\bar{B}(0;p,0)) \\
&\quad+\beta(1-p)\left(\frac{u((1-\theta)AZ\bar{y},\theta AZ\bar{y})}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)Z\bar{y},\theta Z\bar{y})}{(1-\beta)(1-\beta(1-p))}\right) \\
&\quad+\frac{\beta p}{1-\beta}u((1-\theta)Z\bar{y},\theta Z\bar{y})
\end{aligned}
.
$$

2. The expected discounted value of steady state utility at $B=\bar{B}(0;p,0)$ must be higher than that of running up the debt one more time at price βp, repaying if the private sector recovers, and defaulting otherwise,

$$
\begin{aligned}
&u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\bar{B}(0;p,0)) \\
&\quad+\beta(1-p)\left(\frac{u((1-\theta)A\bar{y},\theta A\bar{y}-(1-\beta)\bar{B}(0;p,0))}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(0;p,0))}{(1-\beta)(1-\beta(1-p))}\right) \\
&\quad+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)\bar{B}(0;p,0)) \\
&\quad\geq u((1-\theta)A\bar{y},\theta A\bar{y}+\beta pB'(\bar{B}(0;p,0))-\bar{B}(0;p,0)) \\
&\quad+\beta(1-p)\left(\frac{u((1-\theta)AZ\bar{y},\theta AZ\bar{y})}{1-\beta(1-p)}+\frac{\beta pu((1-\theta)Z\bar{y},\theta Z\bar{y})}{(1-\beta)(1-\beta(1-p))}\right) \\
&\quad+\frac{\beta p}{1-\beta}u((1-\theta)\bar{y},\theta\bar{y}-(1-\beta)B'(\bar{B}(0;p,0)))
\end{aligned}
,
$$

where $B'(B)=\min[\hat{B}'(B),\bar{B}(1;p,0)]$.

If these two conditions are satisfied, the optimal government policy is the solution to the dynamic programming problem

$$
V(B,a)=\max u((1-\theta)A^{1-a}\bar{y},\theta A^{1-a}\bar{y}+\beta B'-B)+\beta EV(B',a')
$$
$$
\text{s.t. } B\leq\bar{B}(0;p,0)
$$

We write Bellman’s equation explicitly as

$$V(B,0) = \max u((1-\theta)A\bar{y}, \theta A\bar{y} + \beta B' - B) + \beta(1-p)V(B',0) + \beta pV(B',1)$$

$$V(B,1) = \max u((1-\theta)\bar{y}, \theta\bar{y} + \beta B' - B) + \beta V(B',1).$$

The first order condition is

$$\beta u_g((1-\theta)A^{1-a}\bar{y}, \theta A^{1-a}\bar{y} + \beta B' - B) = \beta EV_B(B', a'),$$

while the envelope condition is

$$V_B(B,a) = -u_g((1-\theta)A^{1-a}\bar{y}, \theta A^{1-a}\bar{y} + \beta B' - B).$$

The envelope condition implies $V(B,a)$ is decreasing in B. A standard argument — that the operator on the space of functions defined by Bellman's equations maps concave value functions into concave value functions — implies that $V(B,a)$ is concave in B.

Now, using the first order condition, we can use standard arguments to show that the policy function for debt $B'(B,a)$ is increasing in B while the policy function for government spending $g(B,a)$ is decreasing. Using our assumption that

$$u_g((1-\theta)A\bar{y}, \theta A\bar{y} - b) > u_g((1-\theta)\bar{y}, \theta\bar{y} - b)$$

we can argue that $B'(0) > 0$ and that it is impossible for $B'(B) = B$ unless the constraint $B \leq \bar{B}(0; p, 0)$ binds, which implies that $B'(B,0) > B$. We have already argued that $B'(B,1) = B$. Figure 9 illustrates some optimal government strategies as functions of the initial debt.

Optimal debt policy gambling for redemption when $B \leq \overline{B}(0; p, 0)$ binds

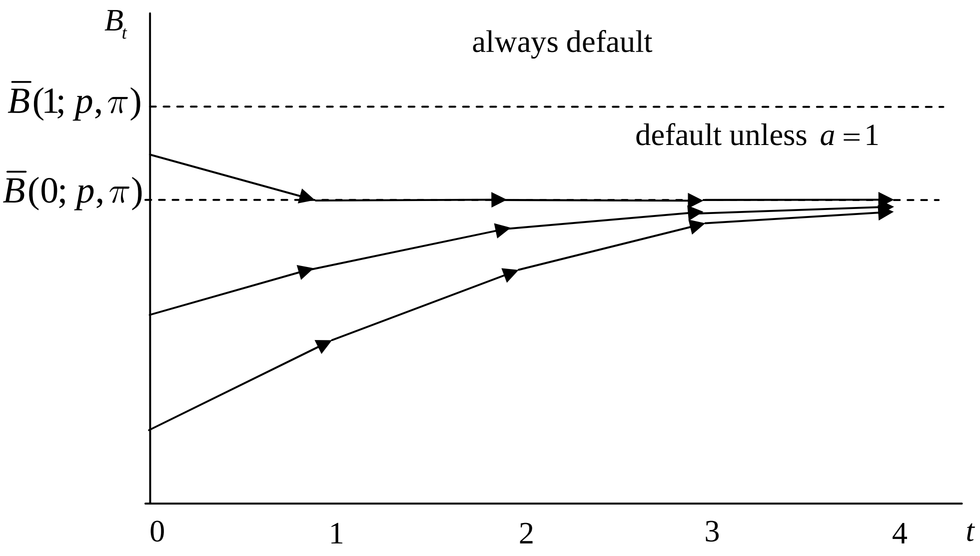

Figure 9

4.2. Equilibrium with eventual default

Let us now consider the case where the government chooses to violate the constraint $B \leq \overline{B}(0; p, 0)$ with its sale of debt in period T, defaulting in period $T+1$ unless the private sector recovers. The optimal government policy along the branch of the uncertainty tree in which $a_t = 0$ is the solution to the finite horizon dynamic programming problem

$$V_t(B_t, 0) = \max u((1-\theta)A\overline{y}, \theta A\overline{y} + \beta B_{t+1} - B_t) + \beta(1-p)V_{t+1}(B_{t+1}, 0) + \beta p \frac{u((1-\theta)\overline{y}, \theta\overline{y} + (1-\beta)B_{t+1}))}{1-\beta}$$

$$\text{s.t. } \overline{B}(1; p, 0) \geq B_{T+1} \geq \overline{B}(0; p, 0)$$

$$B_T \leq \overline{B}(0; p, 0).$$

We solve this by backwards induction with the terminal value function

$$V_T(B_T, 0) = \max u((1-\theta)A\overline{y}, \theta A\overline{y} + \beta p B_{T+1} - B_T) + \beta(1-p)\frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y}))}{1-\beta} + \beta p \frac{u((1-\theta)\overline{y}, \theta\overline{y} + (1-\beta)B_{T+1}))}{1-\beta}$$

$$\text{s.t. } B_{T+1} \leq \overline{B}(1; p, 0)$$

We then choose the value of T for which $V_0(B_0, 0)$ is maximal. As long the constraint $B_{T+1} \geq \overline{B}(0; p, 0)$ binds, we can increase the value of $V_0(B_0, 0)$ by increasing T.

Optimal debt policy gambling for redemption when $B \leq \overline{B}(0; p, 0)$ does not bind

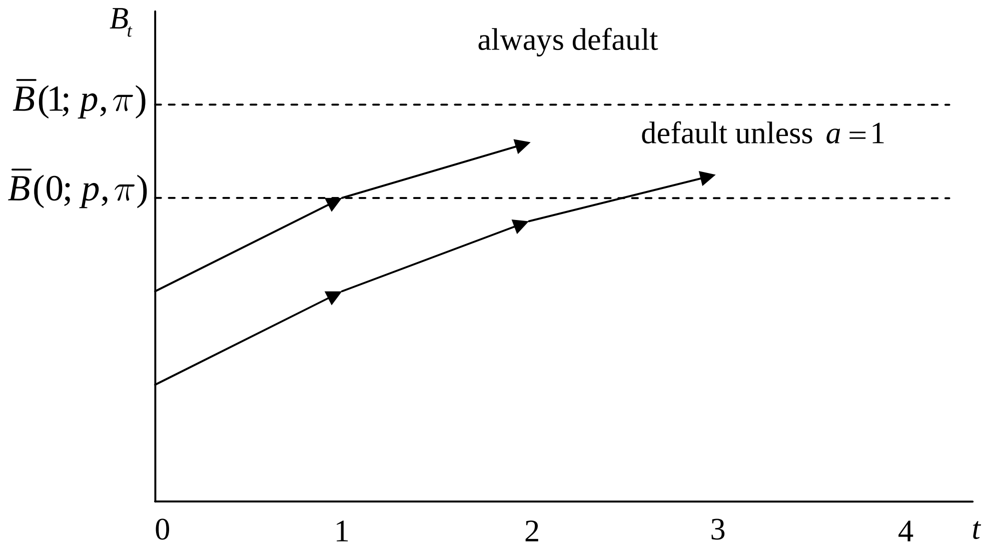

Figure 10

In figure 10, we illustrate two possibilities, which depend on B_0. In one $T = 1$, and in the other $T = 2$.

The algorithm for calculating the optimal policy on a grid of bond levels is a straightforward application of policy function iteration. Since we work backwards from the period in which the government borrows at price βp and will default in the next period unless a recovery of the private sector occurs, let us reverse our labeling of the value functions and define

$$V_0(B, 0) = \max u((1-\theta)A\overline{y}, \theta A\overline{y} + \beta p B' - B)$$
$$+ \beta(1-p)\frac{u((1-\theta)Z\overline{y}, \theta Z\overline{y}))}{1-\beta} + \beta p \frac{u((1-\theta)\overline{y}, \theta\overline{y} + (1-\beta)B'))}{1-\beta}$$
$$\text{s.t. } \overline{B}(0; p, 0) \leq B \leq \overline{B}(1; p, 0).$$

The steps of the algorithm are

1. Solve for the value function $V_0(B,0)$ and the policy function $B_0'(B)$ on a grid of bonds B on the interval $[\underline{B}, \overline{B}(0;p,0)]$. We can set the lower limit $\underline{B}$ equal to any value, including a negative value. In an application with a given initial stock of debt, we could set $\underline{B} = B_0$. We have already solved this problem analytically. The solution is $B'(B) = \min[\hat{B}'(B), \overline{B}(1;p,0)]$ unless $B'(B) < \overline{B}(0;p,0)$, in which case $B_0'(B) = \overline{B}(0;p,0)$. Consequently,

$$B_0'(B) = \max\left[\overline{B}(0;p,0), \min[\hat{B}'(B), \overline{B}(1;p,0)]\right].$$

It turns out that the values of B for which $B'(B) < \overline{B}(0;p,0)$ are those for which it is not optimal to set $T = 0$.

2. Let $t = 0$, and set $\tilde{B}_0 = \overline{B}(0;p,0)$.

3. Solve for the value function $V_{t+1}(B,0)$ and the policy function $B_{t+1}'(B)$ in Bellman's equation $V_{t+1}(B,0) = \max u((1-\theta)A\overline{y}, \theta A\overline{y} + \beta B' - B) + \beta(1-p)V_t(B',0) + \beta p \dfrac{u((1-\theta)\overline{y}, \theta\overline{y} + (1-\beta)B))}{1-\beta}$. Let $\tilde{B}_t$ be the largest value of B for which $V_{t+1}(B,0) \geq V_t(B,0)$.

4. Repeat step 3 until $\tilde{B}_t = \underline{B}$.

Let T be such that $\tilde{B}_T = \underline{B}$. We can prove that $\underline{B} < \tilde{B}_{T-1} < \tilde{B}_{T-2} < \cdots < \tilde{B}_1 < \overline{B}(0;p,0)$. Our algorithm divides the interval $[\underline{B}, \overline{B}(0;p,0)]$ into subintervals $[\underline{B}, \tilde{B}_{T-1})$, $[\tilde{B}_{T-1}, \tilde{B}_{T-2})$,..., $[\tilde{B}_1, \overline{B}(0;p,0)]$. If the initial capital stock B_0 is in the subinterval $[\tilde{B}_t, \tilde{B}_{t-1}]$, then the optimal government policy is to increase B, selling debt B, $\overline{B}(0;p,0) < B \leq \overline{B}(1;p,0)$, in period $t-1$, and defaulting in period t unless the private sector recovers. The optimal sequence of debt is B_0, $B_{t-1}'(B_0)$, $B_{t-2}'(B_{t-1}'(B_0))$ ..., $B_0(B_1(\cdots(B_{t-1}'(B_0))))$.

5. Results for the general model

For the general model in which $a_0 = 0$, $1 > p > 0$, and $1 > \pi > 0$, figure 11 illustrates some of the distinctly different possible patterns of equilibrium dynamics.

Four possible phase diagrams in general model

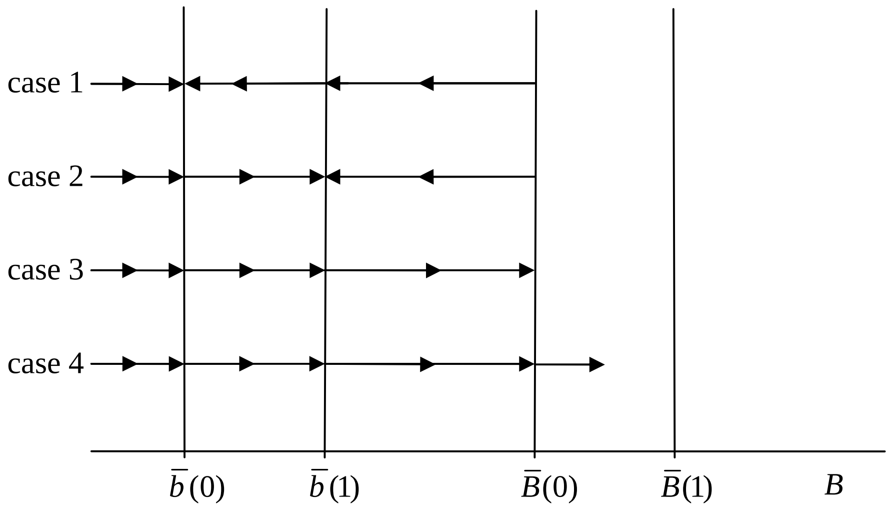

Figure 11

When debt is low enough, $B \le b(0; p, \pi)$, the government necessarily runs up its debt, gambling for redemption. As debt becomes larger, however, the probability of a crisis increases and the price of bonds falls, as in figure 6. For high enough levels of debt, the benefits of running down debt to avoid a self-fulfilling crisis may dominate the benefits of gambling for redemption. For $\pi > 0$ small enough, this never happens, and the government eventually defaults unless the private sector recovers to $a = 1$.

There are also intermediate cases (think of a combination of cases 1 and 3) in which there is some level of debt high enough so that gambling happens above it, while debt goes down for levels below it. In fact, these intermediate cases happen to play a prominent role in our quantitative exercises.

5.1. Calibration of the model and quantitative results

In order to solve for the equilibrium we need to choose a functional form for the utility function. We choose the following specification:

$$u(c,g) = \frac{c^{\rho}}{\rho} + \gamma \frac{\left(g - \bar{g}\right)^{\rho}}{\rho}$$

The parameters we choose for our benchmark scenario are the following:

Table 1: Parameter values in the Benchmark scenario

Parameter	**Value**	**Target**
$\bar{y}$	100	Normalization
A	0.90	Average output loss
Z	0.95	As in Cole and Kehoe (1996)
P	0.20	Average recovery in 5 years
β	0.96	Real interest rate of safe bonds 4%
π	0.03	Real interest rate in crisis zone 7%
γ	0.50	g is 40% GDP when $B = 10$
θ	0.4041	Government revenues as a share of output
ρ	-1.0	Standard in the literature
$\bar{g}$	28	Assumed

We have chosen as a benchmark the steady state of an economy that in normal times has government expenditures of 40% GDP and a stock of debt of 60% GDP, which given an average maturity of 6 years implies that every year the government has to access markets to refinance debt for a value of 10% of GDP. The choice of $\bar{g} = 28$ is arbitrary, and can be interpreted as the level of government expenditures that are committed and can't be changed easily. Clearly, the larger this value the more costly it is to decrease government expenditure and the larger the incentive to gamble for redemption.

Notice that we are abstracting from issues related to the maturity structure of government debt,[1] and that there is no direct mapping between the average maturity of the stock of debt and the amount of debt that has to be refinanced every period. Suppose that every period, the government sells 310 of bonds, divided between 300 1 year bonds and 10 30 year bonds. Then the government has total debt of $300+(30)10=600$. The average maturity is

$$\frac{300(30+29+...+1)/30+300}{600}=\frac{150(30)+300}{600}=8.$$

Every period the fraction of debt that becomes due is $\frac{310}{600}=0.5167$.

Suppose, in contrast, the government sells 40 15 year bonds every period. Then the government has debt of $(15)40=600$, and the average maturity is $\frac{15+14+...+1}{2}=8$, but every period the fraction of debt that becomes due is $\frac{40}{600}=0.0667$. Same average maturity implies very different needs of refinancing every period.

The probability of a recovery implies that on average a recovery happens in 5 years. Some countries could recover very fast, while long-lasting recessions as in Japan are also possible. The choice of a benchmark $\pi=0.03$ is intended to reflect the average values of the risk premia of countries in trouble in the Eurozone.

Now we turn to our quantitative exercises. Consider our benchmark economy. The thresholds that characterize the optimal policy and the optimal debt function are plotted in Figure 12.

Here the lower threshold is slightly above 10%, and default would happen for sure for levels of debt above 32%. In between there is the crisis zone, where the risk premium is three percentage points, and the optimal policy implies reducing the debt in a finite number of periods to avoid the extra cost of financing. This is a simplified version of the Cole and Kehoe (2000) environment.

What would be the impact of a recession in this situation? How the situation changes when facing a recession is pictured in Figure 13.

[1] See Arellano and Ramanarayanan (2012), Broner et al (2012) and Buera and Nicolini (2004) for discussions of the determinants of the maturity of government bonds.

The benchmark economy in normal times

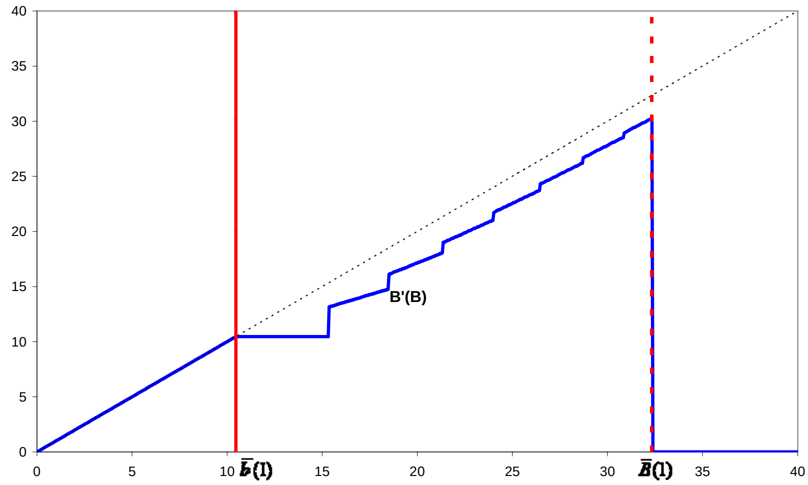

Figure 12

Notice that when the recession hits both the thresholds and the policy function change. First, both thresholds decrease. Therefore, levels of debt (say 10% of GDP in normal times) that were a steady state and perfectly sustainable in normal times now imply that the economy is vulnerable to a crisis and has to pay a risk premium. Also, high levels of debt (say 30%·of GDP in normal times) that would have been sustainable as long as there was no panic and the government had access to refinancing become unsustainable and the government finds it optimal to default immediately.

The impact of a recession

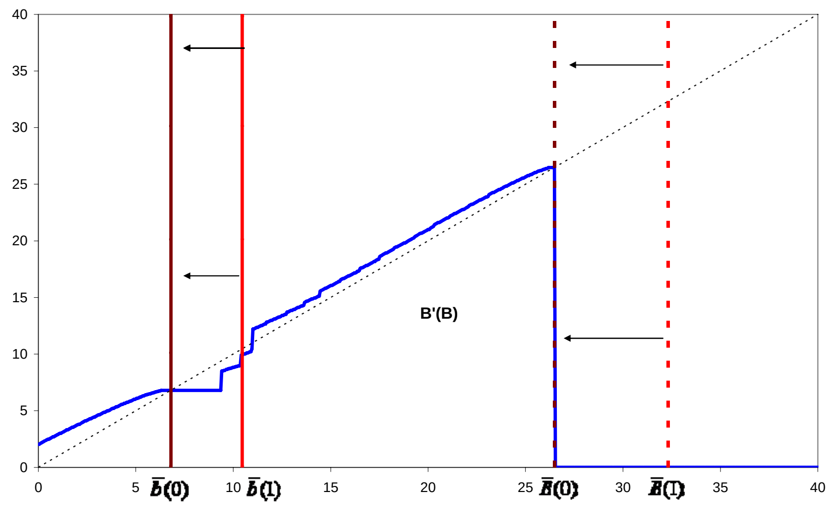

Figure 13

The policy function also changes significantly. For safe levels of debt the government finds it optimal to increase debt until the lower threshold $\bar{b}(0)$. For intermediate levels of debt the government should run down the debt to avoid paying the risk premium. However for levels of debt high enough the government would choose to gamble for redemption. If recovery does not happen, then debt would raise until reaching the upper threshold $\bar{B}(0)$. Notice that even if countries were hit by the recession in exactly the same way and the probability of a panic was the same, still small differences in initial levels of debt would imply radically different paths of debt. In addition, of course, if the severity of a recession or the probability of a recovery differed across countries then we would observe different responses.

Policy implications

Suppose we are in case 1 (the benchmark economy in normal times). Then, if the realization of the sunspot variable signals that a crisis will take place that period, the provision of a loan from a third party at an interest rate higher than

$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

can prevent a crisis but leave the government in case 1.

As in Cole and Kehoe (1996) this is our interpretation of the U.S. President Bill Clinton's 1995 loan package for Mexico. When an economy is in the crisis zone and finds it optimal to reduce its level of debt, a panic would immediately trigger a default. However, the intervention of a third party willing to provide the necessary refinancing would prevent the panic from becoming self-fulfilling.

However, suppose that we are in case 1 or case 2. Then, the provision of a loan from a third party at an interest rate lower than

$$\frac{1}{\beta(p+(1-p)(1-\pi))}-1$$

can push the government into case 3 or 4. The provision of funding at interest rates lower than those implied by market forces might imply that the optimal policy calls for gambling for redemption. In such an environment we could interpret the apparent failure of European Union rescue packages to induce Greece entering a path to sustainable reduced levels of debt.

In general, any policy of a third party that lowers the cost of default or lowers the interest rate on government debt increases the government's incentive to gamble for redemption. In order to illustrate that, we provide the following numerical experiment.

Consider again the situation of our benchmark economy. Suppose now that a third party is willing to provide subsidized lending. In particular, we assume that a third party is willing to lend at the interest rate implied by the absence of panics. That would imply access to credit at $r = 0.04$ even inside the crisis zone.

Figure 14 displays the change in the optimal debt policy associated to such an opportunity. First, it implies an increase in the upper threshold, making default less likely. However, it triggers a change in the optimal debt policy implying that for high levels of debt the optimal response is to increase debt until it reaches the upper threshold.

Notice that this change has happened even in good times. Providing the possibility of refinancing debt at rates below market might trigger a path of increasing indebtness, even in the Cole and Kehoe (2000) environment.

Optimal debt policy in case of bailout at rates below market in normal times

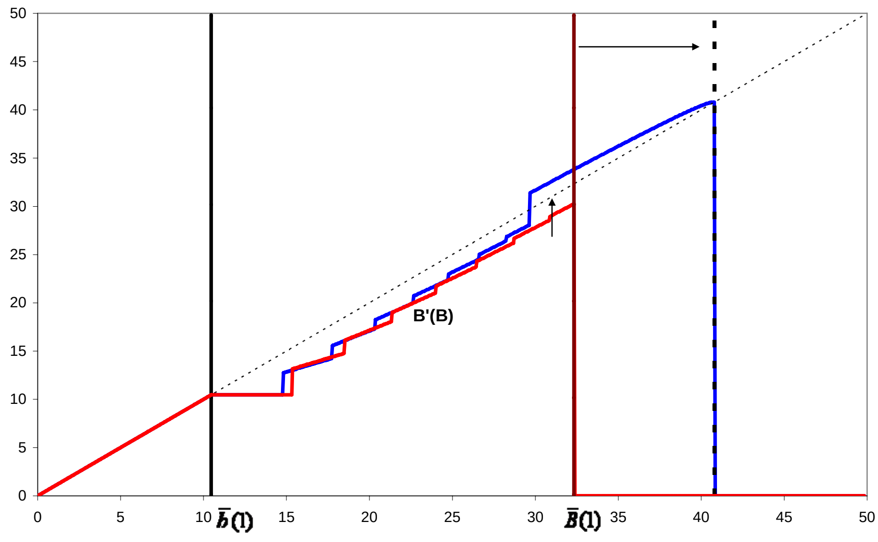

Figure 14

If subsidized lending would happen during a recession, then the incentives to gamble for redemption are much stronger, as shown in Figure 15.

Optimal debt policy in case of bailout at rates below market in recession times

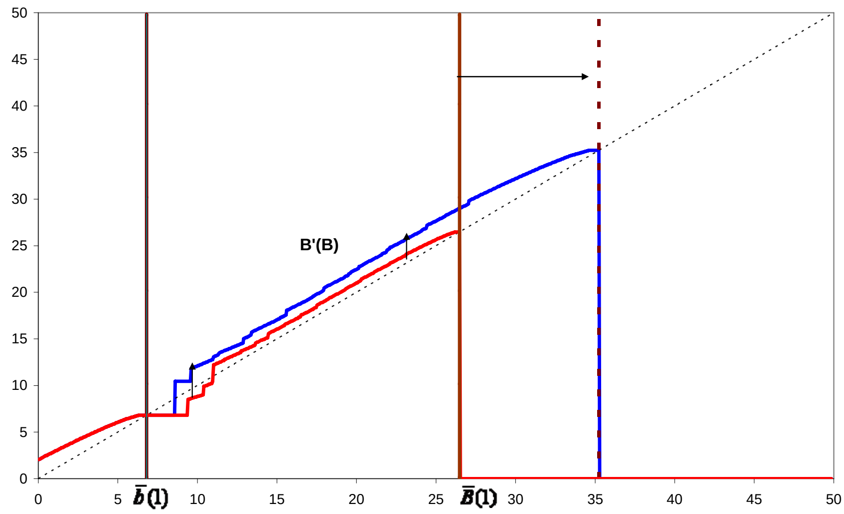

Figure 15

Notice how access to cheaper refinancing allows the government to run up its debt, gambling for redemption until a higher debt threshold.

Sensitivity analysis

In our model environment the probability of a panic, and hence the corresponding risk premium are exogenous and arbitrarily chosen. In order to understand better its impact on optimal debt policy we compare the optimal debt policy under different levels of this value. We compare our benchmark, $\pi = 0.03$, with both a smaller and a larger probability of panics (0.01 and 0.05 respectively). The results are displayed in Figure 16.

Impact of the probability of panics

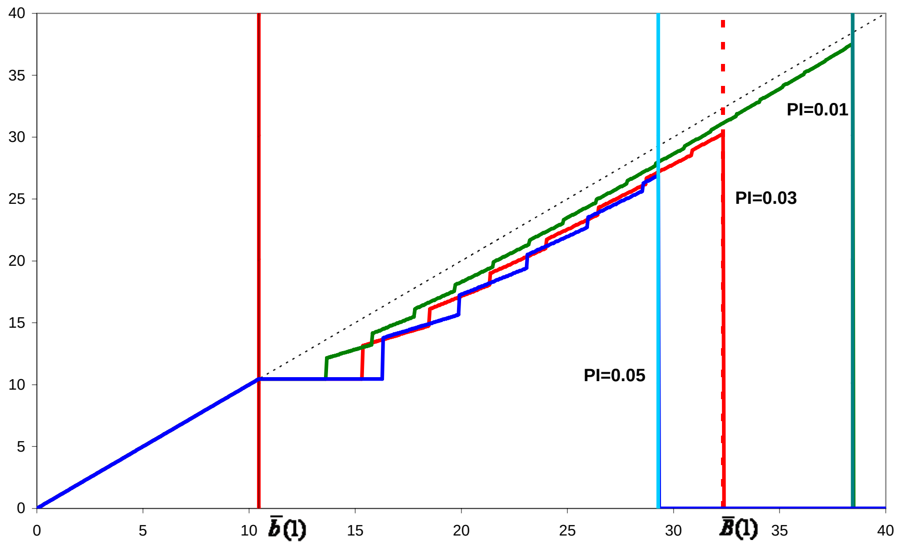

Figure 16

As expected the lower the probability of a panic the higher the upper threshold and the closer the policy function to the 45º line.

Concluding remarks

We have provided a theory of how prolonged recessions of uncertain recovery affect the optimal debt policy when an economy is vulnerable to panics. We show that gambling for redemption (increasing the level of debt in hope of a near recovery) is more than a theoretical possibility. In fact, it is reasonable to expect this kind of behavior in a simple quantitative exercise.

This is important for understanding the design of policy. As in Cole and Kehoe (2000) our results suggest that providing liquidity at market rates to an economy that is subject to a panic is a way to avoid default. However, the provision of liquidity at subsidized rates might induce the borrowing party to gamble for redemption hoping for a recovery.

In our exercise panics are exogenously driven by a sunspot variable. It would be desirable to have a better understanding of what drives panics, and whether investor’s sentiment can truly be linked to fundamentals.

In our analysis we have abstracted from Keynesian features. In our theoretical environment government expenditure only had a role as providing direct utility. If the probability of an economic recovery depended directly on government expenditure, then the incentives to gamble for redemption would even be stronger.

Another feature we have left out of our analysis is what happens in an environment where the expectations of the government and international investors are not aligned. Suppose an environment where the government is more optimistic than the markets (see Cohen and Villemot (2008)), either because its forecast of the probability of a recovery is more favorable or because it has a lower valuation of the probability of a panic. In this environment, a government more optimistic than the market would tend to have lower borrowing because it would perceive that the market is charging too high an interest rate.

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Appendix: The algorithm for computing an equilibrium in the general model

The algorithm computes the four debt thresholds, the value function, and the policy function.

1. Compute the value function $V(B,a,z_{-1},\zeta)$ of being in the default state, where $B=0$ and $z_{-1}=1$:

   The value function in normal times, where $a=1$, is

   $$V(0,1,1,\zeta)=u\left[(1-\theta)Zy,\theta Zy\right]+\beta V(0,1,1,\zeta),$$

   which is just a constant:

   $$V(0,1,1,\zeta)=\frac{1}{1-\beta}u\left[(1-\theta)Zy,\theta Zy\right].$$

   Similarly, in a recession, where $a=0$,

   $$V(0,0,1,\zeta)=u\left[(1-\theta)AZy,\theta AZy\right]+\beta pV(0,1,1,\zeta)+\beta(1-p)V(0,0,1,\zeta),$$

   which is also just a constant:

   $$V(0,0,1,\zeta)=\frac{1}{1-\beta(1-p)}u\left[(1-\theta)AZy,\theta AZy\right]+\frac{\beta p}{(1-\beta(1-p))(1-\beta)}u\left[(1-\theta)Zy,\theta Zy\right]$$

   Notice that theses values are independent of the sunspot ζ.

2. Guess initial values for the thresholds $\bar{b}(0),\bar{b}(1),\bar{B}(0),\bar{B}(1)$, where $\bar{b}(0)<\bar{b}(1)<\bar{B}(0)<\bar{B}(1)$. (We can modify the algorithm to calculate an equilibrium where $\bar{b}(0)<\bar{B}(0)<\bar{b}(1)<\bar{B}(1)$.)

3. Perform value function iteration on a finite grid of values for debt to compute the value function in normal times, where $a=1$:

   Guess an initial value for the value functions if we are in good times and the government has not defaulted in the past: $\tilde{V}(B,1,0,\zeta)$. Then:

   3.1. For low values of initial debt $B\le\bar{b}(1)$:

$\forall\zeta$ the value function is: $V\left(B,1,0,\zeta\right)=\max\left\{V_1\left(B,1,0,\zeta\right),V_2\left(B,1,0,\zeta\right)\right\}$, where $V_1(),V_2()$ are defined as:

$$V_1\left(B,1,0,\zeta\right)=\max_{B'\leq\bar{b}(1)}\left\{u\left[\left(1-\theta\right)y,\theta y+\beta B'-B\right]+\beta\tilde{V}\left(B',1,0,\zeta'\right)\right\}.$$

$$V_2\left(B,1,0,\zeta\right)=\max_{B'\in\left(\bar{b}(1),\bar{B}(1)\right]}\left\{u\left[\left(1-\theta\right)y,\theta y+\beta\left(1-\pi\right)B'-B\right]+\beta\left(1-\pi\right)\tilde{V}\left(B',1,0,\zeta'\right)+\beta\pi V\left(0,1,1,\zeta'\right)\right\}$$

3.2. For intermediate values of initial debt $B\in\left(\bar{b}(1),\bar{B}(1)\right]$:

If $\zeta>1-\pi$ then $V\left(B,1,0,\zeta\right)=V\left(0,1,1,\zeta\right)$.

Else if $\zeta\leq 1-\pi$ then $V\left(B,1,0,\zeta\right)=\max\left\{V_1\left(B,1,0,\zeta\right),V_2\left(B,1,0,\zeta\right)\right\}$, with $V_1(),V_2()$ defined as above.

3.3. For high values of initial debt $B>\bar{B}(1)$: $V\left(B,1,0,\zeta\right)=V\left(0,1,1,\zeta\right)$, $\forall\zeta$.

3.4. If $\max_{B,\zeta}\left|V\left(B,1,0,\zeta\right)-\tilde{V}\left(B,1,0,\zeta\right)\right|>\varepsilon$, then $\tilde{V}\left(B,1,0,\zeta\right)=V\left(B,1,0,\zeta\right)$ and go to 3.1.

Else, go to 4.

4. Perform value function iteration on a finite grid of values for debt to compute the value function in a recession, where $a=0$.

Guess an initial value for the value functions if we are in a recession and the government has not defaulted in the past: $\tilde{V}\left(B,0,0,\zeta\right)$. Then:

4.1. For low values of initial debt, where $B\leq\bar{b}(0)$, the value function is:

$$V\left(B,0,0,\zeta\right)=\max\left\{V_1\left(B,0,0,\zeta\right),V_2\left(B,0,0,\zeta\right),V_3\left(B,0,0,\zeta\right),V_4\left(B,0,0,\zeta\right)\right\},$$

where V_1, V_2, V_3, V_4 are defined as

$$V_1\left(B,0,0,\zeta\right)=\max_{B'\leq\bar{b}(0)}\left\{\begin{aligned}&u\left[\left(1-\theta\right)Ay,\theta Ay+\beta B'-B\right]\\&+\beta pV\left(B',1,0,\zeta'\right)+\beta\left(1-p\right)\tilde{V}\left(B',0,0,\zeta'\right)\end{aligned}\right\}$$

$$V_2\left(B,0,0,\zeta\right)=\max_{B'\in\left(\bar{b}(0),\bar{b}(1)\right]}\left\{\begin{aligned}&u\left[\left(1-\theta\right)Ay,\theta Ay+\beta\left(p+\left(1-p\right)\left(1-\pi\right)\right)B'-B\right]\\&+\beta pV\left(B',1,0,\zeta'\right)+\beta\left(1-p\right)\pi V\left(0,0,1,\zeta'\right)+\beta\left(1-p\right)\left(1-\pi\right)\tilde{V}\left(B',0,0,\zeta'\right)\end{aligned}\right\}$$

$$V_3\left(B,0,0,\zeta\right)=\max_{B'\in\left(\bar{b}(1),\bar{B}(0)\right]}\left\{\begin{array}{l}u\left[\left(1-\theta\right)Ay,\theta Ay+\beta\left(1-\pi\right)B'-B\right]\\+\beta p\pi V\left(0,1,1,\zeta'\right)+\beta p\left(1-\pi\right)\tilde{V}\left(B',1,0,\zeta'\right)\\+\beta\left(1-p\right)\pi V\left(0,0,1,\zeta'\right)+\beta\left(1-p\right)\left(1-\pi\right)\tilde{V}\left(B',0,0,\zeta'\right)\end{array}\right\}$$

$$V_4\left(B,0,0,\zeta\right)=\max_{B'\in\left(\bar{B}(0),\bar{B}(1)\right]}\left\{\begin{array}{l}u\left[\left(1-\theta\right)Ay,\theta Ay+\beta p\left(1-\pi\right)B'-B\right]+\\+\beta p\pi V\left(0,1,1,\zeta'\right)+\beta p\left(1-\pi\right)V\left(B',1,0,\zeta'\right)+\beta\left(1-p\right)V\left(0,0,1,\zeta'\right)\end{array}\right\}$$

4.1. For intermediate values of initial debt $B\in\left(\bar{b}(0),\bar{B}(0)\right]$:

If $\zeta>1-\pi$ then $V\left(B,0,0,\zeta\right)=V\left(0,0,1,\zeta\right)$.

Else if $\zeta\leq 1-\pi$ then

$V\left(B,0,0,\zeta\right)=\max\left\{V_1\left(B,0,0,\zeta\right),V_2\left(B,0,0,\zeta\right),V_3\left(B,0,0,\zeta\right),V_4\left(B,0,0,\zeta\right)\right\}$, with

$V_1(),V_2(),V_3(),V_4()$ defined as above.

4.2. For high values of initial debt $B>\bar{B}(0)$:

$V\left(B,0,0,\zeta\right)=V\left(0,0,1,\zeta\right)$, $\forall\zeta$.

4.4. If $\max_{B,\zeta}\left|V\left(B,0,0,\zeta\right)-\tilde{V}\left(B,0,0,\zeta\right)\right|>\varepsilon$, then $\tilde{V}\left(B,0,0,\zeta\right)=V\left(B,0,0,\zeta\right)$ and go to 4.1. Else, go to 5.

5. Update the threshold values.

5.1. Choose $\bar{b}_{new}(0)$ to be the highest point in the debt grid for which the following condition is satisfied:

$$\left\{u\left[\left(1-\theta\right)Ay,\theta Ay-B\right]+\beta pV\left(0,1,0,0\right)+\beta\left(1-p\right)V\left(0,0,0,0\right)\right\}\geq V\left(0,0,1,\zeta\right)$$

5.2. Choose $\bar{b}_{new}(1)$ to be the highest point in the debt grid for which the following condition is satisfied:

$\left\{u\left[\left(1-\theta\right)y,\theta y-B\right]+\beta V\left(0,1,0,0\right)\right\}\geq V\left(0,1,1,\zeta\right)$

5.3. Choose $\overline{B}_{new}(0)$ to be the highest point in the debt grid for which the following condition is satisfied:

$$V\left(B,0,0,0\right)\geq\left\{u\left[\left(1-\theta\right)ZAy,\theta ZAy+q(B')B'\left(B,0,0,0\right)\right]+\beta pV\left(0,1,1,\zeta'\right)+\beta\left(1-p\right)V\left(0,0,1,\zeta'\right)\right\}$$

where $q(B')=\beta(1-\pi)$ if $B'\left(B,0,0,0\right)\leq\overline{B}(0)$ and $q(B')=\beta p(1-\pi)$ if $B'\left(B,0,0,0\right)>\overline{B}(0)$.

5.4. Choose $\overline{B}_{new}(1)$ to be the highest point in the debt grid for which the following condition is satisfied:

$$V\left(B,1,0,0\right)\geq\left\{u\left[\left(1-\theta\right)Zy,\theta Zy+\beta(1-\pi)B'\left(B,1,0,0\right)\right]+\beta V\left(0,1,1,\zeta'\right)\right\}$$

5.5. If $\left|\overline{b}_{new}(0)-\overline{b}(0)\right|>\varepsilon$ or $\left|\overline{b}_{new}(1)-\overline{b}(1)\right|>\varepsilon$ or $\left|\overline{B}_{new}(0)-\overline{B}(0)\right|>\varepsilon$ or $\left|\overline{B}_{new}(1)-\overline{B}(1)\right|>\varepsilon$, then $\overline{b}(0)=\overline{b}_{new}(0),\overline{b}(1)=\overline{b}_{new}(1),\overline{B}(0)=\overline{B}_{new}(0),\overline{B}(1)=\overline{B}_{new}(1)$ and go to 3. Else, exit.